Chapter 8

Compton-scattering sum rules for vector bosons

In this chapter we enter the relatively unexplored area of sum rules for a spin-1 target. This subject is interesting in connection with the massive gauge bosons of the Standard Model ($W^{\pm}$, Z^0), but the formalism is just as well applicable to the (virtual) massless gauge bosons, such as photons and gluons. The sum rules for the photon structure functions can simply be obtained in the limit of vanishing electromagnetic moments; we will come to that in the last section. We shall start, however, by considering a charged massive spin-1 particle having a magnetic and quadrupole moments, and more generally having arbitrary electromagnetic distributions (or, form factors). The practical applications are therefore not limited to gauge bosons; the resulting sum rules will apply, for example, to the deuteron or vector mesons (ρ, ω, etc) as well. We shall see that consistent electrodynamics of spin-1 fields is only realized within the Yang–Mills theory, which sets the classical values of the electromagnetic moments to the, so-called, natural values. Some extra attention will be paid to the role of the quadrupole moment, which is of course a novel feature of the spin-1 case, as compared with spin-1/2. It, for instance, affects the sum sum rule for the forward spin polarizability. We shall conclude with a look at the spin-1 structure functions and the sum rules for the photon structure functions in particular.

8.1 Electromagnetic moments: natural values

A massive particle with spin S has, in general, $2S + 1$ intrinsic electromagnetic moments. In the case of $S = 1$, these are: the electric charge e, the magnetic dipole moment μ, and the electric quadrupole moment $\mathcal{Q}$. It is interesting that for an elementary (structureless) charged particle the electromagnetic moments are set to the so-called *natural values* [1–4]. For example, in the spin-1 case, they are given by

$$\mu_{\text{nat.}} = \frac{e}{M}, \qquad \mathcal{Q}_{\text{nat.}} = -\frac{e}{M^2}, \tag{8.1}$$

doi:10.1088/978-0-7503-3431-0ch8

where M is the mass. The natural value of the magnetic moment corresponds with the gyromagnetic ratio g (i.e. the ratio of the magnetic moment value, in units of the magneton: $e/2M$, to the value of the spin) equal to 2. In fact, it is the natural value for any spin:

$$g_{\text{nat.}} = 2 = \frac{\mu_{\text{nat.}}}{(e/2M)S}. \tag{8.2}$$

As shown in [3], the natural values arise from the point-like transverse charge density. In simpler terms, this means the charge can only be distributed along the direction of motion. The deviations of the electromagnetic moments from their natural values are called the anomalous moments; we hereby introduce them in the dimensionless fashion:

$$\mu = \mu_{\text{nat.}} + 2\varkappa\frac{e}{2M}, \qquad \mathcal{Q} = \mathcal{Q}_{\text{nat.}} + 2\varsigma\frac{e}{M^2}, \tag{8.3}$$

where $\varkappa$ and ς are, respectively, the magnetic and quadrupole anomalous moments. The normalization is chosen such that $\varkappa = (g - 2)/2$, just as for the spin-1/2 particle.

Let us consider a field-theoretic description of a massive spin-1 particle with the above-mentioned intrinsic moments. In relativistic theory, the particle is described by a bosonic vector field $W^\mu(x)$. This is very similar to the electromagnetic field $A^\mu(x)$, except for a charged particle the field is complex. The free Lagrangians for the electromagnetic and vector-boson fields are similar too[1]:

$$\mathcal{L}_{\text{em}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{8.4a}$$

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{2}W^*_{\mu\nu}W^{\mu\nu} - M^2 W^*_\mu W^\mu, \qquad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu. \tag{8.4b}$$

The only real difference is the mass term; the factor of 2 difference arises because of the convention to write the complex field in terms of two real vectors W_1^μ and W_2^μ:

$$W^\mu = \frac{1}{\sqrt{2}}(W_1^\mu - iW_2^\mu). \tag{8.5}$$

Substituting this into the above Lagrangian, one can see that the normalization factors for the real fields are all the same.

