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Analysis of Turbulence in Fusion Plasmas

Boudewijn Ph van Milligen and Raul Sanchez

Chapter 1

Introduction

In the first quarter of the twenty-first century humanity is facing a number of challenges of enormous scope, impact, and complexity, such as climate change and the degradation of the biosphere, overpopulation, food and water scarcity, health crises and pandemics, disinformation, corruption and bad government, inequalities between regions and population classes leading to tensions, migration, and war, etc, to mention just a few. Many of these problems are related to each other. Some of these issues could be handled better, in principle, if we had access to a plentiful, cheap, and non-contaminating source of energy. Evidently, a source of energy free from greenhouse gas emissions would be enormously beneficial for the control of climate change, and for improving living conditions in crowded cities. In addition, one could think about desalinating sea water, in order to increase the availability of drinking water, reduce disease transmitted through contaminated water, and allow cultivating areas that are currently arid and unproductive.

1.1 Energy policy options

Most people would agree that it is beneficial to reduce the use of fossil fuels, to mitigate climate change, reduce contamination, and avoid exhausting precious resources—bearing in mind, for example, that oil is not only used as an energy source, but also to manufacture plastics and many other materials. When deciding what 'energy mix'[1] to adopt in the near and far future, it is important to consider many factors. One important issue is the flexibility of energy sources to adapt the supply to the fluctuating demand of electricity, regardless of external circumstances. Flexible energy sources are vital to keep power grids operating at times of high load or when other energy sources fail. Fossil fuel power plants are flexible, but have well-known ecological issues. Biofuels could substitute fossil fuels, but they compete with food production. Hydroelectric plants are also flexible, up to a point, but their

[1] Energy mix: a collection of energy sources to cover the energy needs of society.

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availability depends on the local orology, climate, and ecological considerations. Geothermal energy can be very reliable, but is unlikely to be able to meet a significant fraction of the energy demand. Wind and solar energy are not very flexible, as they depend on highly variable external factors that cannot be controlled (the weather)—and the same can be said of many other 'alternative' energy sources (alternative with regard to fossil fuels). If methods are developed to allow massive energy storage, the need for flexible energy sources would be much reduced; but such methods are currently not sufficiently abundant, although hydrogen fuel could potentially take this role [1]. The robustness of the energy supply can also be increased by interconnecting power grids, up to a point. To summarize, the future energy mix should probably consist of a combination of the power sources mentioned and others, but it is not entirely clear what power source can take over the role of fossil fuels as the main flexible power supply. Fusion reactors would fall into this category.

When deciding the energy mix, other factors in addition to the climate, the environment, and flexibility should also be taken into account. These additional considerations are of a more societal nature and include the social acceptance of each option, opportunities in terms of job creation and the economy, the possible creation of additional social inequalities (for example when an option is only available to the rich), etc. Furthermore, one should take into account strategic considerations regarding the potential threats associated with nuclear proliferation or the political stability of regions from which raw materials are obtained. In any case, it appears certain that the decisions that will be taken in the coming years or decades regarding the energy mix will be very complex and have a major impact on society.

1.2 Fusion

Fusion is the energy source that fuels stars, that burn uninterruptedly for billions of years, consuming the most plentiful element of the Universe—hydrogen—and converting it into other elements and energy. It is natural to wonder whether we could tap into this energy source somehow. In a way, some 'alternative' energy sources already do this: they convert solar energy into electricity, either directly or indirectly. For example, photovoltaic panels convert the solar photon flux directly into electricity, wind turbines make use of the air currents associated with weather systems that are driven indirectly by the solar power influx, hydroelectric power plants use the gravitational energy stored in water that was lifted by evaporation driven by solar heat, and so on. One could argue that the same argument would apply to fossil fuels, whose energy content also originates, ultimately, from solar energy; but the difference is, of course, that fossil fuels release solar power that was stored aeons ago.

Research into fusion as a power source is different in the sense that it is an attempt to reproduce the fusion reactions occurring in the Sun directly in the laboratory [2]. Fusion refers to the reaction in which the nuclei of light elements combine to produce a new, slightly heavier element, such as the two hydrogen isotopes that combine to yield helium in figure 1.1. The binding energy of the resulting nucleus is slightly higher than the binding energy of the original two nuclei, and the difference

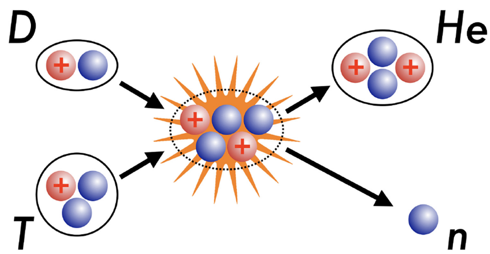

Figure 1.1. Cartoon of a D–T fusion reaction.

is released in the form of the kinetic energy of the fusion products. Nuclear binding energies are associated mainly with the strong nuclear force, one of the fundamental forces of the Universe. As a consequence, nuclear binding energies are orders of magnitude larger than the chemical binding energies associated with the much weaker electromagnetic force, so that the amount of energy released in each fusion reaction is very large in comparison to the chemical reactions occurring when burning fossil fuels.

However, bringing together two light nuclei is not easy because at a relatively large distance, they repel each other due to the fact that the nuclei have a positive electric charge, as indicated by the '+' symbols in figure 1.1. This effect is called Coulomb repulsion, a manifestation of the electromagnetic force. Although it is much weaker than the strong force, it has a much longer range, so that it dominates at relatively large distances. In the Sun, this 'Coulomb barrier' is overcome by the huge pressure in the Sun's core due to its enormous mass, which is compressed by its own gravitational force. On Earth, we cannot apply this method, so it is necessary to overcome the Coulomb barrier by other means, and this means investing energy. The first deliberate fusion reactions on Earth were produced by accelerating deuterons towards a deuteron target [3]. It turns out to be relatively easy to produce fusion reactions. For a moderate amount of money, one can acquire a fusion device called a 'fusor' in which ions are accelerated using electrodes, or even build one at home. However, this type of device is highly inefficient and has never been able to show any net energy gain.

If fusion reactors can be built successfully, there are several important potential benefits: the basic fuel is abundant[2], the energy production per reaction is enormous[3], the reaction product is non-radioactive helium, which is not a green-house gas and not toxic, and a fusion reactor does not, in principle, constitute a nuclear hazard (neither with regard to meltdowns nor proliferation). That being said, there are of course also some drawbacks, such as the activation of the device as a consequence of the neutron bombardment that is produced by the D–T reaction (figure 1.1), in which the fusion of deuterium (D) and tritium (T) produces helium

[2] The fuels for fusion are isotopes of hydrogen, such as deuterium that can be obtained from sea water, and tritium that will need to be bred in the reactor, at least in first-generation reactors based on the D–T reaction.
[3] The energy that may be obtained from 1 kg of deuterium and tritium is equivalent to the energy that can be obtained from burning about ten million liters of gasoline!

