Multiple Grating Moiré

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Moiré fringe patterns can be formed not only in the conventional way, by superposition of two gratings, but also by superposition of several gratings. The advantage is that the moiré fringe spacing and orientation can be independently adjusted. Experiments illustrating this are shown and the zone-plate features of superposed quasiperiodic structures are demonstrated. The microstructure of the multiple grating moiré patterns has favorable properties in applications with visual inspection.

§1. Introduction and Objective

One important factor when making optical measurements is the nature of the light available. Several classes of test methods in optics require operation in coherent light because they rely on interference or diffraction phenomena. Some examples are Schlieren, phase contrast, and interference methods. However, optical disturbances and inhomogeneities can also be studied using incoherent light. The counterpart to interferometry is then the use of moiré techniques.

Moiré methods have been extensively used since 1874 when Rayleigh suggested their use for optical testing.[1,2] Numerous techniques have been developed since then for their use in both incoherent and coherent light.[3] One of the shortcomings of moiré procedures has been the lack of independent parameters. We will see, here, how moiré techniques can be made more flexible by incorporating more than two separate structures to form the composite fringes. I will emphasize that zone-plate-shaped moiré formations are achieved when superposing a multiplicity of distorted grid patterns.

§2. Independent Variation of Frequency and Orientation

When moiré fringes are formed by superposing two gratings, the spatial frequency and orientation of the pattern cannot be varied independently. This is clearly visualized in a Fourier space (sometimes called reciprocal space) representation. The individual gratings then appear as vectors. Their magnitudes correspond to the frequencies and the arguments to the orientations of the grid structures. Figure 1(a) illustrates the case with two superposed gratings, r_1 and r_2. Moiré is a low-pass

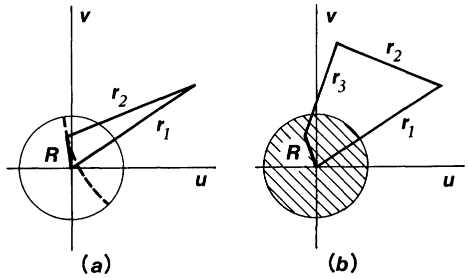

Fig. 1. Fourier space representation of moiré patterns, R; (a) the two-grating case, and (b) the three-grating case. The central circular area represents fringe spacings above the visual threshold.

phenomenon, *e.g.* visual inspection, which means that only frequencies below a certain value are resolvable. The corresponding area in the figure is indicated by a circle, and R is the only beat frequency inside this circle. Figure 1(a) shows that a relative rotation between the gratings will cause R to describe an arc, i.e. the only combinations of frequency and orientation of the moiré pattern that can be achieved.

One way to drastically increase the possible combinations of frequency and orientation of the moiré is to include additional gratings in its formation.[4] The case with three gratings is illustrated in Fig. 1(b). Now, the relative rotations among the gratings can be adjusted to give any R value located inside the circle of resolvable frequencies. We have achieved a situation where the frequency and orientation of the moiré fringes can be independently influenced.

The Fourier-space representation used here was found useful in predicting general characteristics of moiré patterns when a multiplicity of gratings are involved. The simple relationship, $R=\sum r_n$, holds. The diagram of Fig. 1, of course, shows the conditions in one spatial point only. However, the procedure for obtaining information about the spatial variation of R is in most applications simplified because of the undisturbed nature of a couple of the gratings involved.

§3. Generalizations in Display of Moiré Patterns

For testing, a regular grid structure may be photographed via the optical surface, material, or component to be examined. Information about the test subject is then contained as distortion introduced in the image of the original undistorted grating. Superposition of this grating and another regular grating with about the same period results in moiré fringes. With proper rotational adjustment of the gratings, the information wanted is then displayed on a convenient scale as fringe deviation. We mentioned previously some of the restrictions on the possible combinations of frequency and orientation of these moiré fringes. We also showed how complete flexibility is obtained by superposing more than two gratings.

We can illustrate this with some of our experimental results. The patterns shown in Fig. 2 were formed by superposition of three gratings. A distorted grid structure was made by imaging a Ronchi ruling through a pair of lenses. The upper one was an aspheric and the lower one a conventional spherical lens. The others were two identical Ronchi rulings. All had about the same frequency. A light table was used for illumination. The different moiré fringe configurations in Fig. 2 were obtained by different relative settings among the three grating structures. Only slight rotational changes had to be introduced between the patterns shown because the frequencies of the gratings were about five times higher than the highest moiré fringe frequency (Fig. 2(c)). Figure 2 shows that any combination of frequency and orientation in the moiré fringes is possible with a three-grating superposition.

