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Relating Gribov-Zwanziger theory to effective Yang-Mills theory

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Abstract – We consider the Gribov-Zwanziger (GZ) theory with appropriate horizon term which exhibits the nilpotent BRST invariance. This infinitesimal BRST transformation has been generalized by allowing the parameter to be finite and field dependent (FFBRST). By constructing appropriate finite field-dependent parameter we show that the generating functional of GZ theory with horizon term is related to that of Yang-Mills (YM) theory through FFBRST transformation.



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Introduction. – In order to quantize a gauge theory it is necessary to eliminate the redundant degrees of freedom from the functional integral representation of the generating functional. This can be done by modifying the generating functional with the addition of a gauge fixing term [1]. However in non-Abelian gauge theories even after gauge fixing the redundancy of gauge fields is not completely removed in certain gauges for large gauge fields (Gribov problem) [2]. The non-Abelian gauge theories in those gauges contain so-called Gribov copies. Gribov copies play a crucial role in the infrared (IR) regime while it can be neglected in the perturbative ultraviolet (UV) regime [2–4]. This topic has become very exciting currently due to the fact that color confinement is closely related to the asymptotic behaviour of the ghost and gluon propagators in deep IR regime [5].

In order to resolve the Gribov problem, Gribov and Zwanziger proposed a theory, which restricts the domain of integration in the functional integral within the first Gribov horizon [3]. The restriction to the Gribov region Ω can be achieved by adding a nonlocal term, commonly known as horizon term, to the YM action [3,4,6].

The Kugo-Ojima (KO) criterion for color confinement [7] is based on the assumption of an exact BRST invariance of YM theory in the manifestly covariant gauge. But the YM action restricted in the Gribov region (*i.e.* GZ action) does not exhibit the usual BRST invariance, due to the presence of the nonlocal horizon term [8]. Recently, a nilpotent BRST transformation which leaves the GZ action invariant has been obtained and can be applied to KO analysis of the GZ theory [9]. The BRST symmetry in the presence of the Gribov horizon has

great applicability in order to solve the nonperturbative features of confining YM theories [10,11], where the soft breaking of the BRST symmetry exhibited by the GZ action can be converted into an exact invariance [12]. Such a modification is very useful in order to evaluate the vacuum expectation value (VEV) of BRST exact quantity.

In this work we generalize the nilpotent BRST transformation introduced in ref. [9] for GZ theory by allowing the parameter to be finite field-dependent following the method developed by Joglekar and Mandal for pure YM theory [13]. Such a generalized BRST (FFBRST) transformation is nilpotent and leaves the effective action invariant. However, being finite in nature such a transformation does not leave the path integral measure and hence the generating functional invariant. By constructing an appropriate finite field-dependent parameter we show that such FFBRST transformation relates the generating functional for GZ theory to the generating functional in YM theory.

The paper is organized in the following manner. In the second section we illustrate some of the essential features in GZ theory. In the third section we discuss the nilpotent BRST transformation of the multiplicative renormalizable GZ theory. The fourth section is devoted to the discussion of finite field-dependent BRST transformation in the Euclidean space. Connection of GZ theory and YM theory is established in the fifth section. The last section contains discussions and conclusions.

GZ theory: brief introduction. – It has been shown in ref. [4] that the restriction to the Gribov region Ω (defined in such a way that the Faddeev-Popov (FP) operator is strictly positive *i.e.*

$$\Omega \equiv \{A_\mu^a, \partial_\mu A_\mu^a = 0, \mathcal{M}^{ab} > 0\}, \quad (1)$$

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can be imposed by adding a nonlocal term S_h , given in eq. (4) below, to the standard YM action

$$S_{YM} = S_0 + S_{GF+FP}, \quad (2)$$

where S_0 is the kinetic part and S_{GF+FP} is the ghost and gauge (Landau gauge) fixing part of the YM action respectively,

$$S_0 = \int d^4x \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right], \quad (3)$$

$$S_{GF+FP} = \int d^4x \left[B^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu \mathcal{D}_\mu^{ab} c^b \right].$$