(He) and a neutron. Although other fusion reactions are possible, they are more difficult to achieve, so alternative aneutronic reactor designs that would suffer much less from this drawback remain firmly in the conceptual stage, for now. Also, current experimental reactor designs are very large, complex and expensive, although one may hope that future designs based on new technological advances, e.g. regarding magnet coils, could lead to improvements.

Several routes are being explored to achieve efficient fusion on Earth. Currently, the most promising route appears to be magnetic confinement, which is the focus of this book. Nevertheless, we should certainly not omit mentioning that alternative methods are being explored, some financed publicly and some privately. In fact, the last few years have seen a veritable explosion of fusion start-ups [4]. However, most of these efforts are still far from proving their viability and the main competitor to magnetic fusion remains inertial fusion, based on a method in which a pellet of hydrogen fuel is compressed and heated with high power lasers [5, 6].

1.3 Magnetic confinement

The basic idea of fusion is to heat hydrogen fuel to such high temperatures that the associated violent thermal motion is energetic enough to overcome the mentioned Coulomb barrier. Once this barrier is overcome, the two nuclei will fuse, releasing a large amount of energy. The required temperatures are well above the ionization energy of the hydrogen fuel atoms, so that the fuel mixture will be in a fully ionized state, called a plasma (figure 1.2).

Of course, it is not enough to overcome the Coulomb barrier only occasionally. For efficient power generation, the fusion energy production rate should exceed the

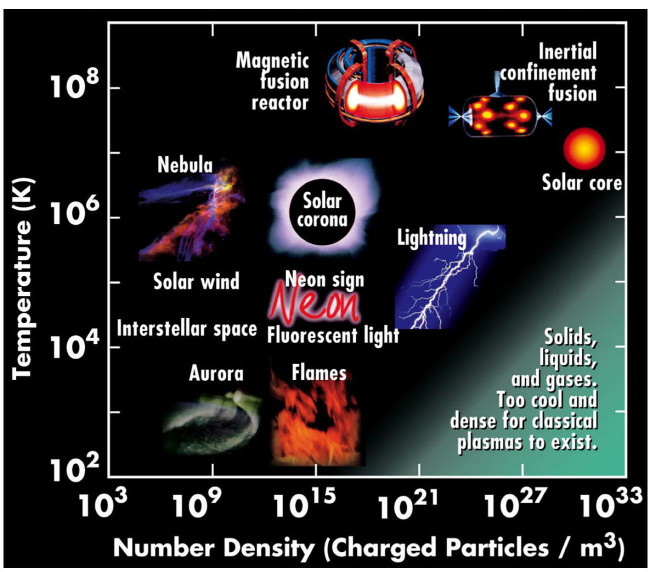

Figure 1.2. The plasma state. (Source: The Contemporary Physics Education Project.) © CPEP https://www.cpepphysics.org/

applied power input needed to maintain the hot plasma state. This requirement translates into a relatively simple criterion for the properties of the confinement system, known as the 'Lawson criterion', which is stated as follows [7]:

$$n\tau_E T \geqslant 3 \times 10^{21} \text{ keV s m}^{-3}. \tag{1.1}$$

According to this criterion, efficient energy production requires confining the fuel at a particle density n (m^{-3}) for a time τ_E(s) and heating it to a temperature T (expressed in keV)[4], such that the above triple product exceeds the specified number. In view of other constraints[5], it turns out that this condition can only be achieved realistically for extremely high temperatures of between 100 and 1000 million degrees K, significantly above the temperature in the Sun's core (around 10 million degrees K). Since our best materials only resist melting or disintegration up to a few thousand degrees, such a plasma cannot be confined in a material vessel. Fortunately, charged particles are subject to electromagnetic forces and can therefore be confined using magnetic fields.

Charged particles moving in a magnetic field $\mathbf{B}$ experience the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. If the magnetic field is strong enough, particles will gyrate around the magnetic field lines in tight helical orbits, so that they become effectively 'tied' to the field line. By bending the magnetic field lines back on themselves, a region could in principle be created in which the particles are confined; since they follow the field lines, they will not leave the mentioned region, to first approximation. This straightforward idea gives rise to the toroidal (donut-shaped) design of magnetic confinement devices. The unavoidable curvature of the field lines and non-uniformity of the field strength induce particle drifts that push the particles away from the field lines. Good confinement therefore requires a careful design of the magnetic trap to compensate for these drifts.

To balance the pressure gradient (∇p) due to the confined plasma, the toroidal[6] magnetic field needs to be supplemented by a poloidal[7] field component. Generally speaking, the poloidal magnetic field can be produced in two ways. In devices called tokamaks, it is mainly produced by means of a net toroidal current flowing in the plasma. In this case, the device design can be essentially axisymmetric[8], so that the whole system can be considered to be two-dimensional, which is an important simplification. An example of such a device is the JET tokamak (see appendix A.5). In devices called stellarators, the poloidal field is mainly produced by currents flowing in external coils, which is incompatible with axisymmetry. An example is the W7-X stellarator (see appendix A.4).

The resulting balance between the outward pressure gradient due to the confined plasma and the inward magnetic force, $\nabla p = \mathbf{j} \times \mathbf{B}$, where $\mathbf{j}$ represents the currents flowing in the plasma, is called magneto-hydrodynamic (MHD) equilibrium.

[4] 1 keV is equivalent to about 10 million °C or K.

[5] The quantum-mechanical collision cross section [2].

[6] Toroidal: the direction that goes the long way around the torus (coordinate ϕ).

[7] Poloidal: the direction that goes the short way around the torus (coordinate θ).

[8] Axisymmetry: symmetry with regard to toroidal rotations.

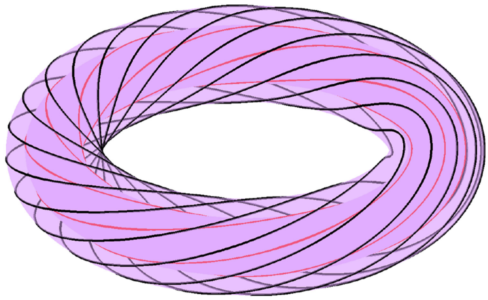

Figure 1.3. Sketch of a toroidal plasma (purple) and two field lines (red and black) on two corresponding flux surfaces. The rotational transform is the winding ratio between poloidal and toroidal turns.

A simplified sketch of the resulting magnetic field is shown in figure 1.3. The figure shows two flux surfaces with embedded field lines. A flux surface is a surface S such that the magnetic field is always directed parallel to the surface ($\mathbf{B} \cdot \mathbf{n} = 0$, where $\mathbf{n}$ is the normal to the surface S). Flux surfaces play a major role in the confinement and transport of magnetically confined plasmas, as particles can move with great freedom along the magnetic field lines (within the flux surfaces), but are strongly restrained in their motion perpendicular to the field lines (perpendicular to the flux surfaces)—in particular, in a well-designed magnetic trap, particle drifts perpendicular to the flux surfaces will be almost vanishingly small. The sketch shows that each field line within the confined region winds both the long and short way around the magnetic flux surface in which it is embedded. The rotational transform is defined as the mean winding ratio between poloidal and toroidal turns of a field line, $\iota\!\!\!-\, = \langle d\theta/d\phi \rangle$.[9] Thus, the black field line, on its corresponding flux surface, has a given rotational transform. The red field line, on a flux surface with a somewhat smaller minor radius, has a slightly different (lower, in this case) rotational transform. By plotting the rotational transform as a function of minor radius r, one obtains the rotational transform profile $\iota\!\!\!-(r)$.