The procedure demonstrated here is, of course, equivalent of formation of moiré between a structure r_1 with information and a regular grid r_2+r_3. In addition to its frequency variation, we can also vary its orientation. This extra parameter was found especially useful since it allows continuous changes of the displayed reference fringe system.

§4. Some Characteristics of Moiré Patterns

Using the Fourier-space representation, we may get some indications of the local spatial configurations of moiré fringe systems. The contributions of the individual structures can be

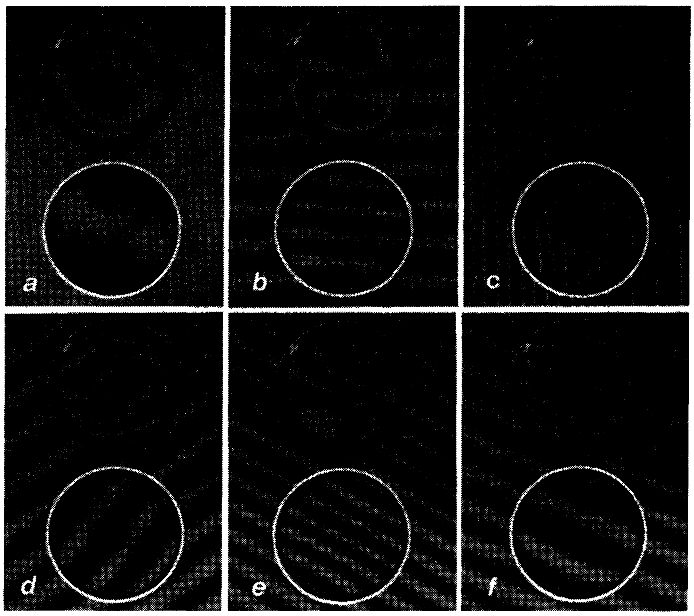

Fig. 2. Illustration of the flexibility of three-grating moiré. The three individual gratings (one with local distortions and two Ronchi rulings) have the same spatial frequency. (a)–(f) show slightly different settings of the two Ronchi rulings.

expressed as Taylor series expansions around a specific location. These series are usually strongly convergent, and only the very first terms of low powers of frequency have appreciable coefficients. Thus, the pattern resulting from superposition of several structures can be represented by terms including zeroth, first, $\cdots$ powers. The first two terms describe an off-axis zone-plate pattern.

When the moiré pattern is formed by three or more individual structures, it is possible as described in §2, to cancel the contributions from the constant terms. Then, terms of first and higher order powers are left. However, the coefficients decrease sharply with increasing power so a pattern with linear variation in frequency occurs. Thus if the superimposed gratings contain disturbances (information) in two dimensions, the moiré fringe system is of Fresnel zone-plate shape.

We may conclude from this simple reasoning that because of the low-pass nature of the moiré phenomenon, the pattern formed will possess some typical characteristics. A zone-plate moiré fringe system is formed independent of the variation in the individual grid structures as long as this variation is reasonably continuous and monotonic. The center of the zone-plate can be selected by adjusting the relative orientations among the several grids to get zero frequency at the position wanted. Both elliptical and hyperbolic shaped zone-plates are to be expected depending on the relative sense of variation of frequency in different directions.[4)]

§5. Zone-Plate Features of Moiré Patterns

The zone-plate character of moiré patterns is easy to demonstrate using a combination of three grid structures. The disturbance (information) in a structure is usually contained as a combination of changes in both frequency and orientation of the grating lines. As concluded in §4, the zone-plate feature is, in a first approximation, not influenced by the particular kind of variation in the individual structures. This is illustrated here by choosing the two extreme types of grating structures: 1) gratings in which only the frequency varies, and 2) only the orientation.

