Part I

Falling bodies and the universality of free fall

IOP Concise Physics

Measuring Nothing, Repeatedly
Null experiments in physics
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Chapter 2

Galileo and free fall

Galileo is famous for having dropped two unequal weights from the Tower of Pisa and observing with his own eyes that they fell at virtually the same rate, striking the ground almost simultaneously: a classic example of a null experiment. The experiment thereby convincingly and dramatically refuted Aristotle's view that bodies of the same material fall with speeds proportional to their weight. Or so claimed Viviani, one of Galileo's students, 12 years after Galileo's death:

> And then, to the dismay of all the philosophers, very many conclusions of Aristotle were by [Galileo] proved false through experiments and solid demonstrations and discourses, conclusions which up until then had been held for absolutely clear and indubitable; as, among others, that the velocity of moving bodies of the same material, of unequal weight, moving through the same medium, did *not* mutually preserve the proportion of their weight as taught by Aristotle, but all moved at the same speed; demonstrating this with repeated experiments from the top of the Campanile of Pisa in the presence of the other teachers and philosophers and the whole assembly of students …. (Viviani, quoted in Cooper (1935, p 26))

Galileo, however, despite a vast amount of published and unpublished material, never mentioned such an experimental demonstration performed at the Tower of Pisa. This has understandably led to considerable skepticism about the historical accuracy of Viviani's report[1]. Whether Galileo actually dropped such 'bodies of the same material' but of 'unequal weight' from the Tower of Pisa, while undeniably charming to reflect upon, is of little consequence since Galileo did conduct

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[1] For an early skeptical view, see Cooper (1935, pp 26–33, 53–55), Cooper (1935, pp 26–33, 53–55); Drake (1978 #1166, pp 19–21, 414–16), however, argues otherwise, while Palmieri (2005a) argues more expansively that the Tower experiment is best construed as the culmination of a long line of what were essentially thought experiments.

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functionally equivalent experiments that he described in his manuscripts, correspondence, and published work. But before reviewing his experiments relating to free fall, some background is in order. To begin, Galileo construed the problem of free fall in a resisting medium (such as air or water) as essentially a problem of buoyancy, as had been dealt with by Archimedes. This meant that Galileo's attempt was to develop a *dynamic* theory on the basis of a *static* theory of equilibrium where the underlying connection is that variations from equilibrium translate into corresponding motions from the equilibrium position. Couple this with having only a Euclidean theory of proportionality to work with, and the result was a highly convoluted developmental history—which, for current purposes, we can by and large, if not totally, ignore[2].

The initial result of Galileo's Archimedean analysis was relatively straightforward, which was that bodies of the same substance, and thus of the same specific gravity or density, but of different size would fall at equal rates. That is, assuming that the specific gravity was greater than that of the medium. If it was not, the body would either float or rise. There was in addition the realization that some compensation would have to be made for resistance to motion caused by the medium rubbing against the body (see Galileo 1989, pp 90–93 [131–4])[3]. The existence of such surface resistance could be used to explain away differences in the free fall velocity of bodies of the same substance. Experimental confirmation—if only in a rough and ready sense—of all this was easy to come by even in so casual a situation as dropping bodies from a conveniently located tourist attraction.

So, for example, in the *Discourse on Two New Sciences*, Sagredo (the neutral observer) claims in response to Simplicio (the Aristotelian defender):

> But I, Simplicio, who have made the test, assure you that a cannonball that weighs 100 pounds (or two hundred or even more) does not anticipate by even one span the arrival on the ground of a musket ball of no more than half [an ounce], both coming from a height of two hundred braccia. (Galileo 1989, p 66 [106–7])

Shortly thereafter, Salviati (Galileo's surrogate) further elaborates:

> Aristotle says: 'An hundred-pound iron ball, falling from the height of 100 braccia [approximately 225 feet], hits the ground before one of just one pound ball has descended a single braccio.' I say that they arrive at the same time. You find, on making the experiment, that the larger anticipates the smaller by two inches; that is when the larger one has strikes the ground, the other is two inches behind it. And now you want to hide, behind those two inches, the ninety-nine braccia of Aristotle, and speaking only of my tiny error, remain silent about his enormous one. (Galileo 1989, p 68 [109])

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[2] For an excellent review of Galileo's reliance on Archimedes, see Palmieri (2005b, pp 346–54).

