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An Introduction to Plasma Physics and its Space Applications, Volume 2

Basic equations and applications

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Chapter 1

Introduction to the physical models for plasmas

The fundamental scheme of theories currently employed for the physical description of large number of interacting particles is discussed in first place. The plasma kinetic theory and statistical mechanics consider a microscopic approach where electrons, ions, neutral atoms and their elementary processes at atomic and molecular level are considered. In the opposite limit are macroscopic fluid models where plasma is described as homogeneous continuum media and transport coefficients such as viscosity or thermal conductivity account for its specific properties.

The plasma state of matter was introduced in section 2.3 of volume 1 as a system composed of a large number of ions, electrons and neutral atoms. The basic physical parameters of *ideal plasmas* were introduced in chapter 4 of volume 1 on the basis of the statistical description of electrically neutral gases in thermal equilibrium. Specifically, the Maxwell–Boltzmann distribution function, equations (3.5) or (3.7) in volume 1, was used to calculate the Debye length that governs the spatial fluctuations of the plasma electric field. This statistical description of gases and plasmas will be generalized in the following to account for time-dependent physical processes and/or transport phenomena where the equilibrium Maxwell–Boltzmann distribution function is no longer valid.

The basic structure of physical theories currently employed to describe a statistical ensemble of interacting particles are outlined in the diagram of figure 1.1. The *microscopic* approaches (statistical mechanics and kinetic theory) make use of statistical averages that take into account specific properties of atoms or molecules. The macroscopic properties that can be measured in the laboratory are calculated from such averages. Alternatively, thermodynamics or fluid dynamics are

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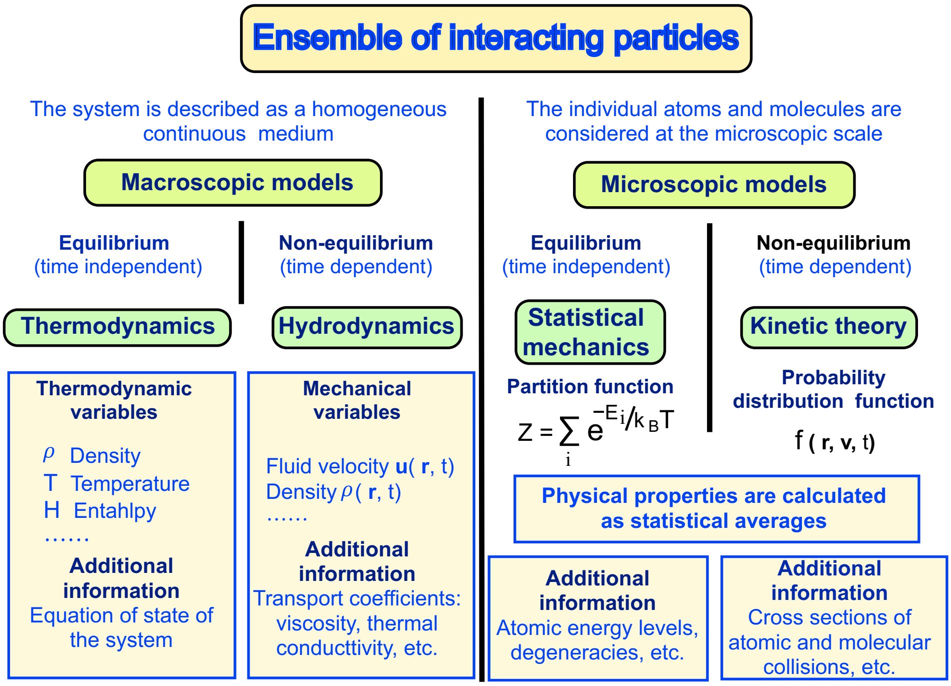

Figure 1.1. Scheme with the more relevant physical theories describing an statistical ensemble of interacting particles. The *additional information* is coefficients and/or equations that cannot be directly derived and need to be provided for the mathematical closure of the physical model.

macroscopic approaches which consider the physical system as a continuum medium, disregarding specific details of its constituent particles at the microscopic level.

Thermodynamic and statistical physics essentially apply to *equilibrium states* of condensed matter, as indicated in figure 1.1. Both formulations are valid when the temporal evolution of the system is much slower than the time scales involved in the physical processes at atomic and molecular levels that relax fluctuations to the equilibrium state. Thus, *time* does not explicitly appear in the equations of thermodynamic and statistical physics because the system is considered always in an equilibrium state and/or very close to it slowly evolving in time.

