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Theoretical search for a π^-nn stable bound state†

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Abstract. The Heitler–London–Pauling variational method is applied to the π^-nn system in a non-relativistic approach and no true bound state is found for conventional parametrisations of the πN interaction.

The enhancement of the attractive 3,3 πN interaction in the π^-nn system raises the intriguing possibility of the existence of a bound state in that system. Since the pion has a relatively small mass, attempts to confine it within a small binding radius will tend to raise the kinetic energy of the system sharply, thus resisting binding (and possibly also obliging a relativistic formulation). On the other hand, the p-wave attractive πN interaction (with $j = t = \frac{3}{2}$) is also expected to grow quadratically with momentum—until cut off by the vertex form factor—and will thus foster binding, while the neutron–neutron interaction is sufficiently attractive to offset most of the nucleon kinetic energy.

If such a bound state exists, it will have the appearance of a particle with $T = A = 2$, $T_z = -2$, and a mass $M \lesssim 2M_n + m_{\pi^-} = 2019$ MeV. This state can then decay only through weak interactions since no negatively charged nucleon exists, and should therefore have a lifetime comparable with that of the charged pion ($\simeq 2{\cdot}6 \times 10^{-8}$ s). One could, of course, consider the charge-reflected form of this state with $T_z = +2$, namely a π^+pp bound state, but here the Coulomb energy will work to the disadvantage of binding; other members of the family with $T_z = \pm 1, 0$ will have broad widths due to hadronic decays such as $\pi^-\text{np} \rightarrow \text{nn}$ and so forth. While the present work was in progress, a related effort by Ericson and Myhrer (1978), in which the possibility of hadronically bound pion states in neutron-rich nuclei is broached, came to our attention. The present calculation may be seen as an extreme extension of that situation in which high neutron densities are generated through the mutual attractions themselves. Earlier studies of πNN bound states were carried out by Gale and Duck (1968) in an approach based on Faddeev equations, but without the inclusion of the attractive and important nucleon–nucleon interaction. (After the present work was completed, there came to our attention a closely related effort by Ueda (1978) in which the Heitler–London–Pauling method is applied to the πNN system, but for a πN force parametrised in terms of a local potential (with a range of about 1 fm) as opposed to the nonlocal form used here with a cut-off which we take as a poorly known input parameter.)

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We attempt to establish the existence of a bound state in the π^-nn system by application of the Heitler–London–Pauling variational method (Pauling 1928, Pauling and Wilson 1935) to the non-relativistic Hamiltonian

$$H = T + V_{\pi 1} + V_{\pi 2} + V_{NN} \tag{1}$$

where the kinetic energy is

$$T = (p^2/M_n) + (q^2/2m_{red}) \tag{2}$$

in coordinates such that the relative nucleon momentum is $\boldsymbol{p} = \frac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2)$, $\boldsymbol{q}$ is the pion momentum and the total momentum $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{q}$ is taken to vanish. The neutron mass is labelled M_n and the pion reduced mass in the system is $m_{red} = 2m_\pi M_n/(2M_n + m_\pi)$. The pion interactions with each of the neutrons are taken in separable p-wave forms which, in the πN centre-of-mass system, are given by

$$\langle \boldsymbol{k}'|V_{\pi i}|\boldsymbol{k}\rangle = c\boldsymbol{k}'.\boldsymbol{k}\frac{\Lambda^{4\nu}}{(k'^2+\Lambda^2)^\nu(k^2+\Lambda^2)^\nu} \qquad i = 1, 2 \tag{3}$$

where $\boldsymbol{k}$ is the centre-of-mass momentum. Unfortunately, fully convincing information on the parametrisation of the πN cut-off does not yet exist, so we have selected two options in (3), linear or quadratic ($\nu = 1$ or 2), and have considered various values of Λ. The coefficient c which gives the interaction strength is determined by using (3) as the potential in a Lippmann–Schwinger equation for the πN scattering amplitude at zero energy and comparing with recent low-energy data (Bertin *et al* 1976) for the 3,3 channel. (The $j = \frac{1}{2}$, $t = \frac{3}{2}$, p-wave channel, though repulsive, is considerably weaker than the $j = t = \frac{3}{2}$ channel. The repulsive $j = \frac{1}{2}$, $t = \frac{3}{2}$, s-wave channel we assume to act only minimally because the two neutrons are expected to be close together and the pion in a relative $l = 1$ state with respect to them for overall $J^\pi = 1^+$.) This yields

$$c = \frac{c'}{1-(\mu c'\Lambda^3/12\pi)} \quad \text{for } \nu = 1, \qquad c = \frac{c'}{1-(\mu c'\Lambda^3/96\pi)} \quad \text{for } \nu = 2 \tag{4}$$

for the linear or quadratic cut-offs, $c' = -0{\cdot}968\ \pi/\mu m_\pi^3$, with $\mu = m_\pi M_n/(M_n + m_\pi)$. Since we anticipate that the two neutrons will be in a relative S state, the Pauli principle ensures that they will form a spin singlet so that the spin term in the πN amplitude averages to zero. When the interaction of (3) is used for the three-body system the translation factor $\exp[i(\boldsymbol{q}' - \boldsymbol{q}).\boldsymbol{r}_i]$ appears for nucleon i at $\boldsymbol{r}_i$ and the momentum $\boldsymbol{k}$ is replaced by the Galilei invariant momentum $(M_n\boldsymbol{q} - m_\pi\boldsymbol{p}_i)/(M_n + m_\pi)$. Lastly, in (1) the neutron–neutron $T = 1$ interaction V_{NN} is taken in the ^{1}S state to be of the Reid (1968) hard-core form.