In the field-theoretic language, the electromagnetic moments arise from the cubic γWW couplings. To construct those, it is good to start with the minimal electromagnetic couplings, obtained by the minimal substitution,

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu, \tag{8.6}$$

into equation (8.4b), leading to the following interaction Lagrangian (containing both cubic and quartic couplings):

[1] This description of the massive spin-1 particle is known as the *Proca model*.

$$\mathcal{L}_{\min} = i\, e\, W^*_{\mu\nu} A^\mu W^\nu - i\, e\, A_\mu W^*_\nu W^{\mu\nu} + e^2 A^2\, W^*_\mu\, W^\mu. \tag{8.7}$$

The minimal substitution ensures the electromagnetic gauge symmetry, exhibited here by the invariance of the full Lagrangian under the following transformations:

$$A_\mu \to A_\mu + \partial_\mu \varphi, \qquad W^\mu \to e^{ie\varphi} W^\mu, \tag{8.8}$$

with φ an arbitrary scalar field. As the result, the charge and electromagnetic current are conserved.

In order to have arbitrary values of the magnetic and quadrupole moments, let us supplement the Lagrangian with the following terms:

$$\mathcal{L}_{\text{extra}} = i\, e\, \ell_1\, W^*_\mu W_\nu\, F^{\mu\nu} + \left(e\ell_2 \big/ 2M^2\right) \left[(D^*_\mu W^*_\nu) W^\alpha \partial_\alpha F^{\mu\nu} + W^*_\alpha (D_\mu W_\nu) \partial^\alpha F^{\mu\nu} \right], \tag{8.9}$$

where ℓ_1 and ℓ_2 are the coupling constants, which contribute to the anomalous moments as follows:

$$\varkappa = \frac{1}{2}(\ell_1 - 1), \qquad \varsigma = \frac{1}{2}(1 + \ell_2 - \ell_1). \tag{8.10}$$

Thus, the natural values of the electromagnetic moments arise when $\ell_1 = 1$ and $\ell_2 = 0$. These are precisely the values corresponding with the Standard Model description of the charged gauge bosons, $W^\pm$. To understand why this choice of couplings is special, let us make a quick detour into the relation between gauge symmetries and the so-called degrees-of-freedom counting.

8.2 Gauge symmetries and spin degrees of freedom

Gauge symmetry is a divine principle of all the relativistic field theories of particles with spin-1 and higher, *viz*, 'higher-spin' field theories. The reason is that the number of field components exceeds the number of spin degrees of freedom (DoF), and the balance is established using gauge symmetries.

For example, a spin-1 particle ought to have three spin DoFs in the massive case and two in the massless[2]. A real four-vector field $W^\mu(x)$, describing it in a relativistic field theory, has four components. Writing the Lagrangian in a way analogous to the scalar theory, i.e. $\partial_\mu W_\nu \partial^\mu W^\nu$, would not work because all four components of the field act independently, giving rise to four spin DoFs; we need fewer. The correct Lagrangian is $\partial_\mu W_\nu \partial^\mu W^\nu - (\partial \cdot W)^2$, since it is invariant under a gauge transformation, $W^\mu \to W^\mu + \partial^\mu \varphi$, which ensures that the number of independent components is reduced to two. The mass term, $M^2 W^2$, raises this number to three, as it should.

The gauge symmetry thus eliminates the extra DoFs. The exact number of eliminated DoFs depends on the form of the gauge transformation. In general, the transformation can be written as $D_\mu \varphi^a(x)$, where D_μ is a differential operator of order d acting on the parameters of the gauge transformation φ^a, with $a = 1, \ldots, n$.

[2] In general, the number of spin DoFs is $2S + 1$ and 2 for, respectively, massive and massless particles with spin S.

The number of eliminated DoFs is then equal to[3] $(d + 1)n$. In the above example of a free spin-1 particle, the transformation $\partial_\mu\varphi$ has $d = 1 = n$, and hence the corresponding symmetry eliminates two DoFs.