The nonlocal term in the 4-dimensional Euclidean space is written as

$$S_h = \int d^4x h(x), \quad (4)$$

where the integrand $h(x)$ is the horizon function. There exist different choices for the horizon function in the literature [9]. One such horizon term is

$$h_1(x) = \gamma^4 \int d^4y g^2 f^{abc} A_\mu^b(x) (\mathcal{M}^{-1})^{ce}(x, y) f^{ade} A_\mu^d(y). \quad (5)$$

$(\mathcal{M}^{-1})^{ce}$ is the inverse of the Faddeev-Popov operator $\mathcal{M}^{ab} \equiv -\partial_\mu \mathcal{D}_\mu^{ab} = -\partial_\mu (\partial_\mu \delta^{ab} + g f^{acb} A_\mu^c)$. The Gribov parameter γ can be obtained in a consistent way by solving a gap equation (also known as horizon condition)

$$\langle h(x) \rangle = 4(N^2 - 1), \quad (6)$$

where N is the number of colors. Another horizon term which gives the correct multiplicative renormalizability of the GZ theory is given as [9,14]

$$h_2(x) = \lim_{\gamma(x) \rightarrow \gamma} \int d^4y \left[(\mathcal{D}_\mu^{ac}(x) \gamma^2(x)) (\mathcal{M}^{-1})^{ce}(x, y) \times (\mathcal{D}_\mu^{ae}(y) \gamma^2(y)) \right]. \quad (7)$$

The nonlocal term (4) corresponding to the horizon function (7) can be localized as [3,4]

$$e^{-S_{h_2}} = \int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} \mathcal{D}\omega \mathcal{D}\bar{\omega} e^{S_{loc}}, \quad (8)$$

with

$$S_{loc} = \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - \gamma^2 \mathcal{D}_\mu^{ca} (\varphi_\mu^{ac}(x) \bar{\varphi}_\mu^{ac}(x)) \right], \quad (9)$$

where a pair of complex conjugate bosonic fields $(\bar{\varphi}_i^a, \varphi_i^a) = (\bar{\varphi}_\nu^{ac}, \varphi_\nu^{ac})$ and anticommuting auxiliary fields $(\omega_i^a, \bar{\omega}_i^a) = (\omega_\nu^{ac}, \bar{\omega}_\nu^{ac})$, with composite index $i = (\nu, c)$, has been introduced. As at the level of the action, total derivatives are always neglected, S_{loc} becomes

$$S_{loc} = \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - \gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc}(x) + \bar{\varphi}_\mu^{bc}(x)) \right]. \quad (10)$$

Here it is concluded that at the local level horizon functions (5) and (7) are same. So that the localized GZ action becomes

$$S_{GZ} = S_{YM} + S_{loc} = S_{YM} + \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - \gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \right]. \quad (11)$$

Thus the local action S_{GZ} and the nonlocal action $S_{YM} + S_h$ are related as the following:

$$\int [D\phi_1] e^{-\{S_{YM} + S_{h_2}\}} = \int [D\phi] e^{-S_{GZ}}, \quad (12)$$

with $\int [D\phi_1] \equiv \int [DADBDCD\bar{c}]$ and $\int [D\phi] \equiv \int [DADBDCD\bar{c}D\varphi D\bar{\varphi} D\omega D\bar{\omega}]$. By differentiating eq. (12) with respect to γ^2 and noting $\langle \partial_\mu \varphi_\mu^{aa} \rangle = \langle \partial_\mu \bar{\varphi}_\mu^{aa} \rangle = 0$, the horizon condition in eq. (6) is recast as

$$\langle g f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \rangle + 8\gamma^2 (N^2 - 1) = 0. \quad (13)$$

The horizon condition can further be written as [9,12]