[3] In order to facilitate reference to other translations, we have indicated page references in brackets to the original text as presented in volume VIII of Favaro's Opere di Galileo Galilei.

While the lack of mention of the Leaning Tower is disappointing, what is important is that the experiments employed balls of the *same material*. Seeking to expand the experimental result to cover bodies of different substances Galileo compares rates of fall for bodies of different substances when made to fall in air and water:

> … you cannot have failed to observe some *frequent and palpable events*, or *to have noticed* two bodies of which one will be moved in water a hundred times faster than the other, while in air, the faster of these does not outrun the other by even one part in a hundred. For instance, a marble egg will fall through water a hundred times as fast as a hen's egg, but through air it will not get four inches ahead in a distance of twenty braccia. One heavy body that takes three hours to get to the bottom in ten braccia of water will pass the same [ten] in air in a pulse beat or two. (Galileo 1989, p 71 [111–2], emphasis added)

So far, none of these 'frequent and palpable events' require much in the way of careful observation or experimentation. The next stage was to consider media, such as mercury, that offered more resistance (understood in terms of Archimedean buoyancy) than water, and here Galileo observed and dramatically concluded that:

> We have seen that the difference of speed in moveables of different heaviness is found to be much greater in more resistant mediums. What now? In mercury as the medium, not only does gold go to the bottom more swiftly than lead, but gold alone sinks, and all other metals and stones are moved upward and float in mercury. Yet balls of gold, lead, copper, porphyry, and other heavy materials differ almost insensibly in their inequality of motion through air. Surely a gold ball at the end of a fall through a hundred braccia will not have outrun one of copper by four inches. *This seen, I say, I came to the opinion that if one were to remove entirely the resistance of the medium, all materials would descend with equal speed.* (Galileo 1989, p 75 [116], emphasis added)

Obviously the transition from observational data in actual media to what would happen 'if one were to remove entirely the resistance of the medium' is far too quick. In response to Simplicio's skepticism, Galileo responds that while 'we lack such a [resistance free] space,' the sequential and continuous nature of the experimental program envisioned makes the equality of free fall in the void 'highly probable.'

> We are trying to investigate what would happen to moveables very diverse in weight, in a medium quite devoid of resistance, so that the whole difference of speed existing between these moveables would have to be referred to inequality of weight alone. Hence just one space entirely void of air—and of every other body, however thin and yielding—would be suitable for showing us sensibly that which we seek. Since *we lack such a space*, let us [instead] observe what happens in the *thinnest and least resistant mediums*, comparing this with what happens in others *less thin and more resistant*. If we find in fact that moveables

> of different weight differ *less and less* in speed as they are situated in *more and more yielding* mediums; and that *finally, despite extreme difference in weight*, their diversity of speed in the most tenuous medium of all (though not void) is found to be *very small and almost unobservable*, then it seems to me that we may believe, *by a highly probable guess*, that in the void all speeds would be entirely equal. (Galileo 1989, p 76 [117], emphasis added)

All very reasonable, but taken to the limit, measurement of the increasingly smaller differences in speed becomes a significant problem for experimental determination because of the 'almost unobservable' differences involved. In short, the gap between the real and the counterfactual can only be reduced to within the limits of what's observable. In an effort to reduce this gap still further, Galileo introduces *a major conceptual innovation*:

> … that a heavy body has from nature *an intrinsic principle of moving toward the common center of heavy objects (that is, of our terrestrial globe) with a continually accelerated movement*, and always equally accelerated, so that in equal times there are added equal new momenta and degrees of speed. This must be assumed to be verified whenever all accidental and external impediments are removed. Among these, there is one that we cannot remove, and that is the impediment of the filled medium that must be opened and moved laterally by the falling moveable. *The medium*, though it be fluid, yielding, and quiet, *opposes that transverse motion now with less, and now with greater resistance*, according as it must be slowly or swiftly opened to give passage to the moveable, which, as I said, goes by nature continually accelerating, and consequently comes to encounter continually more resistance in the medium. (Galileo 1989, pp 77–8 [118–9])

Assuming this is so allows for a significant improvement in experimental procedure[4]. Namely, to *reduce the height through which bodies fall* and thereby *minimize* the effect of the resisting medium and *maximize* the *relative* effect of the 'intrinsic' nature of a body to 'continually accelerate' when unimpeded. But this approach *by itself* is purely theoretical and thus does not affect the practical limits of observational discernment. Indeed, as Galileo explicitly notes, there remains the following *dilemma*:

> [Any] experiment made with two moveables, as different as possible in weight, made to fall from a height in order to observe whether they are of equal speed, *labors under certain difficulties*. If *the height is very great*, the medium that must be opened and driven aside by the impetus of the falling body will be of greater

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[4] There is a connection for Galileo between this 'intrinsic principle' and specific gravity, i.e. 'the excesses of heaviness of the moveable over the weights of [an equal bulk of] the mediums.' See Galileo (1989, pp 79–80 [118–9, 126]) Galileo (1989, pp 79–80, 84 [118–9, 126]), especially the latter section where Galileo describes a method of for determining how to 'weigh air in the void and not in air or any other filled medium.'

> prejudice to the small momentum of a very light moveable than to the force of a very heavy one, and over a long distance the light one will remain behind. But *in a small height* it may be doubtful whether there is really no difference [in speeds], or whether there is a difference but it is unobservable. (Galileo 1989, p 87 [128], emphasis added)

Galileo's response to was to diffuse the second horn of the dilemma by slowing the process such that such small effects are magnified and thus more easily determined. Here, the inclined plane makes its appearance as an experimental apparatus that could do just that:

> In order to employ the slowest speeds possible and thus reduce the change which the resisting medium produces upon the simple effect of gravity it occurred to me *to allow the bodies to fall along a plane slightly inclined to the horizontal*. For in such a plane, just as well as in a vertical plane, one may discover how bodies of different weight behave. (Galileo 1989, p 87 [128], emphasis added)

There were, however, limitations on the range of materials that could be so used. While lead and bronze were ideal experimental candidates, cork and hens eggs were not, and, in fact, Galileo used the inclined plane primarily as a means of determining the rule for the increase in speed as bodies rolled down the plane. See, for example, his report of the use of a bronze ball and various lengths of the plane (Galileo 1989, p 87 [128])[5].

That such a 'hindrance' might affect wooden balls as well as cork and hens eggs is indicated by Stillman Drake's recreation of Galileo's experiments with inclined planes where he obtained results similar to those noted by Galileo in an early unpublished manuscript. But Drake also noticed that 'on a gently inclined grooved plane the large wooden ball rapidly overtook a smaller steel ball simultaneously released in front of it.' Drake suggests that Galileo may have extrapolated this result to the case of freely falling bodies, which would explain his otherwise puzzling claim in the 1590 manuscript *De Motu* that when released from a great height a wooden ball will initially outrun a lead ball (Drake 1973, pp 295–296). Such an extrapolation would make sense to Galileo insofar as he considered inclined planes a *more reliable and accurate method* of data acquisition than just letting things drop from high places. Ultimately though, Galileo apparently abandoned the extrapolation as well as the inclined plane result, which explains why he wanted 'to be free of any hindrance that might arise from contact' with his inclined planes:

> In order to be rid of such hindrances, Galileo came up with a highly prescient idea. So I fell to thinking how one might many times *repeat* descents from

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[5] See, for example, Hahn (2002), who noted that inclined planes were also used as a way of varying the initial velocity for determination of the (parabolic) path an object took once it left the inclined plane onto a horizontal plane and then was left to fly off and return to ground.

