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# Classical Theory of Free-Electron Lasers

A text for students and researchers



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A text for students and researchers

**Eric B Szarmes**

*Department of Physics and Astronomy, University of Hawai'i at Mānoa,  
Honolulu, HI, USA*

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*Dedicated in loving memory of my father, Kornel Rudolf Victor Szarmes*



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# Preface

This textbook grew out of a set of handwritten notes that I originally wrote and compiled as an instructor for Physics 245, Free-Electron Lasers (FELs), at the US Particle Accelerator School in 1996, and have presented as a component of Physics 660, Advanced Optics, at the University of Hawai'i for over ten years. The text focuses on the fully classical theory of FELs with application to FEL oscillators and develops the fundamentals of FEL theory in sufficient depth to provide both a solid understanding of FEL physics and a solid background for research in the field. The topics have evolved over the years and have been reorganized and augmented with new content since their original presentation. Revisions include a correction in the calculation of the small-signal gain, an extended analysis of saturation and new sections on the classical limit, electron beam dynamics, harmonic lasing and helical undulators. All numerical approximations were developed by the author and numerous examples are included throughout to illustrate the application of analytical results. In conformity with most of the early literature on FELs, centimeter-gram-second (CGS) units are employed for all equations and physical quantities except where explicitly noted.

The text is written at a level suitable for advanced undergraduate or graduate students. Students should have taken a course in advanced electrodynamics at the level of Zangwill or Jackson (1975), including exposure to the theory of physical optics and Gaussian beams, and be familiar with relativistic mechanics and electrodynamics. Students should also have a good working knowledge of the techniques of higher mathematics including partial differential equations, Fourier theory, orthogonal functions and complex analysis.

The pedagogic aim of this work is to provide a coherent description of classical FEL physics of practical utility for students and researchers; it is not intended to serve as a review of the literature. For this purpose the references by Brau (1990), Colson (1990) and Friedman *et al* (1988) provide comprehensive overviews of the field and contain extensive references to original research. Specific citations for the current text include Siegman (1986) for general laser and resonator theory, Jackson (1975) for electrodynamics and selected publications from the literature on FEL theory. The analysis of saturation in chapters 11 and 12 and the theory of mode locking in chapter 15 are, to the best of my knowledge, original contributions of the author. Although I cannot claim priority for the other calculations presented in the text, which date from the earliest days of FELs, all results were rederived by the author with special attention to pedagogy and I believe that a number of the derivations are unique. Most of all I hope that the pedagogic organization of the work will be especially helpful to both students and researchers.

All illustrations were created by the author using Canvas and KaleidaGraph for the Macintosh. Special care was taken in the artwork to employ actual mathematical shapes for all relevant functions and curves.

*Honolulu, HI  
September 2014*

E B Szarmes

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# Author biography

## Eric B Szarmes

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Originally from British Columbia, Canada, Eric Szarmes received his Bachelor of Applied Science in Engineering Physics from the University of British Columbia in 1985, and his PhD in Applied Physics from Stanford University in 1992, where he did his doctoral research in high resolution free-electron laser spectroscopy under Professor John Madey. He was a postdoctoral research scientist at the Duke Free-Electron Laser Laboratory from 1992 to 1998, where he made pioneering contributions to the phase-locked and chirped-pulse free-electron laser. In 1998 he joined the faculty of the University of Hawaii where he is currently an associate professor of physics. His current research interests include the theory and design of novel optical resonators for high-resolution free-electron laser spectroscopy, x-ray generation and high-field physics. His greatest passion is for teaching.