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0, \quad (14)$$

with Γ , the quantum action defined as

$$e^{-\Gamma} = \int [D\phi] e^{-S_{GZ}}. \quad (15)$$

We see that the horizon condition (14) is equivalent to

$$\langle 0 | g f^{abc} A_\mu^a \varphi_\mu^{bc} | 0 \rangle + \langle 0 | g f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} | 0 \rangle = -8\gamma^2 (N^2 - 1), \quad (16)$$

which, owing to the discrete symmetry of the action S_{GZ}

$$\begin{aligned} \bar{\varphi}_\mu^{ac} &\rightarrow \varphi_\mu^{ac}, \\ \varphi_\mu^{ac} &\rightarrow \bar{\varphi}_\mu^{ac}, \\ B^a &\rightarrow (B^a - g f^{amn} \bar{\varphi}_\mu^{mc} \varphi_\mu^{nc}), \end{aligned} \quad (17)$$

becomes

$$\langle 0 | g f^{abc} A_\mu^a \varphi_\mu^{bc} | 0 \rangle = \langle 0 | g f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} | 0 \rangle = -4\gamma^2 (N^2 - 1). \quad (18)$$

Further the constant term $4\gamma^4 (N^2 - 1)$ is introduced in S_{GZ} , to incorporate the effect of horizon condition in the action as

$$S_{GZ} = S_{YM} + \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - \gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) - 4(N^2 - 1)\gamma^4 \right]. \quad (19)$$

For the GZ action to be renormalizable, it is crucial to shift the field ω_i^a , [4]

$$\omega_i^a(x) \rightarrow \omega_i^a + \int d^4y (\mathcal{M}^{-1})^{ab}(x, y) g f^{bkl} \partial_\mu [\mathcal{D}_\mu^{ke} c^e(y) \varphi_i^l(y)], \quad (20)$$

so that the complete GZ action becomes

$$S_{GZ} = S_{YM} + \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c - \gamma^2 g \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} + \frac{4}{g} (N^2 - 1) \gamma^2 \right) \right], \quad (21)$$

which is multiplicative renormalizable.

The nilpotent BRST transformations of GZ action. – The complete GZ action after localizing the nonlocal horizon term in D -dimensional Euclidean space can be recast as

$$S_{GZ} = S_{exact} + S_\gamma \quad (22)$$

with S_{exact} , the BRST exact action and S_γ , the action for horizon term, defined as [9]

$$S_{exact} = S_{YM} + \int d^4x \left[\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b - g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right], \quad (23)$$

$$S_\gamma = -\gamma^2 g \int d^4x \left[f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \bar{\varphi}_\mu^{bc} + \frac{4}{g} (N^2 - 1) \gamma^2 \right]. \quad (24)$$

The conventional BRST transformation for all the fields is given by

$$\begin{aligned} \delta_b A_\mu^a &= -\mathcal{D}_\mu^{ab} c^b \Lambda, & \delta_b c^a &= \frac{1}{2} g f^{abc} c^b c^c \Lambda, \\ \delta_b \bar{c}^a &= B^a \Lambda, & \delta_b B^a &= 0, \\ \delta_b \varphi_i^a &= -\omega_i^a \Lambda, & \delta_b \omega_i^a &= 0, \\ \delta_b \bar{\omega}_i^a &= \bar{\varphi}_i^a \Lambda, & \delta_b \varphi_i^a &= 0, \end{aligned} \quad (25)$$

where Λ is usual infinitesimal BRST parameter. But one can check that the BRST symmetry is broken softly for the GZ action [3],

$$\begin{aligned} \delta_b S_{GZ} &= \delta_b (S_{exact} + S_\gamma) = \delta_b S_\gamma = \\ &= \gamma^2 g \int d^4x f^{abc} (A_\mu^a \omega_\mu^{bc} - (\mathcal{D}_\mu^{am} c^m)(\bar{\varphi}_\mu^{bc} + \varphi_\mu^{bc})), \end{aligned} \quad (26)$$

the breaking is due to the presence of the γ -dependent term, S_γ .

To discuss the renormalizability of S_{GZ} , the breaking is treated as a composite operator to be introduced into the action by means of a suitable set of external sources [9].