> *small heights*, and *accumulate many of those minimal differences of time* that might intervene between the arrival of the heavy body at the terminus and that of the light one, so that added together in this way they would make up a time not only observable, but *easily observable*. (Galileo 1989, p 87 [128], emphasis added)

And here Galileo further realized that what he took to be the *isochronal* behavior of the simple pendulum could be employed to not only to enlarge the range of possible test substances but also to more effectively 'accumulate many of those minimal differences of time' and thus allow for precise determinations of differences in the rate of fall for objects of different substances. In more detail:

> Ultimately, one took two balls, one of lead and one of cork, the former being at least a hundred times as heavy as the latter, and I attached them to equal thin strings four or five braccia long, tied high above. Removed from the vertical, these were set going at the same moment, and falling along the circumferences of the circles described by the equal strings that were the radii, they passed the vertical and returned by the same path. Repeating their goings and comings a good hundred times by themselves, they sensibly showed that the heavy one kept time with the light one so well that not in a hundred oscillations, nor in a thousand, does it get ahead in time even by a moment, but the two travel with equal pace. The operation of the medium is also perceived; offering some impediment to the motion, it diminishes the oscillations of the cork much more than those of the lead. But it does not make them more frequent, or less so; indeed, when the arcs passed by the cork were not more than five or six degrees, and those of the lead were fifty or sixty, they were passed over in the same times. (Galileo 1989, pp 87–8 [128–9])

But how exactly is the desired conclusion (that all bodies fall at the same rate in vacuum) supposed to follow? As noted by Simplicio, the conclusion is counter-intuitive because:

> If that is so, why then will the speed of the lead not be [called] greater than that of the cork, seeing that it travels sixty degrees in the time that the cork hardly passes six? (Galileo 1989 #1012, pp 87–88 [128–129])

To answer Simplicio's question (and perhaps that of the reader as well), begin by noting that each arc of swing by the cork corresponds to a fall from the height of the beginning of the arc, and that, as the cork slows because of air resistance, these starting heights become increasingly smaller. Now for each such arc that the cork traverses, there will be an equally sized arc traversed by the lead. This is because the lead loses speed more slowly and so has an abundance of available arc sizes for comparison. Each arc size traversed by the cork therefore defines an experimental comparison of the speeds of cork and lead insofar as the cork and the lead traverse the same length of arc. But, and here's the crucial part, because the pendulum is isochronal, the cork and the lead will traverse the arc *in the same time*. Since this is true for all arcs completed by the cork, this means that Galileo's isochronal pendulum produces in effect *a sequence of experimental comparisons* where the heights dropped become increasing smaller, which thus maximizes the relative effect of specific gravity as compared with medium resistance—to the point of imperceptible differences between the test bodies[6]. That's our reconstruction of the argument. Here's Galileo's functionally equivalent answer to Simplicio:

> And what would you say, Simplicio, if both took the same time in their travels when the cork, removed thirty degrees from the vertical, had to pass an arc of sixty, and the lead, drawn but two degrees from the same point, ran through an arc of four? Would not the cork then be as much the faster? Yet experience shows this to happen. But note that if the lead pendulum is drawn, say, fifty degrees from the vertical and released, it passes beyond the vertical and runs almost another fifty, describing an arc of nearly one hundred degrees. Returning of itself, it describes another slightly smaller arc; and continuing its oscillations, after a great number of these it is finally reduced to rest. Each of those vibrations is made in equal times, as well that of ninety degrees as that of fifty, or twenty, or ten, or of four. Consequently the speed of the moveable is always languishing, since in equal times it passes successively arcs ever smaller and smaller. A similar effect, indeed the same, is produced by the cork that hangs from another thread of equal length, except that this comes to rest in a smaller number of oscillations, as less suited by reason of its lightness to overcome the impediment of the air. Nevertheless, all its vibrations, large and small, are made in times equal among themselves, and also equal to the times of the vibrations of the lead. Whence it is true that if, while the lead passes over an arc of fifty degrees, the cork passes over only ten, then the cork is slower 130 than the lead; but it also happens in reverse that the cork passes along the arc of fifty while the lead passes that of ten or six; and thus, at different times, the lead will now be faster, and again the cork. But if the same moveables also pass equal arcs in the same equal times, surely one may say that their speeds are equal. (Galileo 1989, pp 87–8 [128–9])

While brilliant in its conception, there is a major problem with respect to the practical implementation of the experiment. Galileo's simple pendulums are at best only approximately isochronal, and moreover their relevant variation from isochronism was very likely evident to him. So it appears that Galileo somewhat got ahead of himself—either by an overriding reliance on the formal elegance of

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[6] Assuming isochronism, as Galileo does, these differences are *always* zero and thus effectively imperceptible. Which may be taken to show that the experiment proves too much. (Isochronism is approximately correct only for small oscillations.)

isochronism or an opportunistic selection of periods of near isochronal behavior—in drawing the conclusion of speed equality in the void[7].

In any case, even if construed as a kind of only partially actualized, idealized thought experiment, Galileo's pendulum experiment is nevertheless striking in its suggestive power. What we have in mind are these central features of the experiment. First, that pendulums are an effective way of *accumulating* minimal differences of time and thereby making more readily observable sought after experimental effects. Second, in order to *minimize timing problems* two pendulums with bobs of different substances can be made *to oscillate simultaneously*. Third, that what should be sought as an experimental result is some form of *behavioral equivalency* that *translates* into an equivalency of free fall velocity. As will be seen in the next section, Newton was able to develop along these lines a set of pendulum experiments that not only led to the equality of free fall *acceleration*, but which did so with a *well-defined measure of experimental accuracy*; and all this without having to appeal to isochronism.

References

Abattouy M 2017 The mathematics of isochronism in Galileo: from his manuscript notes to the Discorsi *Soc. Polit.* **11** 23–54

Cooper L 1935 *Aristotle, Galileo and the Tower of Pisa* (Port Washington, NY: Kennikat Press)

Drake S 1973 Galileo's experimental confirmation of horizontal inertia: unpublished manuscript *ISIS* **64** 290–305

Drake S 1978 *Galileo at Work: His Scientific Biography* (Chicago, IL: University of Chicago Press)

Galileo 1989 *Two New Sciences* (Madison, WI: University of Wisconsin Press)

Hahn R 2002 The pendulum swings again: a mathematical reassessment of Galileo's experiments with inclined planes *Arch. Hist. Exact Sci.* **56** 339–61

Naylor R 1977 Galileo's need for precision: the 'point' of the fourth day pendulum experiment *ISIS* **68** 97–103

Palmieri P 2005a 'Spuntar lo scolio piu duro': did Galileo ever think the most beautiful thought experiment in science? *Stud. Hist. Phil. Sci.* **36** 223–40

Palmieri P 2005b Galileo's construction of idealized fall in the void *Hist. Sci.* **43** 343–88

Palmieri P 2009 A phenomenology of Galileo's experiments with pendulums *Brit. Soc. Hist. Sci.* **42** 479–513

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[7] See Palmieri (2005b, pp 371–72) and Palmieri (2009). See also Naylor (1977) for an analysis of the pendulum experiment described in the Fourth Day that was used in an attempt to establish the variable effect of air resistance. Also, see Abattouy (2017) for an extensive analysis of Galileo's developing thought on isochronism.