A Brief Introduction to Topology and Differential Geometry in Condensed Matter Physics (Second Edition)

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Dedicated to Rosangela, Henrique and Guilherme.

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Preface to the first edition

In recent years there have been significant advances in the applications of topology and differential geometry to problems in condensed matter physics. Concepts drawn from topology and geometry have become essential to the understanding of several phenomena in the area. Physicists have been creative in producing models for actual physical phenomena which realize mathematically exotic concepts, and new phases of matter have been discovered in condensed matter in which topology plays a leading role. An important classification paradigm is the concept of topological order, where the state characterizing a system does not break any symmetry, but it defines a topological phase in the sense that specific fundamental properties change only when the system passes through a quantum phase transition.

The primary purpose of this book is to provide a brief, self-contained introduction of some mathematical ideas and methods from differential geometry and topology and to show a few applications in condensed matter. It conveys to physicists the bases for many mathematical concepts, avoiding the detailed formality of most textbooks. The reader can supplement the description given here by consulting standard mathematical references such as those listed in the references.

There are many good books written about the subject, but they present a lot of material and demand time to gain a full understanding of the text. Here, I offer a summary of the main topics, which will provide readers with an introduction to the subject and will allow them to read the specialized literature.

Very little in this text is my original contribution since the goal of the book is pedagogy rather than originality. The material presented was mainly collected from the literature. Some time ago, I used to teach differential geometry in a graduate course about classical mechanics and wrote a book (in Portuguese) on the topic. Now, I have adapted that material and included ideas that appeared in the last years to write the present book.

Chapter 1 is an introduction to path integrals and it can be skipped if the reader is familiar with the subject. Chapters 2–4 are the core of the book, where the main ideas of topology and differential geometry are presented. In chapter 5, I discuss the Dirac equation and gauge theory, mainly applied to electrodynamics. In chapters 6–8, I show how the topics presented earlier can be applied to the quantum Hall effect and topological insulators. I will be mainly interested in the technical details, because there are excellent books and review articles dealing with the physical aspects. In chapter 9, I treat the application of topology to one- and two-dimensional antiferromagnets and the XY model. The same framework presented here can be used to study other systems, such as topological superconductors and quasimetals. The appendices, although important for the application of differential geometry to some problems in condensed matter, are more specifics.

Preface to the second edition

After the publication of the first edition of this book, I have received some suggestions and comments. In the first edition, I aimed to present only the basic formalism of differential geometry and topology for condensed matter. However, I have been told that I should have discussed physics a little more. Therefore, new material has been added to the second edition. I have added new material to all chapters. In particular, chapter 4 has a new section on geodesics, and chapter 6 a new section on the Aharonov–Casher effect. Chapter 8 has been rewritten to include three-band insulators, a study of the checkerboard lattice, the Kane–Mele model, and three-dimensional topological insulators. Chapter 9 is now chapter 15. Chapters 9–14, 16 and 17 are entirely new. In chapter 9, I treat topological phases in one dimension. In chapter 10, I study topological superconductors. Chapter 11 is dedicated to higher-order topological insulators. I consider chapter 12 the principal addition because it treats in detail the classification of topological states of topological insulators and superconductors. In chapter 13, I discuss Weyl semimetals. Chapter 14 presents the Kubo theory for transport and a brief discussion of topological invariants for interacting topological insulators. Chapter 16 explores magnons in topological magnetic insulators, a subject that has caused substantial interest in recent years. I have changed appendix D on *K*-theory to chapter 17 and have added more material. The use of topology in condensed matter physics is still growing and it is almost impossible to keep up with the advances. For instance, the role of topology in non-Hermitian systems is attracting great interest. A lot of new phenomena appear, such as topological phases in single-band systems and anomalous relation between bulk and boundary physics, (Berghholtz *et al* 2021).

Some repetition of the material was made to make easier for the reader to read the chapters. My web site, http://antpires.com will have updates and corrections.

Reference

Bergholtz E J, Budich J C and Kunst F K 2021 Exceptional topology of non-Hermitian systems *Rev. Mod. Phys.* **93** 015005

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Author biography

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Antonio S T Pires graduated from the University of California in Santa Barbara in 1976. He is a Professor of Physics at the Universidade Federal de Minas Gerais, Brazil researching quantum field theory applied to condensed matter. He is a member of the Brazilian Academy of Science, was the Editor of the *Brazilian Journal of Physics*, and a member of the Advisory Board of the *Journal of Physics: Condensed Matter*. He has published the books ADS/CFT correspondence in condensed matter and theoretical tools for spin models in magnetic systems.