Thus embedding the S_γ in to a larger action with introducing 3 doublets of sources $(U_\mu^{ai}, M_\mu^{ai}), (V_\mu^{ai}, N_\mu^{ai})$ and (T_μ^{ai}, R_μ^{ai}) , as

$$\begin{aligned} \Sigma_\gamma &= \delta_b \int d^4x \left(-U_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b - V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b - U_\mu^{ai} V_\mu^{ai} \right. \\ &\quad \left. + g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} \bar{\omega}_i^c \right) = \\ &= \int d^4x \left(-M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b - g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right. \\ &\quad \left. + U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b - N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b - V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b \right. \\ &\quad \left. + g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c - M_\mu^{ai} V_\mu^{ai} + U_\mu^{ai} N_\mu^{ai} \right. \\ &\quad \left. - g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c + g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \right), \end{aligned} \quad (27)$$

whereas the sources involved $M_\mu^{ai}, V_\mu^{ai}, R_\mu^{ai}$ are commuting and $U_\mu^{ai}, N_\mu^{ai}, T_\mu^{ai}$ are fermionic in nature. The above action is invariant under following BRST transformation:

$$\begin{aligned} \delta_b U_\mu^{ai} &= M_\mu^{ai} \Lambda, & \delta_b M_\mu^{ai} &= 0, \\ \delta_b V_\mu^{ai} &= -N_\mu^{ai} \Lambda, & \delta_b N_\mu^{ai} &= 0, \\ \delta_b T_\mu^{ai} &= -R_\mu^{ai} \Lambda, & \delta_b R_\mu^{ai} &= 0. \end{aligned} \quad (28)$$

Therefore, the broken BRST has been restored at the cost of introducing new sources. The different quantum numbers (to study the system properly) of fields and sources, involved in this theory, are discussed in ref. [9]. Still we do not want to change our original theory (24). Therefore, at the end, we have to fix the sources equal to the following values:

$$\begin{aligned} U_\mu^{ai}|_{phys} &= N_\mu^{ai}|_{phys} = T_\mu^{ai}|_{phys} = 0, \\ M_{\mu\nu}^{ab}|_{phys} &= V_{\mu\nu}^{ab}|_{phys} = R_{\mu\nu}^{ab}|_{phys} = \gamma^2 \delta^{ab} \delta_{\mu\nu}. \end{aligned} \quad (29)$$

It follows that $\Sigma_\gamma|_{phys} = S_\gamma$.

The generating functional for the effective GZ action in Euclidean space is defined as

$$Z_{GZ} = \int [D\phi] e^{-S_{GZ}}, \quad (30)$$

where ϕ is generic notation for all fields used in GZ action.

FFBRST transformation in the Euclidean space.

– The properties of the usual BRST transformations do not depend on whether the parameter Λ is i) finite or infinitesimal, ii) field dependent or not, as long as it is anticommuting and space-time independent. Keeping this in mind, Joglekar and Mandal introduced finite field-dependent BRST transformation (FFBRST) [13], which has found many applications in gauge field theories [15–25]. These observations give us a freedom to generalize the nilpotent BRST transformations in eqs. (25) and (28) by the parameter, Λ finite and field dependent without affecting its properties. To generalize the BRST transformations we start with making the infinitesimal parameter field

dependent by introducing a parameter κ ($0 \leq \kappa \leq 1$) and making all the fields, $\phi(x, \kappa)$, κ dependent such that $\phi(x, \kappa=0) = \phi(x)$ and $\phi(x, \kappa=1) = \phi'(x)$, the transformed field.

The usual infinitesimal BRST transformations, thus can be written generically as

$$d\phi(x, \kappa) = \delta_b[\phi(x, \kappa)]\Theta'[\phi(x, \kappa)]d\kappa, \quad (31)$$

where the $\Theta'[\phi(x, \kappa)]d\kappa$ is the infinitesimal but field-dependent parameter. The generalized BRST transformations with the finite field-dependent parameter then can be constructed by integrating such infinitesimal transformations from $\kappa=0$ to $\kappa=1$, to obtain

$$\phi' \equiv \phi(x, \kappa=1) = \phi(x, \kappa=0) + \delta_b[\phi(x)]\Theta[\phi(x)], \quad (32)$$

where

$$\Theta[\phi(x)] = \int_0^1 d\kappa' \Theta'[\phi(x, \kappa')], \quad (33)$$

is the finite field-dependent parameter. Following this method, the modified BRST transformation, in eq. (25), is generalized such that the parameter is finite and field dependent.

