Appendix A

Measuring angles

The degree (°) is a unit of angle. A degree has 60 min of arc (′) and each minute has 60 s of arc (″). In symbols $1° = 60'$ and $1' = 60''$. The terms seconds of arc, arc-seconds, etc, are used to distinguish angle units from the corresponding units of time. It follows from the above that $1° = 60 \times 60'' = 3600''$. A 90° angle is formed between two directions that are mutually perpendicular.

It is common to express angles in decimal form rather than sexagesimal form, i.e. using minutes and seconds. To convert angles in decimal form to sexagesimal, divide the minutes by 60 and the seconds by 3600. Add the results of these divisions to the degrees in the original angle. The sum is the angle in decimal form. For example, convert the angle 30° 20′ 40″ to decimal form. We will work with four decimal places. The number of degrees in the original angle is 30°. To this value we add the number of minutes (20) divided by 60 ($20/60 = 0.3333°$) and the seconds (40) divided by 3600 ($40/3600 = 0.0111°$). Adding all together:

$$30° + 0.3333° + 0.0111° = 30.3444°.$$

$$\text{Therefore,}\ \ 30°\,20'\,40'' = 30.3444°.$$

To convert a decimal to sexagesimal, the whole part of the number is the number of degrees. Multiply the decimal part (right of the decimal point) by 60. The whole part of the result is the number of minutes (′). Next multiply the decimal part times 60. This is the number of seconds (″). For example, convert 18.2462° to sexagesimal. The number of degrees is the whole part, i.e. 18°. The decimal part is 0.2462. We multiply times 60, to obtain $0.2462 \times 60 = 14.772$. The whole part, 14, is the number of minutes. We multiply the decimal part, 0.772 times 60 to obtain $0.772 \times 60 = 46.32$ which is the number of seconds. Therefore, $18.2462° = 18°\,14'\,46.32''$.

A.1 Angles in radians

Although the measurement of angles in degrees is more common, the measurement of angles in radians (which appears as an option in most calculators) is more suitable

doi:10.1088/978-1-6270-5481-2ch10

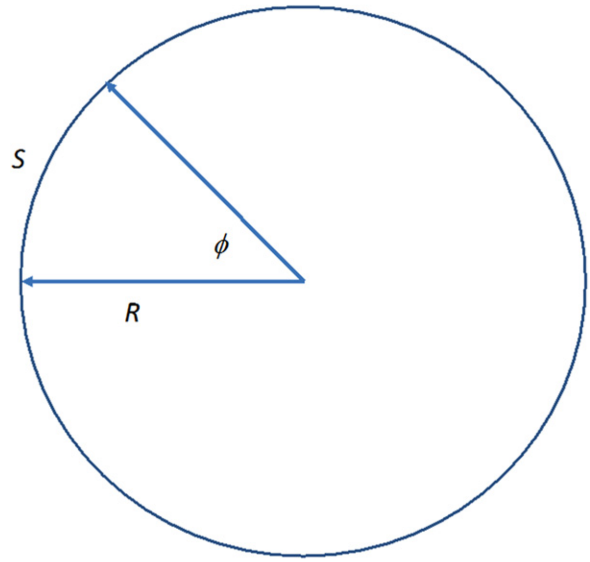

Figure A.1. The angle ϕ in radians is the length of the arc S divided by the radius of the circle R.

in some cases. The radian is a unit that derives from a more rigorous definition of the angle and is more helpful in understanding some important quantities in astronomy.

Figure A.1 shows a circle of radius R. The angle (ϕ) between the two lines can be measured as follows:

Angle (in radians) is equal to the length of arc S divided by radius of circle R. In symbols:

$$\phi = S/R \text{ (radians)}. \tag{A.1}$$

For example, if the angle between the lines is 90° then we have a quarter of a circle. To obtain S the length of the arc of a quarter circle we divide the circumference of the circle ($2\pi R$) by 4, or

$$S = (2\pi R)/4 = \pi R/2.$$[1]

Following the definition above, the angle corresponding to the quarter circle (90°) is

$$\phi = S/R = \pi R/(2R) = (\pi/2) \text{ radians}.$$

Therefore, 90° equals ($\pi/2$) radians or about 1.57 radians. The complete circle (i.e. the 360°) corresponds to 2π radians.

From the above it follows that multiplying the angle in degrees by π and dividing by 180 gives the angle in radians. For example, to convert 30° degrees to radians:

$$30° \times \pi/180 = 0.5236 \text{ radians}.$$

[1] Here π is the familiar number 3.14...