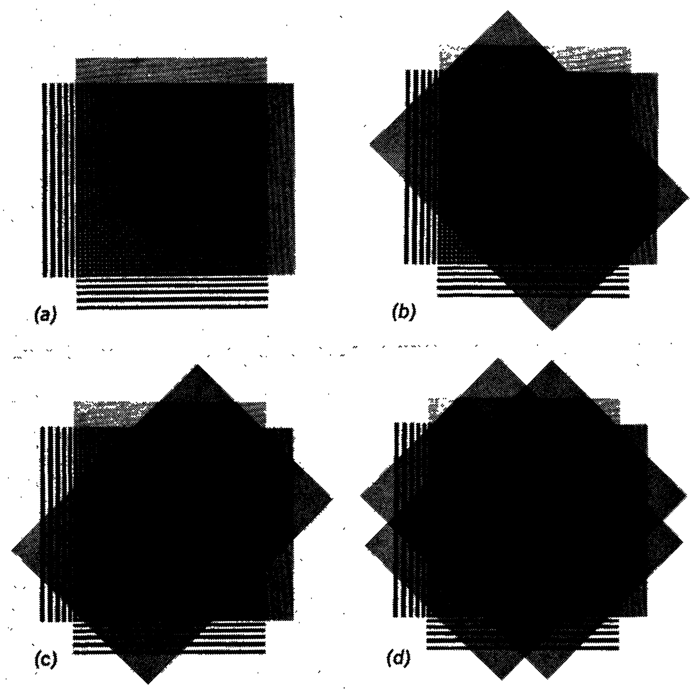

Fig. 3. Formation of two-dimensional, zone-plate patterns from two crossed one-dimensional Fresnel zone-plates: (a) the crossed, one-sided Fresnel zone plates; (b) a regular Ronchi ruling superposed on the configuration in (a); (c) superposition of the same Ronchi ruling as in (b) but rotated 90°; (d) superposition of two crossed Ronchi rulings on the configuration in (a).

Figure 3 shows the case with gratings of varying frequency. Two crossed, one-sided, one dimensional Fresnel zone-plates (gratings with linear variation in frequency) were used. Figure 3(a) shows that the moiré pattern from the crossed Fresnel patterns alone has frequencies too high to be apprehended. The fringe pattern in Fig. 3(b) is obtained if a third structure, a regular Ronchi ruling, is placed onto the crossed structures of Fig. 3(a) and adjusted to give the moiré pattern zero frequency in the center. The circular zone-plate in Fig. 3(b) can be converted into a hyperbolic one by reversing one of the Fresnel zone-plate patterns. A 90° rotation of the Ronchi ruling is, of course, equivalent. The result is shown in Fig. 3(c). If two crossed Ronchi rulings are superposed onto the two crossed Fresnel zone-plate patterns, two independent beat patterns will arise with apprehendable frequencies (see Fig. 3(d)). These are the patterns from Fig. 3(b) and (c).

The other extreme case is when only the orientation in the patterns varies. As a specific example, let us look at moiré patterns formed with grid structures with equally spaced, sinusoidally curved lines (pattern No. 12 of Edmund Sci. moiré kit). Figure 4 shows some moiré patterns obtained with different combinations of these "sinusoidal" gratings and regular Ronchi rulings. The superposition of one "sinusoidal" grating and two Ronchi rulings is shown in Fig. 4(a). The grid structures are arranged so that the spatial variation of the "sinusoidal" grating is displayed. This figure shows that the peak-to-peak fringe deviation in the "sinusoidal" grating is only of the order of the fringe spacing. Two "sinusoidal" gratings and one Ronchi ruling create the moiré pattern in Fig. 4(b). Three superposed "sinusoidal" gratings give the moiré configuration in Fig. 4(c). Here, very few fringes are formed in the circular patterns due to the small amplitude of the "sinusoidal" gratings and the fact that the variation of the grating line orientations is not monotonic. In fact, circular and hyperbolic patterns occur alternately. Thus, each individual pattern in Fig. 4(c) has only two fringes, and it makes no sense to characterize the patterns according to their relative fringe spacings. However, I hope the tendencies of the patterns are clear, *e.g.* the formation of closed fringe systems when grid structure distortions occur in two dimensions.

Further examples of the formation of zone-plate shaped moiré patterns from superposition of several quasi-periodic grid structures are shown in ref. 4.