Now, we can look at the theory considered in the previous section from the point of view of DoF counting. The free theory, equation (8.4), is alright: the kinetic terms of both A_μ and W_μ are gauge invariant. To make the bookkeeping easier, let us introduce A_μ^a, with $a = 1, 2, 3$, and define

$$A_\mu^1 = \frac{1}{\sqrt{2}}\left(W_\mu + W_\mu^*\right), \quad A_\mu^2 = \frac{i}{\sqrt{2}}\left(W_\mu - W_\mu^*\right), \quad A_\mu^3 = A_\mu. \tag{8.11}$$

The above free Lagrangian for the photon and $W^\pm$ is then written simply as

$$\mathcal{L} = -\frac{1}{4}\sum_{a=1}^{3} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2}M^2 \sum_{a=1}^{2} A^a \cdot A^a, \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \tag{8.12}$$

and the first term is invariant under the gauge transformation: $A_\mu^a \to A_\mu^a + \partial_\mu\varphi^a$.

The minimal substitution, equation (8.6), can now be written as $\partial_\mu A_\nu^a \to (\delta^{ab}\partial_\mu - e\varepsilon^{ab3}A_\mu^3)A_\nu^b$, where the summation over repeated indices is understood. Obviously, this substitution spoils some of the gauge symmetries of the free massless theory, retaining only the symmetry under

$$A_\mu^a \to A_\mu^a + (\delta^{a3}\partial_\mu + e\varepsilon^{ab3}A_\mu^b)\varphi^3. \tag{8.13}$$

The easiest way to restore the lost symmetries for $a = 1$ and 2 is to treat all the fields in the same fashion and do the minimal substitution as follows:

$$\partial_\mu A_\nu^a \to D_\mu^{ab} A_\nu^b, \qquad D_\mu^{ab} = \delta^{ab}\partial_\mu - e\varepsilon^{abc}A_\mu^c. \tag{8.14}$$

The resulting theory is none other than the SU(2) Yang–Mills theory, invariant under

$$A_\mu^a \to A_\mu^a + D_\mu^{ab}\varphi^b. \tag{8.15}$$

The more general minimal substitution, equation (8.14), leads to the following extra terms, in addition to the minimal coupling of equation (8.7):

$$\mathcal{L}_{\text{extraYM}} = i.\, e.\, W_\mu^* W_\nu\, F^{\mu\nu} + \frac{1}{2}e^2(|W \cdot W|^2 - |W|^4). \tag{8.16}$$

The first term is the cubic coupling, which fixes the magnetic and quadrupole moments of the W to the natural values (see equations (8.9) and (8.10)). The second term describes the W self-coupling, which emphasizes the non-Abelian nature of the Yang–Mills theory.

---

[3] A more rigorous formulation of this theorem (conveyed to me by Misha Vasiliev, who is world's greatest expert on higher-spin fields) can be found in [5].

To conclude, the consistent electrodynamics of a charged spin-1 particle is only realized within the Yang–Mills theory[4], where, due to the gauge symmetry, the electromagnetic moments are set to their natural values. The coupling (8.9) with arbitrary ℓ_1 and ℓ_2 is nevertheless useful in the sense of effective theory, and particularly in obtaining the low-energy theorems, which we now consider.

8.3 Tree-level unitarity: GDH sum rule

An important step towards deriving the sum rules is the forward Compton scattering. Recall from chapter 4 that the number of independent scalar amplitudes, in case of a real photon scattered off a massive spin-1 particle, is $2S + 2 = 4$. As the four Lorentz-covariant structures it is convenient to choose

$$\begin{aligned} \mathcal{O}_1 &= \varepsilon^* \cdot \varepsilon \, \chi^* \cdot \chi, & \mathcal{O}_2 &= \varepsilon^* \cdot \varepsilon \, q \cdot \chi^* \, q \cdot \chi, \\ \mathcal{O}_3 &= \varepsilon^* \cdot \chi^* \, \varepsilon \cdot \chi - \varepsilon^* \cdot \chi \, \varepsilon \cdot \chi^*, & \mathcal{O}_4 &= \varepsilon^* \cdot \chi^* \, \varepsilon \cdot \chi + \varepsilon^* \cdot \chi \, \varepsilon \cdot \chi^*, \end{aligned} \tag{8.17}$$