To convert radians to degrees, multiply the angle in radians times 180 and divide by π. For example, to convert 1.5 radians to degrees:

$$1.5 \times 180/\pi = 85.9^\circ.$$

A.2 Angular size

Figures A.2(*a*) and (*b*) show two different angles. The angle is decreasing from A.2(*a*) to (*b*). Assume that the observer is at the center of the circle, observing the object represented by the green line. We will call the length of the green line the *size of the object*. The center of the object is at a distance d from the observer. The size of the object is obviously smaller than the length of the arc S and the distance of the object d is smaller than the radius of the circle R. As the angle becomes smaller, the size of the object approaches the length of the arc S and the distance of the object approaches the radius R. Therefore, we can rewrite (A.1) above as:

$$\phi = (\text{size of object})/(\text{distance of object}) \tag{A.2}$$

and the ratio is the definition of the angle between the two lines in radians. This angle is also referred to as the *angular size* of the object. The method is more accurate if the angles are small, i.e. when the size of the object is small compared to the distance, as is the case for astronomical objects.

Example. The angular size of the Moon

The diameter of the Moon is 3474 km and the average distance to Earth is 382 500 km. As the distance is much larger than the actual size of the Moon, we can apply (A.2) above to find the angular size of the Moon in radians, thus:

$$\phi = (3474)/(382\,500) = 0.00908 \text{ radians}.$$

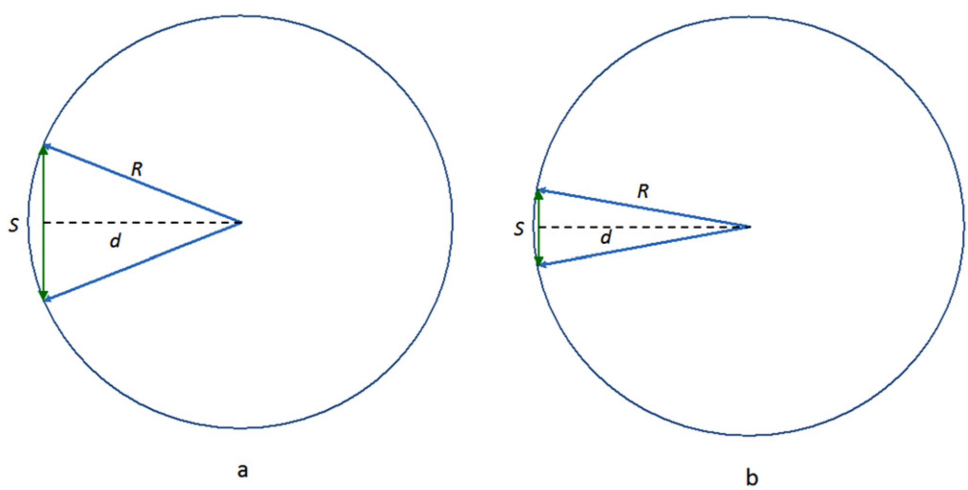

Figure A.2. The angular size of an object as seen by an observer located at the center of the circle. See the text for details.

As above, we convert to degrees by multiplying by 180 and dividing by π to find:

$$\phi = 0.52°.$$

At its closest, the distance of the Moon is 357 000 km and the angular size is

$$\phi = (3474)/(357\,000) = 0.00973 \text{ radians or } \phi = 0.56°,$$

about 7% more than average. When the full Moon phase occurs at about the near distance, the disk appears about 7% larger and is referred to as the *super Moon*.

A.3 The small angle formula

The angle calculated by (A.2) is in radians. We can convert the radians into degrees by multiplying by 180 and dividing by π, and further convert degrees to arcseconds by multiplying times 3600, and since $180 \times 3600/\pi = 206\,265$ we have:

$$\phi = 206\,265 \times (\text{size})/(\text{distance})$$

in arcseconds (″). The above equation is called the *small angle formula* and is important because it allows the calculation of size or distance. For example, the brightest star Sirius has an orbiting companion star, Sirius B. As observed from Earth, the average angular size of the orbit is about 7.5″ and the average distance from Earth is 8.6 light years (ly). We can estimate the average size of the orbit by rearranging the terms in the small angle formula. Thus

$$(\text{size}) = (\text{distance}) \times (\phi)/206\,265 = 8.6 \times 7.5/206\,265 = 0.00031 \text{ ly}.$$

There are 63 271 astronomical units (AU) in one light year, therefore the size of the orbit of the Sirius binary system is 19.6 AU ($= 0.00031 \times 63\,271$), i.e. about 20 times larger than the average size of the Earth's orbit, i.e. slightly larger than the orbit of Uranus (see table 5.2).

The small angle formula is used to define the parsec. As the Earth orbits the Sun, the nearby stars appear to shift with respect to the background of distant stars[2]. The *parallax* is half the angle of shift between two observations from Earth made 6 months apart. We can rearrange the small angle formula to read

$$(\text{distance}) = 206\,265 \times (\text{size})/(\phi).$$

In this case, the angle refers to the parallax in arcseconds and the size is the semi-major axis of the Earth's orbit, i.e. 1 AU. Therefore

$$(\text{distance}) = 206\,265 \times (1 \text{ AU})/(\phi).$$

[2] See appendix B for details.

The parsec is defined as a distance equal to 206 265 AU. In terms of parsecs, the small angle formula reads

$$(\text{distance}) = 1/(\phi)$$

where the angle is in arcseconds.

