Jpn. J. Appl. Phys. Vol. 41 (2002) pp. 3202–3205
Part 1, No. 5B, May 2002

Transmission of Ultrasound from Layered Structures into a Liquid

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(Received November 9, 2001; accepted for publication February 14, 2002)

The transmission of ultrasound through a layered structure into a liquid is controlled by the effective reflection coefficient of the layered structure. Under the conditions of resonant transmission, the effective reflection coefficient coincides with the reflection coefficient in the layer at the end of the layered structure that is in contact with the liquid. When the total number of layers increases, the resonant transmission disappears and the transmission rate of ultrasound oscillates rapidly with respect to the change of frequency. However, in the case of actual transmission measurements, this oscillation is difficult to observe. Furthermore, in the case where the substrate of the layered structure has lower acoustic impedance, the observed transmission rate has a large mismatch with the transmission rate given by the theoretical predictions. [DOI: 10.1143/JJAP.41.3202]

KEYWORDS: ultrasound, layered structure, liquid, effective reflection coefficient, transmission rate

1. Introduction

Ultrasound transmits resonantly from a layered structure (LS) into a liquid.[1)] To express the transmission, we make use of the effective acoustic impedance Z_e of the LS. Under the conditions of resonant transmission, Z_e coincides with the acoustic impedance Z_L of the liquid. Furthermore, the surface vibration has a large magnitude on the surface of the LS immersed in the liquid. However, we do not need the resonance for enhancement of the ultrasound transmission. For example, the ultrasound transmits into liquid efficiently by the quarter wave plates. Both resonant and nonresonant transmissions are expressed concisely by the effective reflection coefficient which is closely related to the effective acoustic impedance.[2)]

An experiment has been previously performed for the liquid-solid interface.[3)] According to the measurements of this experiment, the reflection coefficients have lower values than those of the acoustic mismatch theory. Complex impedance[4)] was discussed to explain the lower values of reflection coefficients. In the present paper, we propose another theory for the transmission of ultrasound between solids and liquid. The theory is based on a novel formation of the reflection rate, and makes use of the effective reflection coefficient which is characteristic of the LS.

2. Transmission Rate

We consider a LS comprising two metals A and B as shown in Fig. 1. The substrate of the LS is assumed to have the same acoustic impedance as B for the sake of brevity, and the liquid is water. The LS has a unit bilayer AB, and the number of bilayers is N. We express this structure as $(AB)_N$.

The ultrasound is emitted from the substrate and penetrates the water. In water, we neglect the reflection from the bottom of the liquid bath. We express the reflection rate in the substrate as R, and the transmission rate into the water as T. They are expressed by the relation $T = 1 - R$. The quantity R is expressed as follows

$$R = \left| \frac{\kappa - \kappa_c}{1 - \kappa \kappa_c^*} \right|^2, \tag{1}$$

where κ is the amplitude reflection coefficient at the interface between the LS surface and water, and κ_c is given by eq. (3)

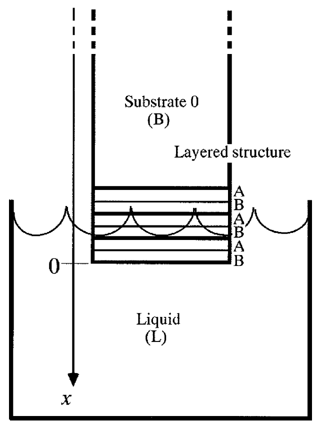

Fig. 1. The LS $(AB)_N$ comprises N bilayers. The unit bilayer comprises materials A and B. In this figure, $N = 3$. For a numerical example in text, A and B are represented as Cu and Ag, respectively. The substrate is assumed to have the same acoustic impedance as material B. In the discussion, we also consider the substrate of Zn which has a lower acoustic impedance than that of Cu or Ag.

defined below.[1)] Because we neglect the reflection in the liquid, κ has the well-known expression: $\kappa = r_{BL}$ and

$$r_{BL} = \frac{1 - Z_L/Z_B}{1 + Z_L/Z_B}, \tag{2}$$

where Z_L is the acoustic impedance of the liquid and Z_B is that of the solid B at the LS surface immersed in the liquid. The quantity κ_c is a complex number which is characteristic of $(AB)_N$ and is given as follows[1)]

$$\kappa_c = \frac{-ie^{-i\beta} Z_m s(\gamma) \sin\alpha}{\cos N\gamma + is(\gamma) g(\alpha, \beta)}, \tag{3}$$

where α (β) is given by the multiplication of the wave number k_A (k_B) in layer A (B) and the thickness d_A (d_B) of layer A (B); i.e., $\alpha = k_A d_A$, and $\beta = k_B d_B$. The quantity Z_A (Z_B) is the acoustic impedance of layer A (B), and $Z_m = \frac{1}{2}(Z_A/Z_B - Z_B/Z_A)$. Furthermore, the functions are given as follows

$$g(\alpha, \beta) = \cos\alpha \sin\beta + Z_p \sin\alpha \cos\beta, \tag{4}$$

$$s(\gamma) = \frac{\sin N\gamma}{\sin\gamma}, \tag{5}$$

where $Z_p = \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)$, and the quantity γ is ob-