where ε and χ are the polarization vectors of, respectively, the photon and the spin-1 target; q is the photon four-momentum. The tree-level forward Compton-scattering amplitude computes (using equations (8.7) and (8.9)) as follows:

$$T^{\gamma W \to \gamma W}(\nu, t = 0) = 2M \sum_{i=1}^{4} \mathcal{A}_i(\nu) \mathcal{O}_i, \tag{8.18}$$

$$\begin{aligned} \mathcal{A}_1(\nu) &= -\frac{e^2}{M}, \\ \mathcal{A}_2(\nu) &= -\frac{e^2}{2M^3} \ell_2 \, (1 + \ell_1) = -\frac{2e^2}{M^3} (1 + \varkappa)(\varkappa + \varsigma), \\ \mathcal{A}_3(\nu) &= -\frac{e^2 \nu}{4M^2} (1 - \ell_1)^2 - \frac{e^2 \nu^3}{16M^4} \ell_2^2 = -\frac{e^2 \nu}{M^2} \varkappa^2 - \frac{e^2 \nu^3}{4M^4} (\varkappa + \varsigma)^2, \\ \mathcal{A}_4(\nu) &= \frac{e^2 \nu^2}{4M^3} \ell_2 \, (1 - \ell_1) = -\frac{e^2 \nu^2}{M^3} \varkappa(\varkappa + \varsigma), \end{aligned} \tag{8.19}$$

where $\nu = (s - M^2)/2M$ is the photon energy in the lab frame, $\mathcal{A}_i$ are scalar amplitudes.

The first amplitude represents the Thomson term. It is interesting that for the natural values ($\ell_1 = 1$, $\ell_2 = 0$), only the Thomson term survives, while the rest of the contributions vanish. In this case, the tree-level amplitude does not diverge at high energy and, therefore, is said to satisfy the 'tree-level unitarity'. The tree-level unitarity is a necessary condition for the theory to be renormalizable at one-loop level.

The various polarizabilities begin to contribute at order ν^2, and hence, as far as the sum rules involving only the electromagnetic moments are concerned, we need to focus on the terms of $O(1)$ and $O(\nu)$. The first one is the Thomson term (note that $\mathcal{O}_2$

[4] A better argumentation of this claim can be found in an insightful paper by Weinberg and Witten [6].

contains photon momenta and hence $\mathcal{A}_2$ contributes at $O(\nu^2)$), which, as already discussed, does not lead to a convergent sum rule. The term of $O(\nu)$, seen in the amplitude $\mathcal{A}_3$, is the anomalous magnetic moment contribution which leads to the GDH sum rule for the spin-1 case:

$$\boxed{\frac{2\alpha}{M^2}\varkappa^2 = -\frac{1}{2\pi^2}\int_0^\infty d\nu\, \frac{\sigma_0(\nu)-\sigma_2(\nu)}{\nu}}. \tag{8.20}$$

The quantity in the numerator on the right-hand side is the helicity-difference photoabsorption cross section. This form is basically the same as in the spin-1/2 case, see equation (4.18), which allows one to conjecture the following expression for the GDH sum rule for any spin S:

$$\boxed{\frac{2\alpha}{M^2}\varkappa^2 S = -\frac{1}{2\pi^2}\int_0^\infty d\nu\, \frac{\sigma_{1-S}(\nu)-\sigma_{1+S}(\nu)}{\nu}}, \tag{8.21}$$

with the anomalous magnetic moment defined universally, for any spin, through the gyromagnetic ratio as $\varkappa = (g-2)/2$.

The anomalous quadrupole moment starts to enter at $O(\nu^2)$ and cannot be disentangled from polarizability contributions; at least, not by means of sum rules derived from real Compton scattering. Conversely, some of the sum rules, which in the spin-1/2 case involved only polarizabilities, may now involve the electromagnetic moments. A remarkable example is provided by the sum rule for the forward spin polarizability (FSP) γ_0. Introducing it for a spin-1 particle in exactly the same way as for the spin-1/2 case results in the following sum rule [7]:

$$\boxed{-\frac{\alpha}{4M^4}(\varkappa+\varsigma)^2 + \gamma_0 = \frac{1}{4\pi^2}\int_0^\infty d\nu\, \frac{\sigma_0(\nu)-\sigma_2(\nu)}{\nu^3}}. \tag{8.22}$$

This has to be compared with the analogous spin-1/2 sum rule, equation (4.19), where the anomalous moment term is absent.