The rotational transform plays an important role in magnetic confinement and stability: when $\iota\!\!\!-$ takes on a rational value (n/m), the field line on the corresponding flux surface closes in on itself after a finite number of turns, making it easier to displace it radially and giving rise to the birth of MHD modes or instabilities when the local plasma conditions are suitable. On the other hand, a flux surface with an irrational $\iota\!\!\!-$ value has a higher stability against such displacements, in principle. If the MHD modes grow sufficiently, they may give rise to topological bifurcations called 'magnetic islands'. An example of such a magnetic island is shown in figure 1.4. The radial deformation of an originally thin rational surface with rotational transform $\iota\!\!\!- = n/m = 3/4$ has produced a helical flux tube with a periodicity matching the field lines of the surface. The poloidal (vertical) cross section shown reveals clearly the $m = 4$ nature of the island through its four-fold symmetry. A similar toroidal (horizontal) cut (not shown) would reveal its $n = 3$

[9] Often, the safety factor q is used, which is the inverse of the rotational transform, $q = 1/\iota\!\!\!-$.

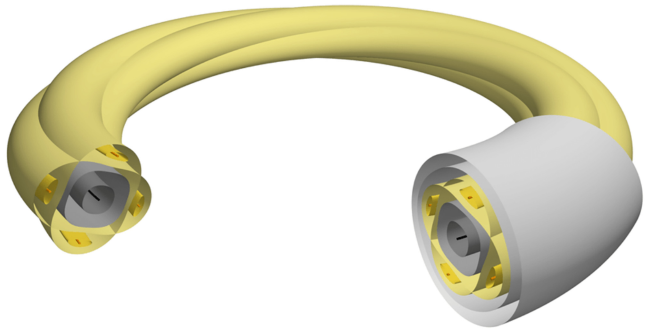

Figure 1.4. The flux surfaces of a magnetic island in a circular plasma configuration. The island (yellow) constitutes a helical flux tube matching the rotational transform of the corresponding flux surface. (Reproduced from [11] © Matthias Hölzl.)

character. The growth of the island leads to the deformation of the flux surfaces immediately outside the island, which however remain nested. Secondary nested flux surfaces appear inside the island, topologically separate from the main plasma. Field lines inside the island never connect to the region outside the island and vice versa. The two disconnected regions are separated by a flux surface called a 'separatrix'. As a consequence, conditions (density, temperature) inside the island may sometimes differ strongly from conditions outside [8]. Such islands have an impact on confinement and transport in fusion plasmas: they may constitute a barrier to radial transport, or they may constitute a kind of radial 'short circuit' through fast transport occurring inside the island, depending on the circumstances [9, 10].

Once the magnetic configuration is decided, the confined plasma needs to be heated and fuelled to achieve the required conditions of density and temperature. The simplest technique is to drive a toroidal current inside the plasma using an external transformer; dissipation due to the finite resistivity of the plasma then leads to heating. However, this technique becomes inefficient as the plasma heats up, as plasma resistivity drops with temperature. Consequently, other techniques are needed. To name a few: the injection of electromagnetic waves at specific absorption frequencies (electron and ion cyclotron resonant heating), the injection of energetic neutral particles (neutral beam injection) that transfer their kinetic energy to the plasma via collisions and contribute ions and electrons after ionization, or the injection of pellets (cryogenic hydrogen or other materials) that are ionized and contribute to the plasma density [12].

1.4 Fusion plasmas: a special class of systems

The goal of heating and fuelling the confined plasma is to create the required conditions for fusion reactions to occur. Some of the injected energy and mass is lost to the surroundings of the plasma, for example through the emission of radiation, or through particles whose orbits exit the plasma and hit the vacuum vessel, thereby transferring mass, momentum, and energy. Thus, fusion plasmas are essential 'open' rather than closed systems.

While extreme conditions are created in the plasma core (e.g. temperatures of many millions of Kelvin), the surrounding laboratory is kept at essentially ordinary ambient conditions (room temperature, sea level pressure, etc) for practical purposes. Thus, fusion plasmas are special in the sense that they are characterized by some of the steepest gradients of any systems in the Universe, in which the temperature drops from the required value for fusion to occur to room temperature in a distance of the order of a metre. Therefore, fusion plasmas are necessarily far from thermodynamic equilibrium. This is an important fact to keep in mind when studying fusion plasmas, as many theoretical approaches are only really valid for systems in thermodynamic equilibrium, and so should be used with considerable caution.

The steep gradients provide 'free energy' that allow the growth of all kinds of instabilities and, consequently, fusion plasmas exhibit an important level of turbulence[10]. This turbulence is not some kind of irrelevant 'noise' that sits on top of an otherwise stable background, but rather it may have a determining role in establishing the global state of the plasma. As noted in the seminal work by Ilya Prigogine[11], open systems far from equilibrium may *self-organize* and spontaneously generate 'dissipative' structures that require energy to sustain them. According to Prigogine, 'in non-equilibrium systems, fluctuations determine the global outcome' [14].

All this makes fusion plasmas rather interesting subjects of study, and it should not be surprising that they exhibit unusual phenomena. One could view turbulence as Nature's frantic attempt to flatten the gradients and try to return to a state of thermodynamic equilibrium. As a result, turbulence tends to work against our efforts to create fusion conditions in the laboratory—with a few exceptions, as we will see. In order to achieve a power source based on magnetically confined fusion plasmas, it is therefore essential to understand how turbulence works and how it can be controlled.

Plasmas are pervaded by many types of instabilities and waves [15]. The current understanding is that two types of instability are the main contributors to radial turbulent transport in fusion devices: drift waves and MHD modes.

Drift waves are collective oscillations driven by a pressure gradient perpendicular to the magnetic field. The motion of electrons and ions is very different: due to their very light mass, electrons move preferentially along field lines to restore quasi-neutrality, whereas the relatively heavy ions mainly experience drifts perpendicular to the field lines. Drift waves can be destabilized by effects interfering with the free ion and/or electron motion caused by, e.g. collisions, particle trapping, or particle drifts. Many different types of drift wave instabilities are distinguished, according to the driving mechanism. The main instabilities that are considered in the framework of turbulent transport are electron temperature gradient (ETG) and ion temperature

[10] Note that it is not easy to provide a concise definition of what turbulence is. For our purposes, a turbulent flow (a) is unpredictable in detail, (b) leads to increased mixing or transport, and (c) involves a broad range of scales [13].

[11] Nobel Prize in Chemistry (1977) 'for his contributions to non-equilibrium thermodynamics, particularly the theory of dissipative structures'.

gradient (ITG) driven instabilities, and the trapped electron mode (TEM) instability. These instabilities are thought to constitute the dominant mechanism for the transport of particles, energy, and momentum across magnetic field lines [16]. A vast amount of literature is devoted to studying their properties and quantifying the turbulent radial fluxes produced by these instabilities in various scenarios.