Now we show that such offshell nilpotent BRST transformations with finite field-dependent parameter are symmetry of the effective action in eq. (21). However, the path integral measure $[D\phi]$ in eq. (30) is not invariant under such transformations as the BRST parameter is finite.

The Jacobian of the path integral measure in Euclidean space for such transformations can be evaluated for some particular choices of the finite field-dependent parameter, $\Theta[\phi(x)]$, as

$$D\phi' = J(\kappa)D\phi. \quad (34)$$

The Jacobian, $J(\kappa)$ in the Euclidean space can be replaced (within the functional integral) as

$$J(\kappa) \rightarrow \exp[-S_1[\phi(x, \kappa)]], \quad (35)$$

iff the following condition is satisfied [13]:

$$\int D\phi(x) \left[\frac{1}{J} \frac{dJ}{d\kappa} + \frac{dS_1[\phi(x, \kappa)]}{d\kappa} \right] \exp[-(S_{GZ} + S_1)] = 0, \quad (36)$$

where $S_1[\phi]$ is local functional of fields.

The infinitesimal change in the $J(\kappa)$ can be written as

$$\frac{1}{J} \frac{dJ}{d\kappa} = - \int d^4x \left[\pm \delta_b \phi(x, \kappa) \frac{\partial \Theta'[\phi(x, \kappa)]}{\partial \phi(x, \kappa)} \right], \quad (37)$$

where the \pm sign refers to whether ϕ is a bosonic or a fermionic field.

Now, we generalize the BRST transformation given in eqs. (25) and (28) by making usual BRST parameter finite

and field dependent as

$$\begin{aligned} \delta_b A_\mu^a &= -\mathcal{D}_\mu^{ab} c^b \Theta, & \delta_b c^a &= \frac{1}{2} g f^{abc} c^b c^c \Theta, \\ \delta_b \bar{c}^a &= B^a \Theta, & \delta_b B^a &= 0, \\ \delta_b \varphi_i^a &= -\omega_i^a \Theta, & \delta_b \omega_i^a &= 0, \\ \delta_b \bar{\omega}_i^a &= \bar{\varphi}_i^a \Theta, & \delta_b \bar{\varphi}_i^a &= 0, \\ \delta_b U_\mu^{ai} &= M_\mu^{ai} \Theta, & \delta_b M_\mu^{ai} &= 0, \\ \delta_b V_\mu^{ai} &= -N_\mu^{ai} \Theta, & \delta_b N_\mu^{ai} &= 0, \\ \delta_b T_\mu^{ai} &= -R_\mu^{ai} \Theta, & \delta_b R_\mu^{ai} &= 0, \end{aligned} \quad (38)$$

where Θ is a finite, field-dependent, anticommuting and space-time-independent parameter. One can easily check that the above FFBRST transformation is also symmetry of the effective GZ action (S_{GZ}).

A mapping between GZ theory and YM theory.

– In this section we establish the connection between the theories with GZ action and YM action by using the finite field-dependent BRST transformation. In particular, we show that the generating functional for GZ theory in the path integral formulation is directly related to that of YM theory with a proper choice of the finite field-dependent BRST transformation. The nontrivial Jacobian of the path integral measure is responsible for such a connection. For this purpose we choose a finite field-dependent parameter Θ obtainable from

$$\begin{aligned} \Theta' &= \int d^4x \left[\bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - U_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b - V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b \right. \\ &\quad \left. - U_\mu^{ai} V_\mu^{ai} + T_\mu^{ai} g f^{abc} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \right] \end{aligned} \quad (39)$$

using eq. (33). The infinitesimal change in Jacobian for above Θ' using eq. (37) is calculated as

$$\begin{aligned} \frac{1}{J} \frac{dJ}{d\kappa} &= - \int d^4x \left[-\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right. \\ &\quad + g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c + M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b - U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b \\ &\quad + g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c + N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b \\ &\quad - g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c + M_\mu^{ai} V_\mu^{ai} - U_\mu^{ai} N_\mu^{ai} \\ &\quad \left. + g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c - g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \right]. \end{aligned} \quad (40)$$