The FSP of the deuteron is conventionally defined to be a factor of two smaller [8] (i.e. γ_0 in equation (8.22) should be replaced with $2\gamma_0$). In addition, the contribution of anomalous moments is assumed to be absent from this sum rule. Fortunately, the contribution of the deuteron anomalous moments into this sum rule is numerically very small and may indeed be ignored[5]. The deuteron is, however, quite a peculiar system. It is a loose bound state of the proton and neutron, and as such, has rather large polarizabilities, enhanced by inverse powers of the binding momentum, which is two orders of magnitude smaller than the deuteron mass. For a 'more elementary'

[5] Using the currently known empirical values of the deuteron anomalous moments and mass (see [7] for details):

$$\varkappa_d \simeq -0.143, \qquad \varsigma_d \simeq 13.5, \qquad M_d \simeq 1.8756\ \text{GeV}, \qquad \alpha \simeq 1/137.036,$$

the correction due to the first term in equation (8.22) is about $-0.0263\ \text{GeV}^{-4} \simeq -4\times 10^{-5}\ \text{fm}^4$, whereas γ_0 of the deuteron is expected to be of the order of a few fm^4, so, five orders of magnitude bigger. See [9] for a first experimental determination of the deuteron FSP.

particle, the interplay between the electromagnetic moments and the polarizability in the above sum rule can be entirely different.

8.4 Forward VVCS and virtual LbL scattering

As in the spin-1/2 case, considered in chapter 5, the doubly-virtual Compton scattering (VVCS) is the place to look for sum rules. The spin structure of the VVCS is of course more complicated for a spin-1 particle. The number of spin structures triples: six instead of two in the previous (spin-1/2) case[6]. The number of sum rules should increase accordingly. In the spin-1/2 case, the lowest order spin-dependent sum rules in the (quasi-)real-photon limit are GDH and Schwinger, so one per each independent spin structure function. Does the number of such sum rules for the spin-1 case triple as well?

A simple answer to this question is 'No.' There are not that many sum rules involving the electromagnetic moments without polarizabilities. A proper description of those sum rules requires us to dwell into the formalism *a la* chapter 5, except three times lengthier, which would certainly be too much for the 'concise' format we are bound to. In what follows we give only the highlights.

A good indication of sum rules for the electromagnetic moments of massive vector bosons can be obtained by looking at the sum rules for the $\gamma^*\gamma^*$ fusion, considered in the previous chapter. The forward LbL scattering is exactly analogous to the VVCS on massive vector bosons, provided the electromagnetic moments are set to zero, whilst the role of polarizabilities is played by the low-energy coefficients c_i from the effective Lagrangian (6.15), (6.16). The sum rules which do not involve polarizabilities appear in the LbL case as the *superconvergence* relations, i.e. the ones where the integral converges to zero.

Let us re-examine two of those relations:

$$\lim_{Q^2\to 0} \int_{\tilde{\nu}_0}^{\infty} d\tilde{\nu}\, \frac{1}{\tilde{\nu}} \tau^a_{\mathrm{TT}}(\tilde{\nu}, K^2, Q^2) = 0, \tag{8.23a}$$

$$\lim_{Q^2\to 0} \int_{\tilde{\nu}_0}^{\infty} d\tilde{\nu}\, \frac{1}{Q} \tau^a_{\mathrm{TL}}(\tilde{\nu}, K^2, Q^2) = 0, \tag{8.23b}$$

where τ^a_{TT} and τ^a_{TL} are cross section quantities describing the fusion of two virtual photons into everything ($\gamma^*\gamma^*\to X$), see [10].[7] The photons have the space-like virtualities K^2 and Q^2, respectively, the invariant $\tilde{\nu} = (s + K^2 + Q^2)/2$, and $\tilde{\nu}_0$ is the inelastic threshold. By definition, $\tau^a_{\mathrm{TT}} = (\sigma_0 - \sigma_2)/2$, and hence the first relation is identified as the GDH sum rule in the absence of electromagnetic moments. To see

[6] Recall from chapter 4 that the number of independent forward VVCS amplitudes is $5S + 3$ for bosons (with nonvanishing spin), and $5S + 3/2$ for fermions. For any S, two of these amplitudes are spin independent. Hence, the number of spin structure functions is 6 for $S = 1$, and 2 for $S = 1/2$.