The validation of the predictions from gyrokinetic numerical models incorporating these instabilities (see the next section) against measurements is currently a very active field of study [17]. In the saturated state, the turbulent fluctuations often lead to the formation of large scale, radially extended structures (called 'streamers'), as well as radially varying poloidal and toroidal plasma flow patterns (called 'zonal flows') that interact nonlinearly, with a significant impact on the final turbulent flux—see figure 1.5. We note that the theory and simulation of these modes nearly always assumes a fixed magnetic geometry, i.e. magnetic fluctuations are not taken into account.

MHD modes, on the other hand, are instabilities that involve magnetic fluctuations in an essential way. These modes are likewise driven by the pressure gradient, particularly in regions of the magnetic configuration with so-called 'bad magnetic curvature'. Often, these instabilities are considered subdominant based on a simple argument: namely, that the relative amplitude of measured magnetic field fluctuations is typically found to be small[12], $\widetilde{B}/B \sim 10^{-4}$, except in the case of major islands. This argument is, however, debatable due to the fact that transport is much faster along than across field lines, so that particles moving along the field lines are very sensitive to small radial magnetic field fluctuations [19]. From an experimental viewpoint, we note that these instabilities are closely associated with the formation of large scale structures (filaments), whose existence is quite evident, particularly in the plasma edge region. Furthermore, as we will discuss in more detail in chapter 4, magnetic field fluctuations may have a long-range, non-local impact via the radial coupling between modes. In addition, these instabilities likely facilitate the

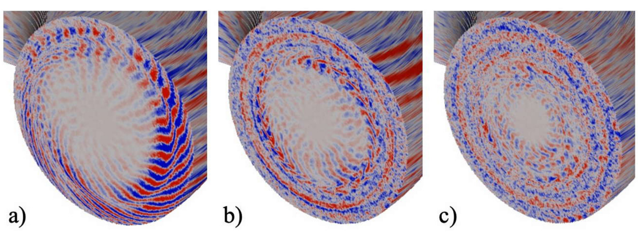

Figure 1.5. Poloidal cross sections of potential fluctuations in a nonlinear global turbulence simulation. (a) Close to the start of the simulation, radially connected structures ('streamers') dominate the system. (b) Zonal flows, spontaneously generated from the ambient turbulence in this strongly magnetised system, rupture the streamers. (c) In the final state, the radial extension of the turbulent structures is strongly reduced [18].

[12] In reversed field pinches, the relative magnetic field fluctuation amplitude is typically very much larger than the cited number.

formation of zonal flows and minor transport barriers, which may have an important indirect impact on plasma transport.

1.5 Transport in fusion plasmas

The efficiency of achieving the conditions required for fusion depends on the balance between the applied drive (fuelling and heating) and the losses. The power injected into the plasma is lost via radiation, convection, and conduction. Losses by radiation are more or less understood (e.g. Bremsstrahlung), and these losses can be controlled by keeping the plasma pure and fully ionized. Here, we will focus on convection and conduction, collectively denoted ‘transport’. Assuming thermodynamic equilibrium, a rather doubtful assumption in view of the above, transport is expected to be dominated by diffusion.

The principle underlying diffusion is well understood: it is caused by the random motion of individual particles [20], namely the well-known Brownian motion, by which the mean square displacement of any particle from a given initial position increases in proportion to the elapsed time[13]. This microscopic, random movement of particles manifests itself collectively, on the macroscopic level, as diffusion. Interestingly, whereas the random motion of the microscopic particles does not have a preferred direction, the resulting macroscopic diffusion does. To see this, consider two neighbouring regions A and B, with different particle densities n_A and n_B (see figure 1.6). Brownian motion will lead to the spreading of particles from A into B and vice versa. Assuming the rate of spreading is the same for both regions, the net particle flux across the interface between A and B will then depend only on the density difference between the regions. Generalizing to a gradually varying density $n(x)$, this implies that the particle flux must depend on the density gradient.

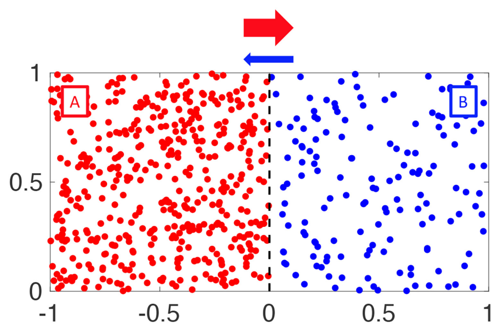

Figure 1.6. Cartoon showing two regions, A and B, with different particle densities, leading to a net diffusive flux across the interface (dashed line) from the high density to the low density region.

[13] The collective effect of particle spreading due to Brownian motion is described by the ‘heat kernel’, a normalized Gaussian distribution whose width increases in proportion to $\sqrt{t}$ (equation (3.14)).

Indeed, Fick's law states that the particle flux, Γ, is proportional to the negative density gradient[14]:

$$\Gamma = -D\nabla n. \tag{1.2}$$

The proportionality constant D is the diffusion coefficient, with a value that is determined only by the characteristics of the microscopic motion. This equation can be supplemented by the conservation law of the particle density n:

$$\frac{\partial n}{\partial t} = -\nabla \cdot \Gamma. \tag{1.3}$$

By combining these equations, one obtains the transport equation for the particle density:

$$\frac{\partial n}{\partial t} = \nabla \cdot (D\nabla n). \tag{1.4}$$

To actually model transport in a given system one would need to add a particle source and specify boundary conditions.

This transport equation is perhaps too simple for many practical applications, but it serves us here to illustrate that the above derivation skips over several important hidden assumptions. While the conservation equation is exact, Fick's law is phenomenological and only valid in limited cases (a homogeneous medium—with fixed D—, no external forces acting on the particles). The fact that the equations are written down in the form of differential equations implies that the random particle motion underlying the diffusion is assumed to consist of infinitesimal steps, i.e. the average particle step size Δx (between 'collisions') is much smaller than any relevant length scale, such as the system size or the gradient length $(\nabla n/n)^{-1}$, so the theory is 'local'. This locality is also reflected in the fact that the flux Γ at location x is assumed to depend only on D and ∇n at that same location x. By imposing locality, one is implicitly denying the relevance of radially extended structures or events for transport. It is also implicitly assumed that the process is 'Markovian', i.e. the next step of a particle is exclusively determined by its current situation (in a probabilistic sense). However, in stochastic magnetic field regions, persistent turbulent eddies, or transport barriers, this assumption may be violated (due to trapping effects), cases in which the history of the particle trajectory becomes important (see sections 3.4 and 4.6). Finally, the whole construct is based on 'standard' Brownian motion, but there are in fact other options consistent with the central limit theorem ('fractional' Brownian motion), which may arise naturally in strongly driven non-equilibrium systems, as we will discuss at some length in chapter 3. The main message of this paragraph is that the diffusive model is not set in stone, and addressing the complexities of transport in fusion plasmas may require one to keep an open mind. A secondary message is that one should never forget that 'transport' is the collective manifestation of the random motion

[14] If D depends on the medium (through the local temperature, say), the Fokker–Planck equation may provide a better approach than Fick's law [21], and one would write $\Gamma = -\nabla(Dn)$.

of individual particles; hence the importance of the underlying random processes for transport.