A.4 A practical way of measuring angles

The precise measurement of angles is of prime importance in astronomy and requires accurate instruments called *goniometers*. For stargazing purposes, a simple yet very practical method can be used, as described below. For most people, the ratio of the width of their hand to the length of their arm is nearly constant. There are several hand positions that can be used: fist, fingers fully splayed, one finger, etc. Each of these positions measures a different angular distance. By using these hand positions, angular distance/size measurements can be made that are adequate for most naked-eye observations. The basic procedure is as follows.

Hold your arm at full length, close one eye and sight along the arm with the other eye. Then, based on how you hold your hand, you can measure different angles.

An angle of 20° corresponds to the span of your open hand at arm's length (see figure A.3). The width of the fountain spans about 20° or, equivalently, the *angular size* of the bowl is 20°. Note that the (linear or actual) *size* of the bowl is about 1.2 m (we are referring to *length* in this case!) but the angular size is an *angle*. If the distance of the object from the observer is increased, the angular size decreases, as shown in the following two figures.

Figure A.3. An open hand at arm's length. The angular size of the fountain is about 20°.

An angle of 10° corresponds to the width your full fist at arm's length (see figure A.4).

An angle of 1° corresponds to the width of the tip of the little finger at arm's length (see figure A.5).

Figure A.4. A fist at arm's length. The angular size of the fountain is about 10°.

Figure A.5. The tip of the little finger at arm's length. The angular size of the fountain is slightly over 1°.

Appendix B

Measuring distance in astronomy

Measuring the distance of celestial objects is a fundamental and difficult task. For nearby stars (less than about 500 light years (ly) away) the method of parallax is used. For more distant objects, more indirect methods are used, which are ultimately based on the parallax method.

B.1 The parallactic shift

The essential idea is that of perspective: the apparent shift of an object's position against a distant background, when the object is observed from different positions. Figure B.1 illustrates the concept. The two photographs were taken from the same distance from the signpost. Figure B.1(*a*) was taken first. The camera was moved 4 m to the left and figure B.1(*b*) was taken. Note that the signpost shifted from left to right relative to the background. The two photographs (observations) were taken from different positions and the result is a *parallactic shift*, or *parallax* for short. There is an inverse relation between the magnitude of the parallactic shift and the distance of the object (signpost) to the observer (camera).

The distance between the two observation points is called the *baseline*. In this example the camera was moved by 4 m, therefore the baseline was 4 m.

Figure B.1. Photographs of a signpost from different positions. See the text for details.

doi:10.1088/978-1-6270-5481-2ch11

Using simple trigonometry one can find the distance of the object if the baseline and the shift are known. The parallax method can be applied to measure the distance of celestial objects. The method is the basis of the High Precision Parallax Collecting Satellite (Hipparcos) mission and also of the Panoramic Survey Telescope & Rapid Response System (Pan-STARRS) in Hawaii. Pan-STARRS can detect approaching comets and asteroids. Pan-STARRS uses four charge coupled device cameras of 1.4 billion pixels (gigapixels) each. The four cameras are mounted on a single telescope to provide four (rather than two) observation perspectives. In what follows we will discuss the application of the parallax method to measuring star distances and the distance to an asteroid.

B.2 A method for measuring the distances of stars

As the Earth moves around the Sun, our position in space is constantly changing. Figure B.2 shows the Earth in two different positions, in January and July.

As shown in figure B.2, our line of sight to a nearby star appears at a different point among the distant stars. The angle between the two lines of sight can be measured by making two observations 6 months apart, for instance in January and July. The baseline is 2 AU. Here we will work with half the angle and half the

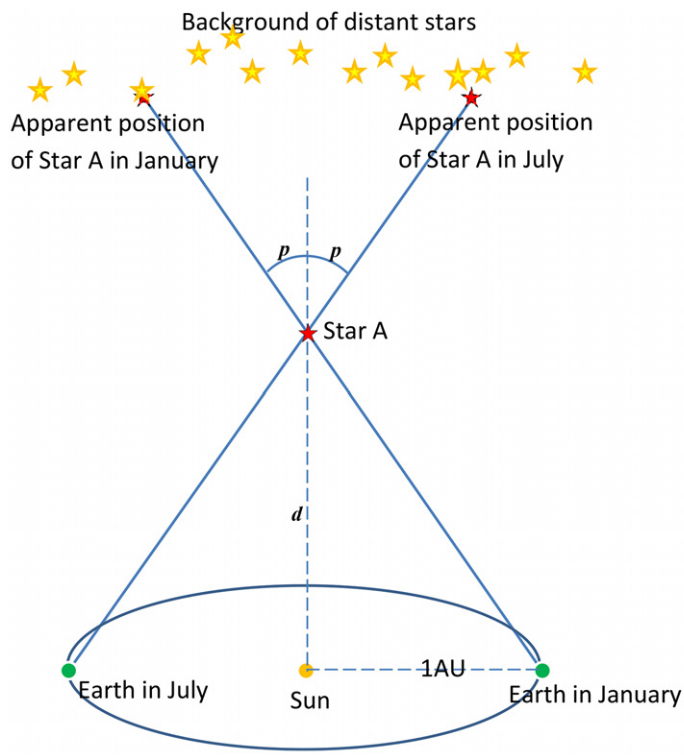

Figure B.2. The shift in the apparent position of a star, as seen from Earth in January and in July. Drawing not to scale.

baseline. The angle p is the *parallax angle*. The distance to the star d in astronomical units (AU) is related to the angle p (half the shift) and the Earth–Sun distance (which is 1 AU) by:

$$d = 206\,265/p$$

where the parallax angle p is measured in seconds of arc[1].