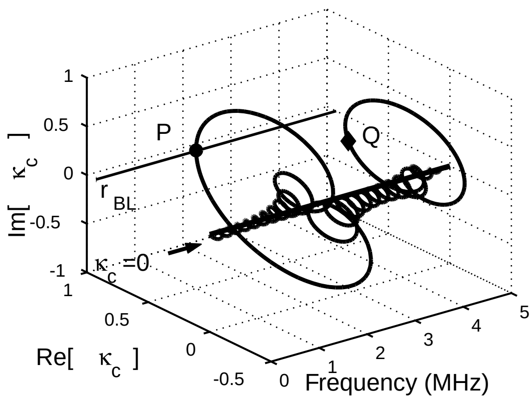

Fig. 2. The effective reflection coefficient κ_c is expressed by a spiral which rotates N times around the straight center line ($\kappa_c = 0$). This figure is of a LS with a unit bilayer comprising Cu and Ag for materials A and B, respectively. The total number of bilayers is eleven, i.e., $N = 11$. The substrate is assumed to have the same material as $B(= \text{Ag})$. At point P, κ_c coincides with r_{BL}, and resonant transmission is set up.

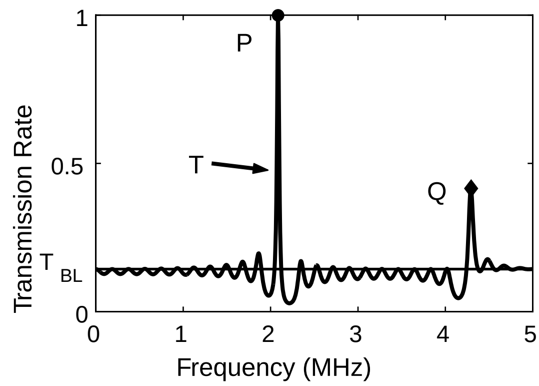

Fig. 3. The transmission rate is shown for the LS consisting of eleven bilayers ($N = 11$) comprising Cu and Ag for the materials A and B, respectively. The frequency for the resonance is 2.085 MHz, and the value of κ_c is indicated by point P which corresponds to the point in the figure for the κ_c spiral.

tained from the dispersion relation[5)] of the unit bilayer as

$$\cos\gamma = \cos\alpha\cos\beta - Z_p \sin\alpha\sin\beta. \tag{6}$$

The above definition of κ_c does not depend on the acoustic impedance Z_L of the liquid. This means that κ_c is characteristic of the LS and is not affected by the liquid in contact with the LS. The quantities α and β depend linearly on the frequency of ultrasound, whereas the other quantity γ does not. However, γ vanishes when the frequency decreases.

We refer to κ_c as the effective reflection coefficient. As an example, we show κ_c in Fig. 2 for the LS comprised of Cu for layer A and Ag for layer B. The thickness of the layer is 0.50 mm for both d_A and d_B. The total number of bilayers is eleven, i.e., $N = 11$. In the figure, we also show r_{BL} as a straight line, which does not depend on the frequency. The acoustic impedances are $Z_A = 44.74$, $Z_B = 38.29$, and $Z_L = 1.48$ in units of $10^6\,\mathrm{kg\,m^{-2}\,s^{-1}}$. The velocities of ultrasound are $5.01\,\mathrm{km\,s^{-1}}$ for layer A (= Cu), and $3.65\,\mathrm{km\,s^{-1}}$ for layer B (= Ag).

At the frequency of point P in Fig. 2, κ_c coincides with the value of $\kappa = r_{BL}$. According to eq. (1), the reflection rate vanishes. Therefore, the transmission rate is enhanced to be as high as the maximum value, i.e., $T = 1$. The resonant transmission occurs at this frequency (2.085 MHz). In this case, the value of γ becomes a complex number, and the magnitude of the localized vibrational mode increases on the LS surface in contact with water. We also see that $\kappa_c \simeq 1$ at this frequency. At the frequency of point Q in Fig. 2, κ_c does not coincide with the value of $\kappa = r_{BL}$. However, it gives a lower reflection rate, and the transmission rate is enhanced to some extent. The frequency is 4.295 MHz. We can see these features of the transmission rate in Fig. 3. In the figure, T_{BL} is the transmission rate from the bulk solid B (= Ag) into water, i.e., $T_{BL} = 1 - |r_{BL}|^2$.