With all these caveats, the diffusive approach can be applied to fusion plasmas. Of course, the situation is somewhat different than the simple picture of Brownian motion sketched above: in hot, strongly magnetized plasmas, the charged particles describe tight helical orbits around the field lines. Nevertheless, the particles experience random 'Coulomb' collisions that cause their gyrocentres to shift from one field line to the next, and the cumulative effect of many such random displacements then gives rise to diffusion. Assuming a static and *uniform* background magnetic field that remains unaffected by the motion of the charged particles, transport equations can be written down for the particle density, particle energy (equivalent to temperature), etc. This approach leads to the 'classical' transport theory [22].

This straightforward theory was rapidly found to be unsatisfactory, underestimating the actual transport measured in magnetically confined plasmas by several orders of magnitude [15]. The geometry of the confining intense magnetic field significantly affects the orbits of the charged particles, giving rise, among others, to the existence of particles trapped in local magnetic well regions. The complex motion of the charged particles in the presence of steep gradients also induces electric fields that lead to particle drifts. All this has a strong impact on the resulting effective transport that cannot be ignored. Subsequently, a sophisticated theory was developed that takes the full impact of the (fixed) magnetic geometry into account, leading to corrected and rather complex expressions for transport. It is known as the 'neoclassical' transport theory [23].

The neoclassical theory is powerful and capable of clarifying many aspects of transport in fusion plasmas. Nevertheless, it still predicts radial fluxes that are often very much lower than what is actually found in experiments. To understand this, one should keep in mind that this is a theory based on several strong assumptions: it assumes a state of thermodynamic equilibrium, it is expressed in terms of differential equations (implicitly assuming small particle excursions, making it a 'local' theory), and, more importantly, neoclassical transport does not consider turbulence, since it ignores the nonlinear interaction between the confining field—assumed to be static—and the motion of the confined particles. Nevertheless, the transport resulting from the latter interaction often accounts for a large fraction of the total transport, called the 'anomalous' or 'turbulent' transport.

So, how can one properly account for turbulence? From the above argument, one clearly should include the evolution of the electric and magnetic fields self-consistently with the particle motion. Since the kinetic equations for the motion of the constituent particles of a plasma are known, as well as the Maxwell equations describing the evolution of the fields, the full plasma dynamics can in principle be simulated using the computer. In practice, this is a problem that not even the largest supercomputer available today can address. It is necessary to simplify the equations, using a minimal set of approximations, to make the calculations tractable. The standard procedure is to average the equations over the gyromotion of the charged particles, thus eliminating the fastest temporal scale of the problem. The resulting 'gyrokinetic' simulations provide the best numerical approximation to the turbulence problem to date [24]. However, these Gyrokinetic simulations are very expensive in terms of computer memory and time, so they are rarely used directly to model the gradually evolving situation of an experimental plasma. Instead, one typically uses Gyrokinetic simulations implemented in simplified geometries, with a magnetic field that is once more taken to be static, to obtain what is described as an 'effective diffusivity' for turbulence, calculated from the ratio between a prescribed gradient and the resulting turbulent flux. This effective diffusivity is then added to the Neoclassical predictions in the form of an additional diffusive term.

There are several problems with this approach, the main one being that the feedback between the turbulence and the background gradients is not taken into account, based on the assumption that there is a clear separation of timescales between the fast evolution of turbulence and the slow evolution of gradients (profiles) due to overall transport. We will see that this separation of scales may not always hold. A second issue is that the modification of the magnetic geometry due to turbulent fluctuations is ignored. On a deeper level, one may also question whether transport due to turbulence can really be described well by (local) diffusive equations, in view of the fact that experimental analyses show that turbulence exhibits many non-local effects: to name a few, intermittent events ('bursts' of the radial fluxes), long-range correlations arising between remote diagnostic systems, and non-Gaussian fluctuation distributions [25, 26]. Paradigmatic examples of long-range correlations are large scale structures emerging spontaneously from the ambient turbulence: e.g. zonal flows, responsible for the important L to H confinement transition, and 'avalanches' or 'streamers'. These important phenomena clearly reflect the complexity of fusion plasmas, as will be discussed in detail in subsequent chapters.

There are further problems, which might collectively be referred to as the 'boundary conditions'. Namely, the above approaches to transport in fusion plasmas are mainly applicable to the interior of the plasma. This plasma region is contained within a 'separatrix' that separates the interior region, where field lines close in upon themselves, from the scrape-off layer, a region where field lines connect to the surrounding material structure, such as the vacuum vessel. Transport in the latter region can be described by specific models that consider the material geometry, the physics of partially ionized gas, and the interaction of particles with the wall. The outcome of such models must then be interfaced with transport models for the interior region. This book will not dwell on the latter issues, but rather focus on the understanding of turbulence, mostly in the interior region.

1.6 How important is turbulence?

Before delving into a large effort to understand turbulence and its effect on the plasma, one would like to have some idea of how important it really is [27, 28]. To do so, one would like to compare the transport (of particles, heat, etc) between the experiment (where turbulence is present) and a hypothetical situation that is identical with one exception, namely the absence of turbulence. The difference is sometimes referred to as 'anomalous transport'[15], which is a bit of a misnomer, as if to deny the origin of the transport enhancement (namely, turbulence).

The fact that it is hard to model even non-turbulent transport, due to the mentioned 'boundary conditions' and other factors, makes this into a complicated task. Admitting the difficulty of comparing the absolute values of global transport coefficients between experiment and model, one is led to explore other options.

One important way to test transport hypotheses is to perform perturbative experiments [29]. In such experiments, instead of focusing on the absolute value of transport coefficients, one introduces a perturbation (such as a change in the heating source) and observes the reaction of the local plasma state. From this response, one can estimate the relevant transport coefficients [30] (essentially given by the ratio between a given flux and the corresponding driving gradient) which can be compared to local estimates from neoclassical transport, obtained from calculations based on the measurement of local profiles. The advantage of this approach is that the effects can be studied locally, so that the boundary conditions (hopefully) play only a minor role.

Another purely experimental method is measuring the net fluxes caused by temporally fluctuating fields explicitly, and comparing the result to the total flux from power balance estimates[16] [31]. In these studies, one typically finds, in the edge plasma where such measurements are feasible, that the fluctuating (turbulent) flux accounts for a large fraction of the total flux.