B.3 The parsec

For star distances it is convenient to define a unit of length equal to 206 265 AU. This unit is the parsec (pc).

$$1 \text{ pc} = 206\,265 \text{ AU}$$

and the distance formula becomes

$$d = 1/p$$

where the angle p is expressed in seconds of arc and the distance d in parsecs. As expected, the parallax angle is inversely related to the distance, i.e. the parallax is smaller for the more distant stars. The star Pollux (one of the twins of Gemini) has a parallax of about 0.1 arcseconds. Using the formula, we find that the distance of Pollux is 10 (= 1/0.1) pc. The star Acrux, the brightest in the Southern Cross, has parallax angle of about 0.01 arcseconds. Using the formula we find the distance of Acrux to be 100 (= 1/0.01) pc. Stars at distance of 1000 pc would have a parallax angle of 0.001 arcseconds, which is a very small angle (about 10 000 times smaller than a human hair held at arm's length) and beyond present measurement capabilities.

The parsec and the light year are commonly used units to express star distances. One can convert parsecs to light years using the formula:

$$1 \text{ pc} = 3.26 \text{ ly}.$$

B.4 Measuring the distance of nearby objects

The parallax method can be used for measuring the distance of nearby objects, such as asteroids, comets and the path of meteorites[2]. The principle of the method is the same as for stars: two observations are made from two different locations and the distance is determined by knowing the baseline and measuring the parallax angle. Nearby objects shift position in the sky faster than stars. Therefore, the two observations must be done simultaneously.

For nearby objects we can use two simultaneous observations from two different parts of the Earth, as shown in figure B.3.

[1] The number 206 265 is used to convert the radians to seconds of arc. See appendix A.

[2] The distances of the planets and the Moon are currently measured using radar techniques.

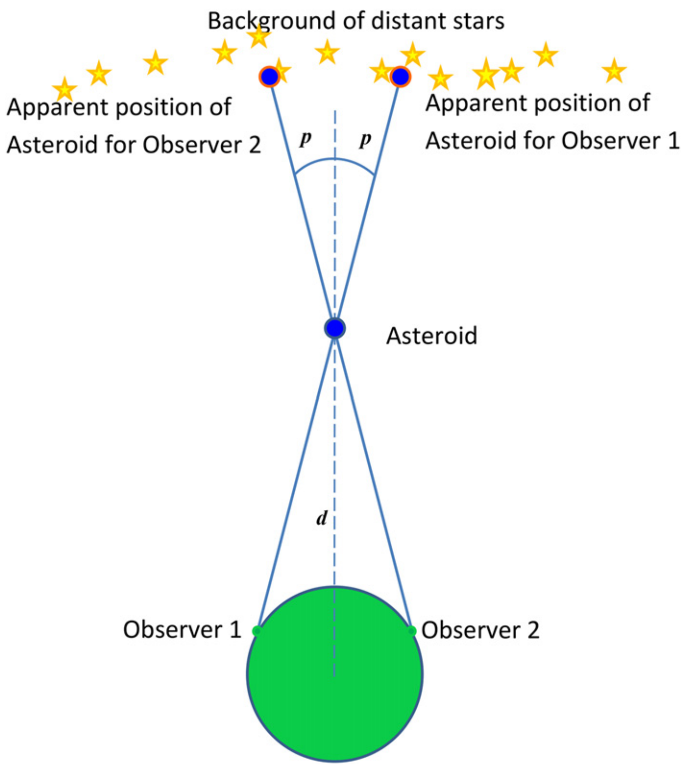

Figure B.3. The shift of a nearby object (e.g. an asteroid) as seen simultaneously by two observers on Earth. Distances not to scale.

Knowing the (linear) distance between the two observers, we can calculate the distance of the object *d*. The precision of the method increases if the distance between observers is large, e.g. 500 km. To determine the height of meteorites, photographs taken by two observers separated by a distance of approximately 20 km are adequate.

Appendix C

Time keeping

The repeating cycles of celestial motions have been used to mark time and make calendars since ancient times. The Earth's rotation provides one such measure, although seasonal variations made several refinements necessary. One complete rotation of the Earth with respect to the Sun is a solar day. It can be measured by marking two successive transits of the Sun through the meridian of the observer. An ordinary clock can be used to measure the time interval between two successive noon positions as marked by a sundial. This is known as the *apparent solar* day.