§6. Further Comments on Multiple Grating Moiré

The familiar microstructure in the moiré pattern from two Ronchi-ruling type structures consists of rows of diamond shaped areas. Each period is made up of one row and the longest diagonal of the diamonds is perpendicular to the moiré fringes. On the other hand, the microstructures of the multiple structure moiré pattern, differ from these characteristics in several respects. The micropatterns are now composed of polygons which change geometry even within the same moiré fringe. Furthermore, the period of the moiré fringes is now built up of several rows of microstructure components. This is illustrated for the case of three superposed grid structures in Fig. 5.

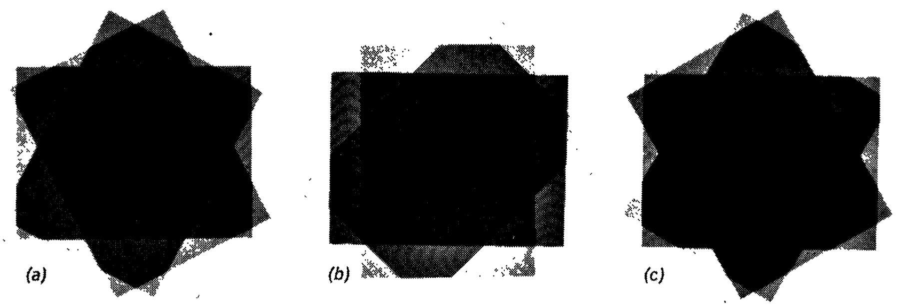

Fig. 4. Demonstration of three-structure moiré with gratings in which the orientation of the lines varies spatially. (a) one "sinusoidal" line grating and two Ronchi rulings; (b) two "sinusoidal" line gratings and one Ronchi ruling; (c) three "sinusoidal" line gratings.

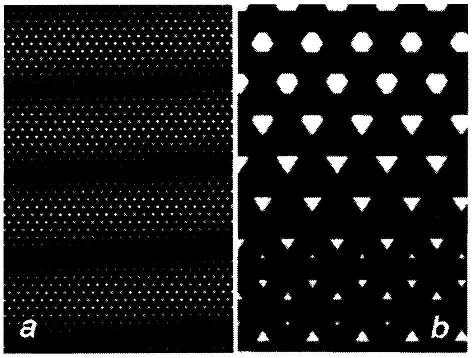

Fig. 5. Visualization of characteristics of three-grating moiré: (a) shows three superposed Ronchi rulings; and (b) a magnified portion of the pattern in (a).

The contrast of the moiré fringes decreases with the number of individual structures superposed to form them. However, four grids still give acceptable contrast as was shown in ref. 4. A number of moiré beat patterns occur when n structures are superposed; there are $\{n!/m!(n-m)!\}2^{m-1}$ beats between combinations of m structures of the total n superposed structures. For example in Fig. 3(d), two of the sixteen moiré beat patterns from combinations of three ($m=3$) of the four ($n=4$) superposed structures have low enough frequencies to be clearly apprehended.

As indicated in connection with Fig. 1(b), only fringe patterns with frequencies corresponding to the circular shaded area are visually apprehended. However, another factor besides the frequency is also important and that is the relative contrast among the moiré beat fringes. In this respect, multiple structure moiré as shown here have some characteristics which are advantageous in applications. When the orientations among the superposed gratings are spread relatively even (cf. Fig. 5 with about 60° between gratings) the local microstructure is rather isotropic. Among the moiré beat frequencies in general, only one is low, and the corresponding fringe pattern is seen with high contrast. The others are of low contrast, and their angular orientations are evenly distributed.

§7. Conclusions and Applications

The addition of a third grid structure when forming moiré patterns by superposition of periodic and/or quasiperiodic structures increases the number of independent parameters. For example, the moiré fringe spacing and fringe orientation can be varied continuously as well as independently. With this extension, moiré techniques possess flexibility comparable to interferometric procedures for displaying optical information. From a practical point of view, the multiple structure moiré patterns were found to be contrasty enough for visual inspection. Furthermore, the character of the microstructure is such that averaging procedures are in general not required. In conclusion: the properties of this type of moiré procedure make it highly recommended for applications in metrology.

The examples also indicated zone-plate features in moiré patterns formed by superposition of quasiperiodic grid structures. This general characteristic, on the other hand, makes it difficult to extract information about the individual structures, thus caution should be used in drawing generalized conclusions. However, knowledge of the individual gratings and use of Fourier space representation allow easy prediction and information of the corresponding moiré configurations.

Acknowledgement

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