Under specific circumstances, turbulence can be largely suppressed at specific radial locations and times. These circumstances are known as 'H-mode' barriers [32] and internal transport barriers [33, 34]. Here, turbulence suppression is a consequence of the formation of local sheared radial electric fields, generated by the plasma itself [35]. The resulting plasma state, however, differs from the plasma state prior to the formation of the transport barriers not only in that turbulence is suppressed, but many profiles (plasma potential, temperature, and density) are also changed significantly, so that a clean comparison between states with/without turbulence is difficult. Something similar happens with plasmas in which a radial electric field is imposed by external means: turbulence can be suppressed, but profiles change [36]. On the other hand, it definitely seems to be the case that turbulence suppression is correlated with improved confinement, both at the edge and well inside the plasma, so that it seems reasonable to attribute low confinement to turbulent transport.

The level of turbulent transport can nowadays be predicted with a high degree of confidence using gyrokinetic simulations [24], even though not all situations can be handled (e.g. internal transport barriers). In any case, these calculations confirm that turbulence plays a major role in the understanding of transport in fusion devices.

[15] Balescu makes a distinction between 'anomalous' transport, a transport component due to turbulence that nevertheless behaves diffusively, and 'strange' transport, a component of transport that does not behave according to the standard diffusive equations [25].

[16] The total outward (heat or particle flux) at a given radius can be estimated from the net total power or number of particles deposited inside that radius, in steady state.

Summarizing, as far as we know, in the edge plasma, where gradients tend to be steep and fluctuations tend to be large, turbulent transport dominates overall transport. In the core region, where gradients tend to be small and fluctuations are relatively small, turbulent transport appears to be generally subdominant. For the intermediate region, no general statements can be made, but turbulence is expected to play a major role. In any case, the edge region has a very important weight in the overall confinement, which may justify the conclusion that 'turbulence matters for fusion'.

1.7 Unusual transport phenomena

The preceding section focused on the *value* of the transport coefficients. However, that is not the whole story. This section will describe some observations of remarkable phenomena that are difficult to explain on the basis of strictly local, diffusive models or using the cited approach in which turbulence is fully decoupled from the background transport and only considered as an additional term.

1.7.1 Power degradation and size scaling

In the absence of a complete transport theory that can predict the performance of a fusion device, one has turned to scaling laws. A scaling law is an engineering tool to predict the value of a system parameter as a function of some other significant variables in the absence of detailed knowledge of the underlying physics.

One of the main figures of merit of magnetically confined plasmas and one of the factors of the Lawson criterion for fusion is the energy confinement time, τ_E, which measures the efficiency of energy confinement in the device. It is defined as

$$\tau_E = \frac{W}{P - dW/dt}, \tag{1.5}$$

where W is the global plasma energy content and P the applied heating power.

This quantity is studied as a function of engineering parameters[17]:

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \overline{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{\rm cr}^{\alpha_S} M^{\alpha_M}, \tag{1.6}$$

where C is a constant, I (MA) is the plasma current, B (T) is the toroidal magnetic field, $\overline{n}$ (10^{19} m^{-3}) is the central line averaged density, P (MW) is the absorbed power, R (m) is the major radius, κ is the elongation, ϵ is the inverse aspect ratio, $S_{\rm cr}$ is the cross-sectional area (proportional to a^2, where a is the minor radius), and M is the hydrogen isotope mass. The parameters α are the corresponding scaling exponents, which are determined from a fitting procedure [37].

Decades of study in a huge range of devices, including different designs (tokamaks [38] and stellarators [39]) and machine sizes, have shown a remarkable consistency of the exponents.

[17] Using dimensional engineering parameters implies that the constant C must have a dimension that depends on the scaling exponents. This can be avoided through the use of appropriately normalized, dimensionless parameters.

One surprising result is the fact that the power scaling parameter systematically has the value $\alpha_P = -0.6 \pm 0.1$. In other words, as the heating power P is raised, the confinement quality as measured by τ_E degrades. This implies, necessarily, that the transport coefficients increase as the power is raised and gradients become steeper. It is worth noting that this result is largely independent from the type of toroidal confinement device, its size, the working gas, or the fuelling or heating method used, suggesting this is a feature common to all types of magnetically confined plasmas.

Various explanations have been suggested for this scaling behaviour. One idea is based on the assumption that heat transport tends to be of the 'gyro-Bohm' type, in which the radial diffusion is due to radial particle excursions of size $\rho_i = m_i|v_i|/eB$ and timescales of the order of the ion diamagnetic drift time. This would lead to an effective thermal diffusivity $\chi_{GB} \propto T^{1.5}/(aB^2)$, where T is the temperature, a the minor radius and B the magnetic field strength. It is evident that χ_{GB} increases with T, leading to a decrease of confinement time with T and hence heating. The final effect of this assumption on the scaling of τ_E can be assessed by making two further assumptions, namely: (i) the energy content of the plasma, W, can be estimated from $W \propto a^2 R\bar{n}_e T \propto P\tau_E$ (with this equation, T can be removed in favour of P) and (ii) $\tau_E \propto a^2/\chi_{GB}$. Combining results, one finds $\tau_E \propto P^{-0.6}$, quite close to the experimental result [40]. It should be noted that the apparent success of this straightforward argument to produce the correct exponent for the scaling with P is offset by its failure to produce the correct scaling for other parameters (such as B). Therefore, and also in view of the fact that plasmas do not always behave according to the gyro-Bohm expectation (particularly outside the core region [41]), this argument is not conclusive.

A second possible explanation is that an increase of heating power P will lead to an increase of (temperature and density) gradients, and thus an increase of the 'free energy' available to instabilities and turbulence. Steeper gradients will subsequently activate or enhance instabilities that increase radial transport [42], implying that turbulence is behind this scaling behaviour. We know that different circumstances (gradients) may trigger different types of turbulence, but apparently this does not matter to the scaling result, suggesting that the plasma somehow self-organizes towards a preferred end state, activating and deactivating turbulence as needed.

Another remarkable point is the scaling with system size. Assuming transport is dominated by diffusion (standard diffusion, non-fractional the sense of [43]), one expects τ_E to increase linearly with the cross-sectional area (or the square of the linear system size, L, see [44] for an argument). In actual fact, the scaling systematically falls short of this expectancy: building a bigger machine helps, but not as much as one would think. A possible explanation of this behaviour could be that transport is not strictly local, but somehow 'feels' the presence of the boundary, via long-range events (avalanches or streamers).

1.7.2 Profile consistency and stiffness

In the nineteen eighties it was observed that, often, a significant change in input electron cyclotron heating power P_{ECRH} only led to a moderate change of the resulting electron temperature profiles $T_e(r)$. The observation that profiles (of temperature, density, and pressure) often tend to adopt roughly the same shape, regardless of the applied heating and fuelling profiles, was dubbed profile consistency (or profile resilience) [45]. The resulting profiles, usually having a very similar shape, are known as canonical profiles [46, 47]. Like power degradation, the phenomenon has been observed over an extraordinarily broad range of devices and experimental conditions [48], although this does not mean that profiles always adopt the 'canonical' shape.