[7] The response function τ^a_{TL} is interchanged with τ_{TL} in [11], as compared with [10], because of a different convention for the longitudinal polarization vector: $\varepsilon_0^* = \varepsilon_0$ versus $\varepsilon_0^* = -\varepsilon_0$, respectively. Here we assume the latter convention.

that the other one is the analogue of the Schwinger sum rule, let us introduce the spin-1 structure functions, if only very schematically.

The structure function formalism, analogous to the spin-1/2 case, was developed in the context of the deuteron by Hoodbhoy, Jaffe, and Manohar [12]. The Lorentz-covariant form of the forward VVCS amplitude splits again into the symmetric and antisymmetric parts, $T^{\mu\nu} = T_S^{\mu\nu} + T_A^{\mu\nu}$, which contain six and two amplitudes, respectively. The imaginary parts of those amplitudes are given by the following eight structure functions,

$$\text{scalar: } F_1(x,\, Q^2),\ \ F_2(x,\, Q^2), \tag{8.24a}$$

$$\text{vector: } g_1(x,\, Q^2),\ \ g_2(x,\, Q^2), \tag{8.24b}$$

$$\text{tensor: } b_i(x,\, Q^2),\ \ i = 1,\, \dots,\, 4. \tag{8.24c}$$

They are referred to, respectively, as the scalar, vector, and tensor structure functions. The latter are new compared to the spin-1/2 case. They involve the longitudinal polarization of the spin-1 target. Just as the scalar ones, the tensor structure functions enter through the symmetric part of the VVCS tensor, whilst the vector ones through the antisymmetric part.

These structure functions can be matched to the cross section quantities,

$$\sigma_{\rm TT},\ \sigma_{\rm TL},\ \sigma_{\rm LT},\ \sigma_{\rm LL},\ \tau_{\rm TT},\ \tau_{\rm TL},\ \tau_{\rm TT}^a,\ \tau_{\rm TL}^a, \tag{8.25}$$

used to describe the $\gamma^*\gamma^*$ collisions in chapter 7. Of course, one needs to keep in mind that one of the virtual photons in the LbL case corresponds with the massive boson, hence, the virtuality must match the mass: $K^2 = -M^2$. The energy invariants are matched as $\tilde{\nu} = M\nu = Q^2/2x$. The precise matching of the structure functions can be worked out by comparing the helicity amplitudes for the two cases. As the result we, in particular, have

$$g_1 \sim \frac{Q}{\nu}\tau_{\rm TL}^a + \tau_{\rm TT}^a, \tag{8.26a}$$

$$g_2 \sim \frac{\nu}{Q}\tau_{\rm TL}^a - \tau_{\rm TT}^a, \tag{8.26b}$$

and the analogy with the spin-1/2 case becomes apparent, see equation (5.4). The superconvergence relation (8.23b) thus corresponds with the Schwinger sum rule.

To show this more explicitly, let us cast the above superconvergence relations into sum rules for spin structure functions of a real photon, denoted as $g_i^\gamma(x,\, Q^2)$. For this, we interchange the virtualities, $K^2 \leftrightarrow Q^2$, such that K^2 is set to zero whereas Q^2 is arbitrary. The exact relation between g_i^γ and τ in this case becomes:

$$g_1^\gamma = \frac{1}{4\pi^2\alpha} \lim_{K^2\to 0} \frac{\tilde{\nu}}{\sqrt{1 - K^2Q^2/\tilde{\nu}}}\left(\frac{KQ}{\tilde{\nu}}\tau_{\rm TL}^a + \tau_{\rm TT}^a\right), \tag{8.27a}$$

$$g_2^\gamma = \frac{1}{4\pi^2\alpha} \lim_{K^2\to 0} \frac{\tilde{\nu}}{\sqrt{1 - K^2Q^2/\tilde{\nu}^2}} \left(\frac{\tilde{\nu}}{KQ} \tau_{\mathrm{TL}}^a - \tau_{\mathrm{TT}}^a \right). \tag{8.27b}$$