In contrast, the physical description of *non-equilibrium* macroscopic properties explicitly considers the time evolution of the system. The hydrodynamics and kinetic theory are more involved and concern *non-equilibrium* states where the physical system explicitly changes in time, as shown in the scheme of figure 1.1. The fluid transport equations for a plasma can be derived by considering the medium as a continuum using the mass, momentum and energy conservation principles, together with the Maxwell equations (2.6) of volume 1 for the macroscopic electromagnetic fields. However, the mathematical closure of the problem needs additional physical magnitudes, such as the thermal conductivity or viscosity (called transport coefficients) that are introduced as additional phenomenological expressions.

The starting point of kinetic theory is the *velocity distribution function* $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ for each plasma species (α = e, i, a, electrons, ions and neutral atoms) that will be introduced in chapter 2. In the general case, it depends on the velocity $\boldsymbol{v}$, position $\boldsymbol{r}$ and also evolves in time. It is proportional to the probability of finding at instant t one particle at point $\boldsymbol{r}$ moving with $\boldsymbol{v}$ velocity. As we shall see, this distribution can be calculated as the solution of the integro-differential *kinetic* or *Boltzmann equation* which accounts for the average forces acting on plasma particles at the microscopic level.

The time-dependent macroscopic properties, such as energy, pressure or average speed are calculated as statistical averages using $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ and appropriate operators for these physical magnitudes. This procedure was previously employed in section 3.3 for neutral gases using the equilibrium distributions (3.5) and (3.7). This Maxwell–Boltzmann distribution for neutral atoms is recovered from the Boltzmann integral in the limit corresponding to the thermodynamic equilibrium state.

The complex statistical approach of kinetic theory is appealing when the relevant collisional mean free path λ_c and/or collision time $\tau_c = 1/\nu_c$ can be compared to the characteristic length L_s or the time scale τ_s of the problem under consideration. Equivalently, this can also be when $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ appreciably changes along the typical length L_s and/or on plasma time scale τ_s of the system. However, the diagram of figure 1.1 shows the mathematical closure of kinetic formulation also requires *supplementary information*, such as the cross sections of particle collisions introduced in chapters 5 and 6 of volume 1.

In the *hydrodynamic* approach plasmas are regarded as *macroscopic continuum media* where a huge number of particles are contained within small volumes with typical size l much smaller $L_s \gg l \gtrsim \lambda_D$ than the length scale L_s of the system, but larger than the Debye length. The plasma hydrodynamic equations consider three mutually inter-penetrating fluids of neutral atoms, electrons and ions which motions are coupled by electromagnetic interaction and particle collisions and are more involved than those of ordinary fluids. In essence, the more tractable fluid transport equations apply when the relevant mean free path λ_c for collisions is $L_s \gg l \gg \lambda_c$ much smaller than characteristic length L_s of the problem and then, the plasma is dominated by collisions.

As discussed in section 4.2 of volume 1, this hydrodynamic approach is valid when collisions between particles relax the random fluctuations on a fast time scale to a *local* probability distribution function, as was the case of LTE plasmas discussed in section 4.3 of volume 1. Equivalently, when the distribution function $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ changes smoothly along typical distances $l \gg \lambda_c$ much longer than the collisional mean free paths λ_c. As we shall see in chapter 3, in this case, the hydrodynamic equations can be alternatively derived from kinetic theory assuming an unspecified velocity distribution function.

Finally, the collisions between plasma species connect both hydrodynamic and kinetic formulations of plasma state. The energy transfer between particle species by elastic collisions, the ionization and recombination rates, etc, are incorporated in the kinetic equations by means of *collision operators* that account for the changes introduced in $f_\alpha(\boldsymbol{r}, \boldsymbol{v}, t)$ by elementary processes. We will see in chapter 3 that their macroscopic average will provide us with expressions for the energy and momentum exchange that are needed in the three-fluids hydrodynamic description to account for the mechanical coupling between ion and electrons fluids, etc.

The basis of kinetic description of plasmas is introduced in chapter 2 where this *microscopic* approach will show the connection between the elementary processes and the non-equilibrium probability distribution functions discussed in chapters 5 and 6 of volume 1. The connection between the statistical averages and the individual particle properties is a complex theory and in appendix B is outlined a more rigorous derivation of the Boltzmann or kinetic equation, which is the basis of the kinetic description of chapter 2 where we will follow a more intuitive approach.

The hydrodynamic approach will be introduced in chapter 3 where the fluid equations are derived from the kinetic description. As we shall see, macroscopic transport coefficient such as plasma thermal conductivity or viscosity is also connected with collisional processes at the microscopic level. The fluid equations are derived from statistical averages of the mechanical magnitudes of interest such as energy and momentum. However, we shall see some physical assumptions are required for the closure of macroscopic transport equations.