Now the Jacobian for the path integral measure in the generating functional (30) can be replaced by e^{-S_1} iff condition (36) is satisfied. We consider an ansatz for S_1 as

$$\begin{aligned} S_1 &= \int d^4x \left[\chi_1(\kappa) \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \chi_2(\kappa) \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right. \\ &\quad + \chi_3(\kappa) g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c + \chi_4(\kappa) M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b \\ &\quad + \chi_5(\kappa) U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b + \chi_6(\kappa) g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \\ &\quad + \chi_7(\kappa) N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + \chi_8(\kappa) V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b \\ &\quad + \chi_9(\kappa) g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c + \chi_{10}(\kappa) M_\mu^{ai} V_\mu^{ai} \\ &\quad + \chi_{11}(\kappa) U_\mu^{ai} N_\mu^{ai} + \chi_{12}(\kappa) g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\ &\quad \left. + \chi_{13}(\kappa) g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \right]. \end{aligned} \quad (41)$$

where $\chi_j(\kappa)$ ($j = 1, 2, \dots, 12, 13$) are arbitrary functions of κ and satisfy the following initial conditions:

$$\chi_j(\kappa = 0) = 0. \quad (42)$$

The condition (36) with the above S_1 leads to

$$\begin{aligned} & \int D\phi(x) e^{-(S_{eff} + S_1)} [\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b (\chi'_1 + 1) \\ & + \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b (\chi'_2 - 1) + g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c (\chi'_3 - 1) \\ & + M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b (\chi'_4 - 1) + U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b (\chi'_5 - 1) \\ & + g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c (\chi'_6 + 1) + N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b (\chi'_7 - 1) \\ & + V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b (\chi'_8 - 1) + g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c (\chi'_9 + 1) \\ & + M_\mu^{ai} V_\mu^{ai} (\chi'_{10} - 1) + U_\mu^{ai} N_\mu^{ai} (\chi'_{11} + 1) \\ & + g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c (\chi'_{12} - 1) \\ & + g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c (\chi'_{13} + 1) \\ & + g f^{abc} \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{bd} c^d \varphi_i^c \Theta(\chi_1 + \chi_3) \\ & - \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \Theta(\chi_1 + \chi_2) \\ & - g f^{abc} \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{bd} c^d \omega_i^c \Theta(\chi_2 - \chi_3) \\ & + g f^{abc} M_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \Theta(\chi_4 - \chi_5) \\ & - M_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b \Theta(\chi_4 + \chi_6) \\ & + g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \omega_i^c \Theta(\chi_5 + \chi_6) \\ & - N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b \Theta(\chi_7 - \chi_8) \\ & - g f^{abc} N_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \Theta(\chi_7 + \chi_9) \\ & + g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \Theta(\chi_8 + \chi_9) \\ & + M_\mu^{ai} N_\mu^{ai} \Theta(\chi_{10} + \chi_{11}) \\ & + g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \Theta(\chi_{12} + \chi_{13})] = 0, \end{aligned} \quad (43)$$

where the prime denotes the differentiation with respect to κ . Equating the coefficient of the terms $\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b$, $\bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b$, $g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c$, $M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b$, $U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b$, $g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c$, $N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b$, $V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b$, $g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c$, $M_\mu^{ai} V_\mu^{ai}$, $U_\mu^{ai} N_\mu^{ai}$, $g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c$ and $g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c$ from both sides of the above condition, we get following differential equations:

$$\begin{aligned} \chi'_1 + 1 &= 0, & \chi'_2 - 1 &= 0, & \chi'_3 - 1 &= 0, & \chi'_4 - 1 &= 0, \\ \chi'_5 - 1 &= 0, & \chi'_6 + 1 &= 0, & \chi'_7 - 1 &= 0, & \chi'_8 - 1 &= 0, \\ \chi'_9 + 1 &= 0, & \chi'_{10} - 1 &= 0, & \chi'_{11} + 1 &= 0, & \chi'_{12} - 1 &= 0, \\ \chi'_{13} + 1 &= 0. \end{aligned} \quad (44)$$