3. Transmission for Multilayered Structures

Next, we consider the same system as that discussed in the preceding section with the exception of its total number of bilayers. When the number N is 50, the effective reflection coefficient κ_c is that shown in Fig. 4. In a frequency region from 0 MHz to the first enhancement frequency 2.085 MHz, or in a region between adjacent frequencies for the transmission enhancement, *e.g.*, from 2.085 MHz to 4.295 MHz, the spiral of κ_c rotates N times as the frequency changes in each region. The spiral passes on a straight line expressed by $\kappa_c = 0$ and on two curves expressed by κ_e and κ_f. The expression of the curve κ_e is obtained under a condition $\mathrm{Im}[\kappa_c] = 0$ as follows

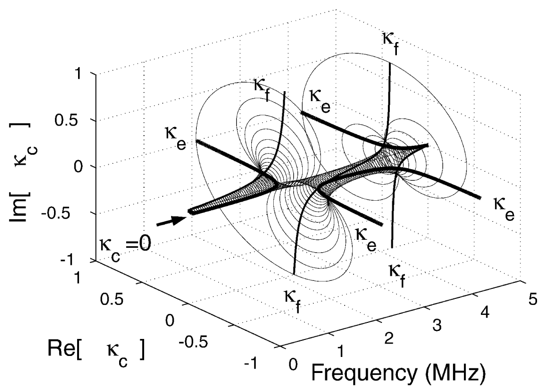

Fig. 4. This figure shows the effective reflection coefficient κ_c with $N = 50$. The LS consists of a unit bilayer of Cu and Ag. The substrate is assumed to have the same acoustic impedance as Ag. The κ_c spiral rotates around the center straight line ($\kappa_c = 0$) and also passes on the curves of κ_e and κ_f which do not depend on N.

$$\kappa_e = \frac{-Z_m \sin\alpha\cos\beta}{g(\alpha, \beta)}. \tag{7}$$

The expression of the curve κ_f is obtained under a condition $\mathrm{Re}[\kappa_c] = 0$ as follows

$$\kappa_f = \frac{i Z_m \sin\alpha\sin\beta}{g(\alpha, \beta)}. \tag{8}$$

The condition for the straight line $\kappa_c = 0$ is expressed as "$s(\gamma)\sin\alpha = 0$," and this also holds true for the conditions $\mathrm{Im}[\kappa_c] = 0$ and $\mathrm{Re}[\kappa_c] = 0$. An interesting feature is that these expressions do not depend explicitly on N. Therefore, the pitch of the spiral rotation becomes higher when we increase N, as is obvious from a comparison of Fig. 2 and 4. This feature has not yet been considered, according to the literature on ultrasound. Therefore, we discuss its details in the following.

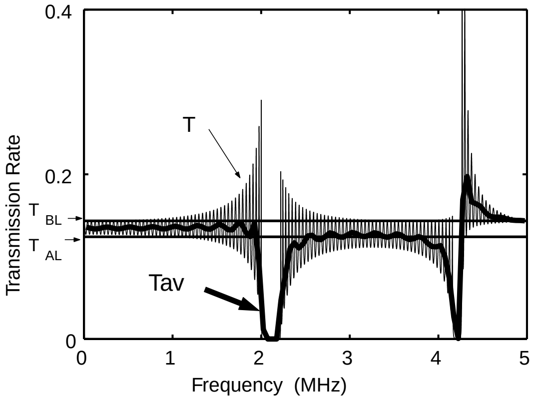

Fig. 5. The transmission rate oscillates with a higher pitch when the number of bilayers is fifty, i.e., $N = 50$. The LS comprises a unit bilayer of Cu and Ag, and the substrate is also assumed to have the same acoustic impedance as Ag.

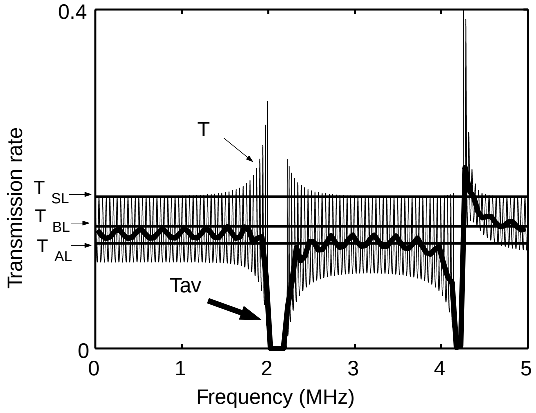

Fig. 6. The transmission rate oscillates with a larger amplitude when the substrate has a lower acoustic impedance than that of Cu or Ag. The number of bilayers is set to be fifty ($N = 50$). The bilayer comprises Cu and Ag, but the substrate consists of Zn.