The length of the apparent solar day is not the same from day to day. This is primarily the result of the tilt of the Earth's rotation axis with respect to its orbit and the elliptical shape of the orbit. The apparent solar day in December could be as much as 50 s longer than in September[1]. The cumulative effect of a succession of shorter days could lead to a difference of several minutes between the Sun transit as marked by the sundial and noon as indicated by an ordinary clock.

The time as measured by the 24 h clock is referred to as the *mean solar time* and the mean solar day starts at midnight. All locations with the same longitudes have the same mean solar time. This is different from *time zones*, as discussed below. The numerical difference:

'apparent solar time' – 'mean solar time'

is referred to as the *equation of time* and is shown in figure C.1.

To understand the significance of the numbers in the vertical axis of figure C.1, let us assume an observer at the prime meridian at Greenwich, using standard time (as opposed to daylight savings time; explained below). For January 1, the number of minutes indicated on the vertical axis of the figure is −4, meaning that the Sun will cross the meridian at 12:04 according to the clock. For the end of October, the number of minutes is 16, meaning that the Sun will cross the meridian 16 min

[1] The apparent solar day should not be confused with the daylight hours.

doi:10.1088/978-1-6270-5481-2ch12

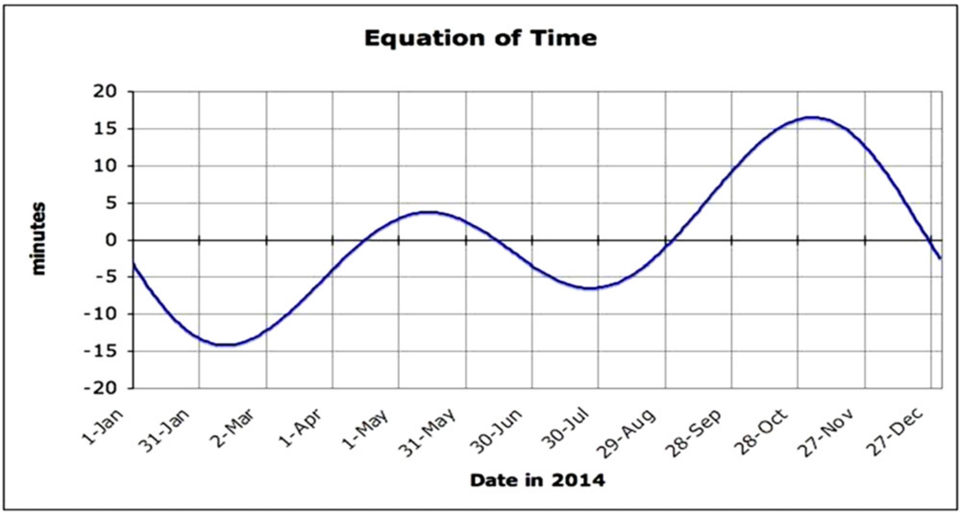

Figure C.1. The equation of time. Credit: Astronomical Applications Department, U.S. Naval Observatory, http://aa.usno.navy.mil/faq/docs/eqtime.php.

before the clock reads 12 noon, i.e. the Sun will cross the meridian when the clock reads 11:44.

In summary, to find when the Sun crosses the meridian read the number of minutes from figure C.1:

– if the sign is negative, the number of minutes is added to 12 noon;
– if the number of minutes is positive, it is subtracted from 12 noon.

The curve in figure C.1 crosses the 0 line at four points, meaning that the Sun crosses the meridian at 12:00 mean solar time four times every year.

C.1 Civil time and the 24 h day

As the mean solar time is different for each longitude, a more standardized and uniform *civil time* system became necessary. The mean solar time at the Greenwich Observatory in the UK is referred to as the Greenwich Mean Time (GMT) and was the first international standard time to be adopted. It served as the basis for standard time and time zones used across the globe. The time zones are approximately 15° wide, which is the equivalent of 1 h.[2] The civil time is the same for the entire time zone and coincides with the mean solar time of the nominal center of the zone.

Greenwich (longitude 0°) is the center of time zone 0 which extends from longitudes 7.5° E to 7.5° W. Time zone +1 is centered at longitude 15° E, extending from longitude 7.5° E to 22.5° E. If the time in Greenwich is 06:00 (6 a.m.) then the civil time in zone +1 is 07:00 (7 a.m.). In zone +2 it is 08:00 (8 a.m.), etc. Similarly time zone −1 is centered at longitude 15° W, extending from longitude 7.5° W to 22.5° W. If the time in Greenwich is 06:00 (6 a.m.) then the civil time in zone −1 is 05:00 (5 a.m.).

[2] See the discussion of RA in chapter 3.

In zone −2 it is 04:00 (4 a.m.), etc. There are notable irregularities in the boundaries to accommodate for national and provincial borders, as well as zones with fractional time changes (30 min or 15 min).

The *daylight savings* time affects civil time by moving the clocks one hour forward in the spring (daylight savings time). In the autumn the clocks are moved back one hour and this is the *standard time*.