Solving these for τ and substituting in equation (6.20) with $\tilde{\nu} = Q^2/2x$ we, respectively, have

$$\boxed{\int_0^{x_0} dx\, g_1^\gamma(x,\, Q^2) = 0} \tag{8.28a}$$

$$\boxed{\int_0^{x_0} dx \left[g_1^\gamma(x,\, Q^2) + g_2^\gamma(x,\, Q^2) \right] = 0}. \tag{8.28b}$$

Given that the elastic contribution is absent in the case of LbL scattering, we can extend the upper limit of integration to unity, and thus these two sum rules imply as well the BC sum rule: $\int_0^1 dx\, g_2^\gamma(x,\, Q^2) = 0$.

The third superconvergence relation (7.23b), derived originally in [11], has no spin-1/2 analogue. When extended to the massive boson case, it may, potentially, yield a sum rule for the quadrupole moment. Some work in this direction has been done, e.g. for the tensor structure function b_1, see [13] and references therein. Yet, a sum rule involving a quadrupole moment (and no polarizabilities!) has not been written down. For now, this task is given away as an exercise to this chapter.

References

[1] Weinberg S 1970 *Lectures on Elementary Particles and Quantum Field Theory* (Brandeis University Summer Institute 1970 vol 1) (Cambridge, MA: MIT Press)

[2] Ferrara S, Porrati M and Telegdi V L 1992 $g = 2$ as the natural value of the tree level gyromagnetic ratio of elementary particles *Phys. Rev.* D **46** 3529–37

[3] Alexandrou C, Korzec T, Koutsou G, Lorce C, Negele J W, Pascalutsa V, Tsapalis A and Vanderhaeghen M 2009 Quark transverse charge densities in the Δ(1232) from lattice QCD *Nucl. Phys.* A **825** 115–44

[4] Lorce C 2009 Electromagnetic properties for arbitrary spin particles: natural electromagnetic moments from light-cone arguments *Phys. Rev.* D **79** 113011

[5] Pascalutsa V and Timmermans R 1999 Field theory of nucleon to higher-spin baryon transitions *Phys. Rev.* C **60** 042201

[6] Weinberg S and Witten E 1980 Limits on massless particles *Phys. Lett.* B **96** 59–62

[7] Hagelstein F 2014 Sum rules for electromagnetic moments and polarizabilities of spin-1 particles in massive Yang–Mills QED *Master's thesis* University of Mainz

[8] Ji X and Li Y 2004 Sum rules and spin dependent polarizabilities of the deuteron in effective field theory *Phys. Lett.* B **591** 76–82

[9] Ahmed M W, Blackston M A, Perdue B A, Tornow W, Weller H R, Norum B, Sawatzky B, Prior R M and Spraker M C 2008 Near-threshold deuteron photodisintegration: an indirect determination of the Gerasimov–Drell–Hearn sum rule and forward spin polarizability (γ_0) for the deuteron at low energies *Phys. Rev.* C **77** 044005

[10] Budnev V, Ginzburg I, Meledin G and Serbo V 1975 The two photon particle production mechanism. Physical problems. Applications. Equivalent photon approximation *Phys. Rep.* **15** 181–281

[11] Pascalutsa V, Pauk V and Vanderhaeghen M 2012 Light-by-light scattering sum rules constraining meson transition form factors *Phys. Rev.* D **85** 116001

[12] Hoodbhoy P, Jaffe R L and Manohar A 1989 Novel effects in deep inelastic scattering from spin 1 hadrons *Nucl. Phys.* B **312** 571–88

[13] Close F E and Kumano S 1990 A sum rule for the spin dependent structure function $b - 1(x)$ for spin one hadrons *Phys. Rev.* D **42** 2377–9