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Lectures on General Relativity, Cosmology and Quantum Black Holes

Lectures on General Relativity, Cosmology and Quantum Black Holes

Badis Ydri

Annaba University, Annaba, Algeria

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Cover image: Conceptual illustration of gravity waves created by merging black holes. Credit: David Parker / Science Photo Library

To my father for his continuous support throughout his life...

Saad Ydri

1943–2015

Also to my ...

Nour

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Appendix: Differential geometry primer

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Preface

These lecture notes originated from a formal course of lectures delivered during the academic years 2012–2013 (general relativity and classical black holes), 2014–2015 (cosmology), and 2016–2017 (quantum black holes) to second year Master’s students of theoretical physics at Badji Mokhtar Annaba University.

The choice of topics and references is a personal one. The main references used in these notes include: (i) Wald (general relativity and differential geometry), (ii) Carroll (classical black holes and advanced cosmology), (iii) Mukhanov (inflationary cosmology: maybe the best book on cosmology, especially for a theoretical physicist), (iv) Birrell and Davies (quantum field theory (QFT) on curved backgrounds; one of the best QFT books I have ever seen), and (v) Hartle (elementary exposition of cosmology and observational cosmology). I should also mention Weinberg (on issues related to gauge fixing) and Dodelson (on observational facts). Of the lucid and pedagogical lectures that were also crucial I mention ’t Hooft, Jacobson, Liddle, Harlow, and Baumann.

This is not a book on general relativity per se, and thus our treatment of this fundamental topic is brief and many interesting topics are left out. The required differential geometry background is summarized in an appendix. The primary goal of this book is to understand in a critical way two pillars of modern theoretical physics: (i) inflationary theory and (ii) quantum black holes and the information loss problem. Hence, we found it necessary to treat black holes and cosmology extensively and in great detail before we can touch upon the important issues found in inflation and the information loss problem. In particular, I think that our treatment of the information loss problem (the final chapter) is quite systematic and more or less complete, and it would have been further improved had it been possible to include current topics such as ER = EPR, firewalls, etc. As for inflationary theory, we have reached the point where the calculation of the first acoustic peak observed in the cosmic microwave background spectrum can be performed.

Other important topics discussed in great detail in this book include the cosmological constant and its relation to dark energy. The vacuum energy is computed in various situations using varying methods and an introduction to QFT on curved backgrounds is given.

We have also included a detailed presentation of the ADM formalism and provide a brief introduction to quantum gravity as exemplified by Hořava–Lifshitz gravity. Indeed, for a successful treatment of the problem of quantizing gravity, we think that Hořava–Lifshitz gravity is a very promising and serious candidate, which adheres to the spirit as well as to the tradition of renormalizable QFT. The references on this topic are the original papers by Hořava.

This book is organized as follows. Chapter 1 is an extensive summary of the essentials of general relativity. Chapter 2 contains a detailed discussion of classical black holes: Schwarzschild, Reissner–Nordström, Kerr, and Kerr–Newman. Chapter 3 contains a detailed discussion of observational facts about the Universe and various

cosmological models, in particular the hot Big Bang model. Chapter 4 contains a detailed presentation of inflationary cosmology. Chapter 5 gives an introduction to QFT on curved backgrounds and the calculation of the vacuum energy. Chapter 6, which is the most important chapter of the book, contains a systematic derivation from first principles of Hawking radiation, the information loss paradox, and black hole thermodynamics.

Again, we stress that the list of topics and references included in these lectures only reflects the choice, preference, and prejudice of the author and is not intended to be complete, exhaustive, or thorough in any sense whatsoever.

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Author biography

Badis Ydri



Badis Ydri—currently a professor of theoretical particle physics, teaching at the Institute of Physics, Badji Mokhtar Annaba University, Algeria—received his PhD from Syracuse University, New York, USA and in 2011 his Habilitation from Annaba University, Annaba, Algeria.

His doctoral work, titled ‘Fuzzy Physics’, was supervised by Professor A P Balachandran. Professor Ydri is a research associate at the Dublin Institute for Advanced Studies, Dublin, Ireland, and a regular ICTP associate at the Abdus Salam Center for Theoretical Physics, Trieste, Italy. His post-doctoral experience comprises a Marie Curie fellowship at Humboldt University Berlin, Germany, and a Hamilton fellowship at the Dublin Institute for Advanced Studies, Ireland.

His current research directions include: the gauge/gravity duality; the renormalization group method in matrix and noncommutative field theories; non-commutative and matrix field theory; quantum gravity as emergent geometry, emergent gravity and emergent cosmology from matrix models.

Other interests include string theory, causal dynamical triangulation, Hořava–Lifshitz gravity, and supersymmetric gauge theory in four dimensions.

He has recently published three books. His hobbies include reading philosophic works and history of science.