The mean solar time varies within the zone. For each degree west of the center of the zone, the mean time is 4 min behind the civil time of the zone. For each degree east of the center of the zone, the mean solar time is 4 min ahead of the civil time of the zone. For example, if the center of the zone is at longitude 30° E and the civil time of the zone is 10:00 (10 a.m.), then the mean solar time in a location at longitude 31° E is 10:04 and the mean solar time in a location at longitude 29° E is 09:56, assuming of course that the locations are not separated by an irregular boundary. A 24 h interval is the *mean solar day*, and is measured from midnight to midnight. Note that in the common a.m. and p.m. designations, noon is 12 p.m. and midnight is 12 a.m.

At present, the *Universal Coordinated Time* (UTC) is a 0–24 h clock and is the current basis for civil time. It is an extension of the GMT in that it refers to the same meridian (Greenwich) and combines data from several timekeeping centers worldwide. The UTC makes use of timers based on the so-called atomic clocks. Atomic clocks allow time measurement with a precision of one billionth of a second. As a result, minor irregularities in the UTC are corrected by including 'leap' seconds.

The UT1 system tracks the Earth's rotation using signals from powerful radio sources in space (quasars). The difference between UTC and UT1 is less than 1 s. For most practical purposes the difference between various time systems is of little importance. The time of astronomical events (equinoxes, eclipses, Moon's phases, etc) are usually posted in UTC. The conversion methods outlined in the beginning of this chapter apply to UTC as well.

Online access to UTC, UT1 and others is available at http://time.gov/HTML5/ or http://www.usno.navy.mil/USNO/time.

C.2 Sidereal time

For astronomical measurements, a convenient way of marking time is based on successive transits of a star through the local meridian. This time interval is known as the *sidereal day*. The word sidereal derives from the Latin *sidus*, meaning *star*. By convention, the sidereal day is marked by the transits of the March equinox. The length of one sidereal day is approximately 23 h 56 min and 4 s. It should be noted that the rotation of the Earth is slowing down because of the Moon's gravitational interaction. The Earth's rotation is also affected by shifts of material in the Earth's crust and interior. In addition, as the sidereal time is marked by the transit of the March equinox, the sidereal day is also affected by the precession of the equinoxes and small irregularities in the Earth's spin axis[3].

[3] See the discussion of precession in chapter 2.

The *local apparent sidereal time* (LAST) equals the hour angle (HA)[4] of the March equinox, corrected for precession and other fluctuations of the spin axis. As the right ascension (RA) of stars is measured relative to the March equinox, all stars that have a RA equal to the LAST cross the local meridian at the same time at that location. For example, both Iota Orionis and Lambda Orionis have RA 5 h 35 m. Both these stars will cross the meridian of a location when the sidereal clock in that location reads 5 h 35 m. In other words, a star will cross the meridian of the observer when the local sidereal time at the observer's location equals the RA of the star. This is the main reason why the RA is measured in units of time rather than in units of angle.

There is a simple relation between LAST, the HA of a star and the RA of a star:

$$\mathrm{HA} = \mathrm{LAST} - \mathrm{RA}.$$

Therefore, if we know the local sidereal time and the RA of a star, we can locate the meridian of the star (the hour circle of the star, defined in chapter 3). If the star's declination is also known, then the position of the star in the sky is completely defined.

Listings of the sidereal time can be downloaded from http://aa.usno.navy.mil/data/docs/siderealtime.php.

An instant reading of the local sidereal time is available on line at http://tycho.usno.navy.mil/sidereal.html.

[4] For HA and RA see the discussion of equatorial coordinates in chapter 3.

Appendix D

Star magnitude systems and the distance modulus

The brightness of the stars we see in the sky is determined by the amount of light emitted from the star and the distance of the star. We use the *apparent* and *absolute magnitude* systems to describe how bright the star appears to an observer on Earth and how bright the star actually is. Both systems use a scale that is inverse: the larger the number the dimmer the star. Also, the numbers used in both systems indicate ratios rather than differences. The two magnitudes are related to each other. The relation between the two magnitudes is given by the *distance modulus* and involves the distance of the star, as will be described below.

D.1 The apparent magnitude system

The *apparent magnitude* system refers to the *apparent brightness*, i.e. the brightness as perceived by an observer on Earth. The system was introduced by the Greek astronomer Hipparchos over 2000 ago and in its original form used six magnitudes: 1 through 6. The brightest stars were assigned magnitude 1. The dimmest stars were assigned magnitude 6.

The symbol m is used to indicate the apparent magnitude. The magnitude scale works as follows. If the magnitudes of two stars differ by 1 unit, then the star with the smaller m is 2.51 times brighter than the star with the larger m[1].

For example:

- If star A has $m = 1$ and star B has $m = 2$ then star A appears 2.51 times brighter than star B.
- If star A has $m = 2$ and star B has $m = 3$ then star A appears 2.51 times brighter than star B.

[1] The number 2.51 is used here approximately for the fifth root of 100, which is 2.511886…

doi:10.1088/978-1-6270-5481-2ch13

If the magnitudes of two stars differ by 2 units, then the star with the smaller m is 2.51^2 times (approximately 6.3 times) brighter than the star with the larger m.