The Θ -dependent terms will cancel separately and comparing the coefficients of Θ -dependent terms, we

$$\begin{aligned} \chi_1 + \chi_2 &= \chi_1 + \chi_3 = \chi_2 - \chi_3 = \chi_4 - \chi_5 = 0, \\ \chi_4 + \chi_6 &= \chi_5 + \chi_6 = \chi_7 - \chi_8 = \chi_7 + \chi_9 = 0, \\ \chi_8 + \chi_9 &= \chi_{10} + \chi_{11} = \chi_{12} + \chi_{13} = 0. \end{aligned} \quad (45)$$

The particular solution of eq. (44) subjected to the condition (42) and eq. (45) is

$$\begin{aligned} \chi_1 &= -\kappa, & \chi_2 &= \kappa, & \chi_3 &= \kappa, & \chi_4 &= \kappa, \\ \chi_5 &= \kappa, & \chi_6 &= -\kappa, & \chi_7 &= \kappa, & \chi_8 &= \kappa, \\ \chi_9 &= -\kappa, & \chi_{10} &= \kappa, & \chi_{11} &= -\kappa, & \chi_{12} &= \kappa, \\ \chi_{13} &= -\kappa. \end{aligned} \quad (46)$$

Therefore, the expression for S_1 in term of κ is

$$\begin{aligned} S_1 &= \int d^4x \left[-\kappa \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \kappa \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right. \\ & + \kappa g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c + \kappa M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b \\ & + \kappa U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b - \kappa g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \\ & + \kappa N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + \kappa V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b \\ & - \kappa g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c + \kappa M_\mu^{ai} V_\mu^{ai} \\ & - \kappa U_\mu^{ai} N_\mu^{ai} + \kappa g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\ & \left. - \kappa g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c \right]. \end{aligned} \quad (47)$$

The transformed action is obtained by adding $S_1(\kappa = 1)$ to S_{GZ} as

$$S_{GZ} + S_1 = \int d^4x \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + B^a \partial_\mu A_\mu^a + i \bar{c}^a \partial_\mu \mathcal{D}_\mu^{ab} c^b \right]. \quad (48)$$

We left with the YM effective action in Landau gauge:

$$S_{GZ} + S_1 = S_{YM}. \quad (49)$$

Note the new action is independent of the horizon parameter γ , and hence the horizon condition ($\frac{\partial \Gamma}{\partial \gamma^2} = 0$) leads to a trivial relation for S_{YM} . Thus using FFBRST transformation we have mapped the generating functionals in the Euclidean space as

$$Z_{GZ} \left(\int [D\phi] e^{-S_{GZ}} \right) \xrightarrow{FFBRST} Z_{YM} \left(\int [D\phi] e^{-S_{YM}} \right), \quad (50)$$

where Z_{YM} is the generating functional for the Yang-Mills action S_{YM} .

Conclusion. – The GZ theory which is free from Gribov copies as the domain of integration is restricted to the first Gribov horizon, is not invariant under usual BRST transformation due to the presence of the nonlocal

horizon term. Hence the KO criterion for color confinement in a manifestly covariant gauge fails for GZ theory. A nilpotent BRST transformation which leaves GZ action invariant was developed recently and can be applied to KO analysis for color confinement. This nilpotent BRST symmetry is generalized by allowing the transformation parameter finite and field dependent. This generalized BRST transformation is nilpotent and symmetry of the GZ effective action. We have shown that this nilpotent BRST with an appropriate choice of finite field-dependent parameter can relate GZ theory with a correct horizon term to the YM theory in the Euclidean space where the horizon condition becomes a trivial one. Thus we have shown that theory free from Gribov copies (*i.e.* GZ theory with proper horizon term) can be related through a nilpotent BRST transformation with a finite parameter to a theory with Gribov copies (*i.e.* YM theory in the Euclidean space). The nontrivial Jacobian of such finite transformation is responsible for this important connection.

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