We show the transmission rate T for the above system in Fig. 5. Corresponding to the high pitch of κ_c rotation, the oscillation of the transmission rate becomes rapid against the frequency. At the frequency for point P in Fig. 3, κ_c no longer coincides with r_{BL} and the transmission rate is a low value. On the contrary, near the frequency for point Q, the transmission rate is a higher value, because the effective reflection coefficient approaches the reflection coefficient r_{BL}.

The actual pulses of ultrasound have a finite width in this frequency region. For example, when we assume that the width is 50 kHz, then the observed transmission rate has the value T_{av}, as shown in Fig. 5, which is obtained by averaging the theoretical transmission rate T in the frequency width. For the sake of comparison, we also show the transmission rates T_{BL} and T_{AL}. The rate T_{BL} is for the transmission from the bulk solid B (= Ag) into water, and T_{AL} is for the transmission from the bulk solid A (= Cu) into water. Generally, T_{av} becomes a smooth curve and falls in a value range of less than T_{BL} and greater than T_{AL}.

4. Discussion

In the same system as that discussed above, the transmission rate for the ultrasound incident from the liquid into the LS substrate has exactly the same rate as those in Figs. 3 and 5. In these cases, the reflection waves do not exist in substrate material B. This means that $1/R = 0$, where R is given by eq. (1). Therefore, it is necessary that $\kappa = 1/\kappa_c^*$, i.e., the reflection coefficient κ at the LS surface becomes a complex number.[2] An experiment for a similar system has already been conducted,[3] and has shown that the reflection coefficient differs from that of the ordinary acoustic mismatch theory. The authors of that study have concluded that the difference is caused by the complex reflection coefficient[4] taking into consideration the loss of the acoustic energy. However, we can consider a complex reflection coefficient, as shown in the present paper, without considering the loss, and predict that the reflection rate is different from that of the ordinary acoustic mismatch theory for the bulk-liquid interface.

When we substitute a material with lower acoustic impedance for the LS substrate, the oscillation of transmission rate has a large amplitude. In the case where the system for Fig. 5 has a Zn substrate instead of an Ag one, the transmission rate becomes as shown in Fig. 6. The value indicated as T_{SL} is the transmission rate calculated for the ultrasound transmission from the bulk solid of Zn into water. The acoustic impedance of Zn is $30.06 \times 10^6\,\mathrm{kg\,m^{-2}\,s^{-1}}$, which is less than that of Ag or Cu. The transmission rates indicated as T_{AL}, T_{BL}, and T_{av} are calculated in the same way as those for Fig. 5. If the substrate consists of water, then the transmission rate oscillates almost completely from zero to unity.

In the cases of both Figs. 5 and 6, the averaged transmission rate T_{av} has values almost the same as those of T_{BL} and T_{AL}. This is interpreted to be the transmission of wave packets through the interface with the liquid being determined by the solid-liquid interface rather than the whole LS, because the wave packets are localized near the interface. However, T_{av} has a lower value than those of T_{BL} or T_{AL} when we increase the total number of layers in the LS. For a LS with $N = 200$ and with a substrate consisting of water, the value of T_{av} is about 0.07. This value cannot be explained by only the solid-liquid interface for the wave packets. This problem is left for further investigation.

If we were to extend the discussion in this section, we could elucidate the low thermal conductance in an LS.[6,7]

5. Conclusions

To discuss the transmission rate for the finite-size LS, we have derived a novel idea of the effective reflection coefficient κ_c.[1] The κ_c spiral passes on one straight line $\kappa_c = 0$ and on the two curves κ_e and κ_f. These two curves do not depend on the total number N in the LS. When we increase the number N, the κ_c spiral rotates with a higher pitch in a complex plane pictured for frequency changes. This feature of ultrasound transmitting through LS is discussed for the first time in the present paper.

Many enhancement peaks of transmission rate appear in a certain range of ultrasound frequencies, when we increase the total number of layers in the LS. In actual usage, the ultrasound has a finite frequency width. Therefore, the observed transmission rate has a mismatch with the transmission rate of the theoretical predictions. The mismatch becomes large when we increase the number of layers in the LS and decrease the acoustic impedance of the LS substrate.

Acknowledgements

This work was supported in part by a Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology (Grant No. 11650062).

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