For example:

– If star A has $m = 1$ and star B has $m = 3$ then star A appears 2.51^2 (approximately 6.3) times brighter than B.
– If star A has $m = 4$ and star B has $m = 6$ then star A appears 2.51^2 times brighter than B.

More generally, if the magnitude of object A is mA and the magnitude of object B is mB then object A appears $2.51^{(m\mathrm{B}-m\mathrm{A})}$ times brighter than object B.

For example, if $m\mathrm{A} = 1$ and $m\mathrm{B} = 6$ then star A is brighter than star B by $2.51^{(6-1)} = 2.51^5 = 100$. Therefore, a decrease of magnitude by 5 units means a 100 times increase in brightness.

Before the invention of the telescope, the system was based on observations with the naked eye. The invention of the telescope allowed observation of stars dimmer than 6, therefore magnitudes larger than 6 were introduced. The use of modern instrumentation makes the magnitude scale more precise, so the units are subdivided into decimals. For example, the magnitude of the star Deneb is 1.25. The system also extends to negative magnitudes. For example, the magnitude of the star Sirius is −1.46.

Table D.1 shows the brightness ratio (brightness of larger m to brightness of smaller m) for given differences in magnitude.

An increase of 5 units in m corresponds to a decrease in brightness by a factor of 100. An increase of 10 units in m corresponds to a decrease in brightness by a factor of 10 000.

Table D.2 shows the reduction in brightness as m increases in steps of 5.

The brightest stars in the sky have magnitudes around −1 to 0. A magnitude of 25 is about the limit of what can be detected using telescopes at present. Stars of magnitude 25 appear 10^{10} (ten billion) times dimmer than the brightest stars! The Sun appears as the brightest object in the sky. For the Sun $m = -26.7$.

Magnitudes apply also to objects that do not emit their own light, for example planets and the Moon, although the values may change because of their changing distances from Earth. The full Moon on average reaches $m = -12.7$ and Venus at her brightest can reach $m = -4.8$.

Table D.1. The brightness ratio for given differences in apparent magnitude.

Difference in m	1	2	3	4	5
Brightness ratio	0.40	0.16	0.06	0.03	0.01

Table D.2. The brightness ratio for given differences in apparent magnitude.

Difference in m	5	10	15	20	25
Brightness ratio	10^{-2}	10^{-4}	10^{-6}	10^{-8}	10^{-10}

D.2 Absolute magnitude

The Sun appears brighter than any star in the sky. But in terms of actual light output (so-called *luminosity*)[2], the Sun is an average star. For example, the star Deneb emits 50 000 times more light than the Sun, meaning that if Deneb were in place of the Sun, we would receive 50 000 times more light! The absolute magnitude is a system that compares the actual light output of stars. The basic idea is as follows.

The star Arcturus (apparent magnitude $m = -0.04$) is about 37 light years (ly) away and emits 200 times more light than the Sun. If we could place the Sun at 37 ly away from Earth, it would look about 200 times dimmer than Arcturus. Using the same thought experiment, we can place all the stars at the same distance from Earth and compare their brightness at this equal setting. This method would eliminate the dependence on distance. For the standard distance we choose 10 parsecs (pc)[3].

We define the *absolute magnitude* as the magnitude the star would have if set at a distance of 10 pc from Earth. The absolute magnitude is designated by M. For example, for the Sun $M = 4.9$ which means that if the Sun was set at a distance of 10 pc (33 ly) away from Earth, it would be a star of apparent magnitude 4.9, i.e. barely visible to the naked eye!

As a rule, if the absolute magnitude is smaller (and remember -100 is smaller than -10) than the apparent magnitude, then the star is farther away than 10 pc.

The reverse is also true: if the apparent magnitude is smaller (and remember -100 is smaller than -10) than the absolute magnitude, then the star is closer than 10 pc. There is a direct relation between the numerical difference between the two magnitudes and the distance, which is discussed next.

D.3 The distance modulus

The relation between absolute magnitude M and apparent magnitude m is

$$M = m + 5 - 5\log(d)$$

where d is the distance of the star in parsecs and log is the logarithm to base 10.

For example, Arcturus has $m = -0.04$ and is about 37 ly away, i.e. in parsecs $d = (37/3.3) = 11$ pc.

Therefore, for Arcturus, $M = -0.04 + 5 - 5\log(11) = -0.04 + 5 - 5 \times 1.05 = -0.29$.

Note that $\log(10) = 1$, therefore, if a star is already at a distance of 10 pc, its absolute and apparent magnitudes are the same. In the example above, Arcturus is at a distance of 11 pc, very close to 10 pc, therefore it is not surprising that M and m for Arcturus are about the same. If the star is farther away than 10 pc, the absolute magnitude is a smaller number than the apparent magnitude. Remember that -5 is smaller than -4 and that smaller in the magnitude systems means brighter.

[2] See the discussion of luminosity in chapter 8.

[3] One parsec equals about 3.3 ly. See the discussion on the small angle formula in appendix B.