The explanation of this observation is found in the flux–gradient relation. In a standard diffusive model, the heat flux is linearly proportional to the temperature gradient, and similarly for other fields. In this case, no 'profile consistency' would occur, as profiles would evolve gradually as the sources are modified. Figure 1.7 provides a sketch of a strongly nonlinear flux–gradient relation. For small gradients, the flux increases linearly with the gradient, as one would expect for diffusion. But for larger gradients, the flux increases rather steeply over a small gradient interval, and returns to a more reduced slope for even higher gradient values. The solution of the transport problem, which consists of balancing sources and sinks by adjusting the flux would then lead to a very similar gradient for a broad range of fluxes (the blue area in the cartoon). Thus, in this range of fluxes, the profile (equal to the integral of the gradient) would look very similar, and one would thus obtain 'profile consistency'. In other words, the gradient would tend to be clamped to the range where the flux–gradient relation is steep over the relevant (broad) range of fluxes. This clamping of profiles is often called 'profile stiffness'.

What is the physical mechanism underlying this nonlinear flux–gradient relation? Several possible explanations have been put forward, but the main candidate appears to be the 'critical gradient' hypothesis [49]. This hypothesis assumes that there exists a critical gradient value, above which an instability is triggered, thus activating a new (fast) transport channel. It would explain the enhanced flux when the critical gradient value is exceeded, thus leading to a nonlinear flux–gradient relation as described above. This type of behaviour can be justified on the basis of the calculated growth rates of specific instabilities, changing from negative to positive at the critical gradient.

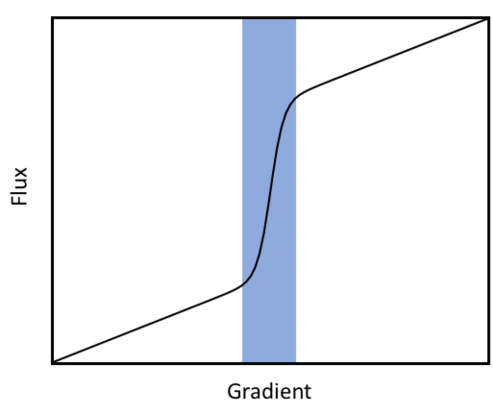

Figure 1.7. Cartoon of a 'critical gradient' type flux–gradient relation.

Therefore, turbulence again seems to be the essential ingredient to explain this phenomenon. It should be noted that the explanation is based on a feedback between profiles and turbulence, which poses an extraordinary challenge to turbulence simulations, as long and short temporal and spatial scales must be resolved simultaneously and evolved in time to reproduce the phenomenon.

1.7.3 Rapid transport phenomena and uphill transport

Changes in sources or sinks lead to adjustments of the transport fluxes and consequently the profiles. In 'ordinary' diffusive-type transport models, the time-scales of these large scale profile adjustments are typically of the order of the confinement time. However, experiments reveal that sometimes phenomena occur on much shorter timescales.

A paradigmatic example was reported at the TEXT tokamak [50]. The edge of the ECRH heated plasma was cooled through the injection of an impurity. Figure 1.8 shows the experimental time traces of the electron temperature, T_e. The graph shows that the core T_e responds almost immediately to the edge cooling, i.e. within 0.5 ms, much faster than could be explained by diffusive processes. This observation led this group to postulate an ad hoc model in which the local transport coefficient is modified globally on this rapid timescale (reflecting a 'non-local' transport mechanism in which a modification at the plasma edge has an immediate impact on the

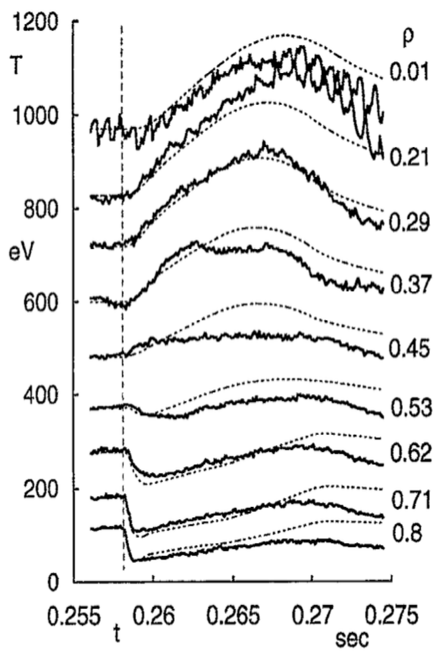

Figure 1.8. Electron temperature from ECE from a number of radii across the plasma, following strong edge cooling induced by impurity injection. (Reproduced with permission from [50]. Copyright (1992) by the American Physical Society.)

transport coefficient near the core). The resulting simulated plasma response (shown in the figure as dashed lines) approximately matched the observations, but the physical justification of this model remained unclear.

So-called 'uphill' transport (i.e. transport characterized by fluxes that go against the direction of the negative thermodynamic gradient) was observed in many experiments. Figure 1.9 shows an example in which strong off-axis heating leads to on-axis T_e profile peaking [51]. Standard diffusive models would require unrealistically intense inward convection to explain such behaviour, that would flow against the gradient ('uphill').

These remarkable phenomena are related and can be explained, at least qualitatively, on the basis of transport models with a strongly nonlinear flux–gradient relationship in combination with a supercritical superdiffusive transport channel [52], both of which have their roots in turbulence, as argued above. The nonlinear flux–gradient relation would produce the profile stiffness effect mentioned above, whereas the superdiffusive transport component would allow the unheated core region to 'fill up' [25].

1.7.4 Zonal flows

Although turbulence in fusion plasmas occurs in three-dimensional physical space, the presence of a strong magnetic field has the effect of reducing the effective dimension to near two. Unlike three-dimensional turbulence, two-dimensional turbulence is characterized by inverse energy cascades, implying the spontaneous formation of large scale structures from small scale turbulence. While noting that

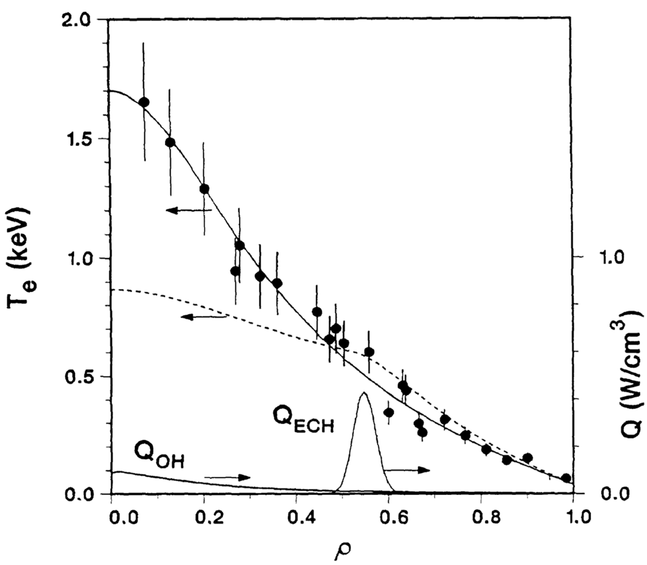

Figure 1.9. Electron temperature profile (dots) and power deposition from Ohmic heating (Q_{OH}) and ECR heating (Q_{ECH}). The dashed curve is a prediction. (Reproduced with permission from [51]. Copyright (1992) by the American Physical Society.)

this is an extreme simplification of the issue, it is still true that the presence of a strong magnetic field can lead to the spontaneous formation of large scale structures from the ambient turbulence, known as zonal flows [35]. One of the physical mechanisms for zonal flow production is Reynolds stress [53]. This effect is similar, in some respects, to the formation of 'bands' in the atmosphere of the planet Jupiter, where the dimension-reducing role of the magnetic field is taken over by the planet's fast rotation.