The Heitler–London–Pauling variational method (Pauling 1928) uses as a trial wavefunction a superposition of the two wavefunctions involving a 'bound state' for the exchanged particle at each nucleon site. For our case there is, of course, no πN bound state in the 3,3 channel, but we use the method as suggestive for the trial wavefunction

$$\psi_\lambda(\boldsymbol{q}, \boldsymbol{r}) = \chi_{NN}(r)(c_1\phi_\lambda(\boldsymbol{q}, \tfrac{1}{2}\boldsymbol{r}) + c_2\phi_\lambda(\boldsymbol{q}, -\tfrac{1}{2}\boldsymbol{r})) \tag{5}$$

where $\boldsymbol{r}$ is the internucleon separation vector and

$$\phi_\lambda(\boldsymbol{q}, \boldsymbol{\rho}) = \frac{N_\pi q_\lambda \exp(i\boldsymbol{q}.\boldsymbol{\rho})}{(q^2+\eta^2)(q^2+\Lambda^2)^\nu} \qquad \nu = 1, 2;\ \lambda = 1, 2, 3. \tag{6}$$

Here λ refers to the three components of the p-wave πN function, N_π is a normalisation constant such that

$$\int |\phi_\lambda|^2 \, \mathrm{d}\boldsymbol{q}/(2\pi)^3 = 1,$$

and η is a variational parameter. The cut-off is taken linearly or quadratically in parallel to the choice made in (3) and with the same value of Λ. The nucleon–nucleon part of the trial wavefunction is taken to be of the form

$$\chi_{\mathrm{NN}}(r) = \begin{cases} 0 & r \leqslant r_c \\ N_{\mathrm{N}} \exp(-\alpha r)(1 - \exp[-(r - r_c)/a]) & r > r_c \end{cases} \tag{7}$$

where r_c is the Reid hard-core radius (Reid 1968), a is fixed here arbitrarily at 0·63 fm by rough comparison with the deuteron wavefunction (Moravcsik 1958) since it depends primarily on the short-range features of the system (it could be treated—and indeed was here to some degree—as yet another variational parameter), N_{N} is the normalisation factor such that $\int \chi_{\mathrm{NN}}^2 \, \mathrm{d}r = 1$ and α is a variational parameter, as are $c_{1,2}$ of (5).

Varying $c_{1,2}$ yields (Pauling and Wilson 1935) energy extrema

$$E_{\pm} = (H(1) \pm H(-1))/(1 \pm \Delta) \tag{8}$$

where

$$\bar{H}(\sigma) = \int \chi_{\mathrm{NN}}(r) \, \phi_\lambda^*(\boldsymbol{q}', \tfrac{1}{2}\boldsymbol{r}) \; H \, \phi_\lambda(\boldsymbol{q}, \tfrac{1}{2}\sigma \boldsymbol{r}) \chi_{\mathrm{NN}}(r) \, \mathrm{d}\boldsymbol{r} [\mathrm{d}\boldsymbol{q}'/(2\pi)^3] [\mathrm{d}\boldsymbol{q}/(2\pi)^3] \qquad \sigma = \pm 1 \tag{9}$$

and

$$\Delta = \int \chi_{\mathrm{NN}}(r) \, \phi_\lambda^*(\boldsymbol{q}, \tfrac{1}{2}\boldsymbol{r}) \phi_\lambda(\boldsymbol{q}, -\tfrac{1}{2}\boldsymbol{r}) \, \chi_{\mathrm{NN}}(r) \, \mathrm{d}\boldsymbol{r} \, [\mathrm{d}\boldsymbol{q}/(2\pi)^3] \tag{10}$$

is the overlap integral. Our procedure was to evaluate all but the final radial integral on $|\boldsymbol{r}|$ analytically and to perform this last quadrature numerically. This was done for various values of the parameters η and α, as well as for the cut-off Λ, which in principle is a fixed input parameter, but in practice is known only rather loosely.

No bound state is found when the parameter η is varied between 0 and 2000 MeV and α is varied between 0·05 and 3·5 fm^{-1}, with a = 0·63 fm and the cut-off Λ taken from 300 MeV to 2500 MeV. Minimal expectation values for the energy are about 120 MeV and occur for $\Lambda \sim$ 1500–1700 MeV, with small η and $\alpha \sim$ 1·5 fm^{-1}, the latter corresponding to a system with $\langle r^2 \rangle^{1/2} \sim$ 1·8 fm. These may possibly indicate quasibound states at about 120 MeV or higher, much as found by Ueda (1978). (Ueda used a local πN interaction to generate an additional NN force due to non-virtual pion 'exchange', and thus was not restricted—as we are in the variational approach—to true bound states, but could search the resulting effective potential for resonances and quasibound states.) The experimental observation of a π^-nn bound state may be possible through reactions such as $\pi^- + \mathrm{d} \to \pi^+ + (\pi^-\mathrm{nn})$ or $\pi^- + \mathrm{t} \to \mathrm{p} + (\pi^-\mathrm{nn})$, in which two-body final-state kinematics are achieved if a bound state exists.

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