For example Canopus, the second brightest star in the sky, has apparent magnitude $m = -0.72$ and distance 310 ly ($310/3.3 = 94$ pc). The absolute magnitude of Canopus is:

$$M = m + 5 - 5\log(d) = -0.72 + 5 - 5 \times \log(94) = -0.72 + 5 - 5 \times 2 = -5.7.$$

In other words, if Canopus was 10 pc away it would outshine Venus at her best (for Venus $m = -4.8$).

On the other hand Sirius, the brightest star in the sky, has $m = -1.46$ and a distance of 8.6 ly ($8.6/3.3 = 2.6$ pc). The absolute magnitude of Sirius is:

$$M = m + 5 - 5\log(d) = -1.46 + 5 - 5 \times \log(2.6) = -1.46 + 5 - 5 \times 0.42 = 1.44.$$

Thus if both Sirius and Canopus were placed at 10 pc from Earth, Canopus would appear $2.5^{(5.7+1.44)}$, over 700 times brighter than Sirius!

The difference $m–M$ is the *distance modulus*, because it is a direct measure of the distance of the star. This can be seen by rearranging the relation between m, M and d above. From $M = m + 5 - 5\log(d)$ it follows that:

$$\log(d) = 1 + (m - M)/5.$$

Therefore, the larger the distance modulus $(m - M)$ the larger the distance is. If $m = M$ the star is at 10 pc, because $\log(10) = 1$.

We can extract the distance from the logarithm, recalling that $a = \log(x)$ means that $x = 10^a$. For example if $\log(x) = 3$, then $x = 10^3$. From $\log(d) = 1 + (m - M)/5$ we find the formula for the distance in terms of m and M:

$$d = 10^{1 + (m-M)/5}.$$

We can use this formula directly to calculate the distance of a star in parsecs, once the apparent and absolute magnitude are known. For example, Polaris has $m = 1.96$ and $M = -3.65$; therefore

$$d = 10^{1+(1.96+3.65)/5} \quad \text{or} \quad d = 10^{1+1.122} = 10^{2.122} = 132 \text{ pc}.$$

The distance formula is very useful, because we can always find m by directly observing the star, and we can 'guess' M from the HR diagram. As discussed in chapter 8, the spectral type of the star and the luminosity class allow us to approximately locate the star in the HR diagram. From the point representing the star in the HR diagram we can read on the vertical axis the luminosity of the star in units of the Sun's luminosity. From the luminosity we can calculate M. Inserting M and m in the above equation yields the distance of the star in parsecs. This method of determining the distance is known as *spectroscopic parallax*, although there is no angle involved in the procedure.

Appendix E

Bibliography

A detailed and easy to follow stargazing guide for the entire sky can be found in:

Moore P 2001 *Stargazing: Astronomy without a Telescope* 2nd edn (Cambridge: Cambridge University Press)

Constellation maps, figures, coordinates and transit times and other details useful in observational work are contained in:

Bakich M E 1995 *The Cambridge Guide to the Constellations* (Cambridge: Cambridge University Press)

More advanced discussion of coordinate systems and spherical trigonometry equations can be found in:

Roy A E and Clarke D 2003 *Astronomy: Principles and Practice* 4th edn (London: Taylor and Francis)

Birney D S, Gonzalez G and Oesper D 2006 *Observational Astronomy* (Cambridge: Cambridge University Press)

Introduction to the celestial sphere, Kepler's laws, the solar system, the physical properties of planets and stars, in a very accessible level of discussion can be found in:

Chaisson E and McMillan S 2013 *Astronomy Today* 8th edn (Reading, MA: Addison-Wesley)

Koupelis T 2014 *In Quest of the Universe* 7th edn (Jones and Bartlett)

Arny T T and Schneider S E 2013 *Explorations: Introduction to Astronomy* 7th edn (New York: McGraw Hill)

For a more advanced introduction, see:

Roy A E and Clarke D 2003 *Astronomy: Principles and Practice* 4th edn (London: Taylor and Francis)

Holliday K 1999 *Introductory Astronomy* (John Wiley & Sons)

Optics of telescopes and imaging, as well as an introduction to methods of analysis, in a level suitable for students intending to pursue a major in astronomy are discussed in:

Kitchin C R 2013 *Telescopes and Techniques* 3rd edn (Berlin: Springer)

Birney D S, Gonzalez G and Oesper D 2006 *Observational Astronomy* (Cambridge: Cambridge University Press)

doi:10.1088/978-1-6270-5481-2ch14

An interesting and enjoyable account of the development of calendars and measuring time is given in:

Steel D 1999 *Marking Time: The Epic Quest to Invent the Perfect Calendar* (John Wiley & Sons)

Two classic works of popularized astronomy:

Rudaux L and De Vaucouleurs G 1962 *Larousse Encyclopedia of Astronomy* (Prometheus Press)

Flammarion C 1964 *The Flammarion Book of Astronomy* (Simon & Schuster)

A scholarly work on the origin and meaning of star names is the re-edition of the classic work:

Allen R H 1963 *Star-names and Their Lore and Meaning* Dover Edition