The radial variation of the zonal flow velocity ('sheared flow') generated by this effect then leads to turbulence suppression, as the radially varying flow speed tears apart turbulent eddies, thus reducing their size. In this way, a feedback loop is created: turbulence growth–zonal flow generation–sheared flow enhancement–turbulence suppression [54]. This is an example of self-organization with the amazing result that turbulence, driven by steep gradients, produces an effect that … reduces turbulence. The end state of the turbulence and the profiles then includes a transport barrier with steep gradients, associated with the sheared flow (with names such as H-mode barrier or internal transport barrier). Needless to say that the discovery of these spontaneous transport barriers was welcomed by the fusion community as a magnificent tool to improve the efficiency of future fusion reactors!

1.7.5 The impact of rational surfaces on transport

It has long been suspected that the topology of the confining magnetic field has an impact on plasma transport. Specifically, we are referring to the fact that minor transport barriers are often formed that are associated with low-order rational surfaces, which will be an issue that we will come back to often in this book. A legitimate question is why, if this effect is supposedly important, studies on this issue do not abound in the literature on transport in fusion devices—although some notable studies are available, as discussed below. A localized transport barrier (i.e. a local reduction of heat flux) implies a local change of *slope* of the temperature profile. Given the general turbulent state of the plasma and the prevailing measurement resolution and errors, such rather localized changes of slope are usually not easy to detect. As a result, many transport models completely ignore their possible existence and do not contemplate any effects that explicitly depend on the rational values of the rotational transform.

In 1997 an astounding report appeared that combined measurements using a very high resolution Thomson scattering diagnostic with a cleverly designed experiment in which the microwave power deposition position was scanned through the plasma [55]. Figure 1.10 shows one of the most surprising results from that report, reflecting clear evidence that a succession of minor transport barriers existed in the RTP tokamak. Several years later, a heuristic transport model was proposed to explain these results, the so-called 'q-comb' model, in which relatively sharp reductions of the electron heat transport coefficient were associated with the main low-order rational surfaces [56]. This simple hypothesis was sufficient to reproduce the observed behaviour.

Apart from the mentioned minor transport barriers, plasmas occasionally, and under special circumstances, can develop so-called internal transport barriers (ITBs) [33].

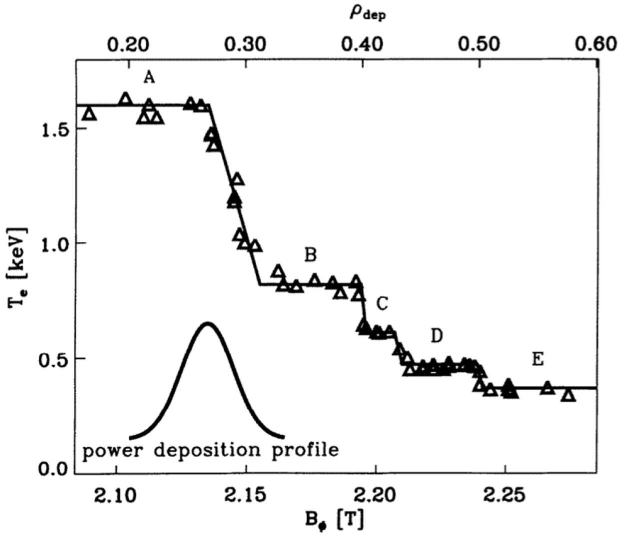

Figure 1.10. Stepwise response of the central electron temperature $T_e(0)$ as the deposition location of ECRH power is scanned: evidence of minor transport barriers. (Reproduced with permission from [55].)

ITB arise only transiently, but are much stronger than the 'minor transport barriers' that are assumed to be semi-permanent. In tokamaks, strong ITBs can be established by creating a core reversed magnetic shear region. The location of such ITBs appears to be correlated with integral values of the (minimum of the) safety factor, q [57]. The impact of ITBs on heat transport has been studied in some detail at, e.g. Alcator C-Mod [58] and JET [59, 60], showing that the heat diffusivity drops strongly in the ITB region. ITBs have also been obtained and studied in stellarators [61] and, here too, a relationship with the magnetic configuration is suggested. The existence of ITBs is widely acknowledged and supported by experimental evidence on many machines.

It seems undeniable, therefore, that the magnetic topology has a non-trivial impact on transport. Minor transport barriers and ITBs are likely established through a similar mechanism. In the neighbourhood of low-order rational surfaces, it may be easier for sheared flow regions to form as a consequence of Reynolds stress forces associated with MHD turbulence—a type of turbulence that is sensitive to the rotational transform. This is not to say that *all* transport barriers are necessarily of this type; there may be regions where other types of turbulence dominate and where sheared flow regions can form without immediate relation to the local rotational transform. At this time, insufficient data are available to provide a clear statement about this issue.

1.8 Summary

The study of turbulence is intellectually and computationally challenging. In this chapter, we have tried to show, without going into too much detail, that fusion plasmas are rather extreme systems, far from thermodynamic equilibrium, subject to strong turbulence and riddled with remarkable phenomena. Estimates of the relative importance of turbulent transport as compared to regular (diffusive, neoclassical) transport, based on both theory and experiment, systematically indicate that turbulence is important and often even constitutes the dominant component of radial (heat and particle) transport. In addition, some of the mentioned unusual[18] phenomena can be ascribed unambiguously to turbulence, either through the non-linearity of the flux–gradient relation, or through its non-locality. Some other phenomena are likely associated with turbulence (e.g. the formation of minor transport barriers near rational surfaces via zonal flow formation, driven by turbulence), but remain to be clarified further. It is therefore clear that the study of turbulence in fusion plasmas is a rich subject with many open questions that require urgent answers.

One important observation is that the often assumed separation between scales may not hold: turbulence is not confined to small radial scales and short times, but its impact can be felt on the large scales of the system size and the confinement time. This impact may occur via large scale structures that form spontaneously from small scale turbulence (an example being the zonal flow), or via the mutual modification of 'background' profiles and turbulent transport in a feedback loop. This observation has a major impact on the understanding of this phenomenon, and implies that it cannot be simply dismissed as an additional contribution to diffusion (i.e. a local and instantaneous mechanism). A significant effort has therefore been devoted to the detection and analysis of the long-range (spatial and temporal) effects of turbulence, as will be discussed in this book.

There are also practical motives for undertaking the study of turbulence. An improved understanding of turbulence may lead to its control or mitigation, with potentially enormous economic benefits. In addition, the incorporation of the full complexity associated with turbulence in predictive models may allow optimizing plasma configurations, leading to improved fusion device designs [62].

The remainder of this book is mainly dedicated to efforts to improve our understanding of turbulence through data analysis and modelling.

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