Self-propagating miniature device based on shape memory alloy

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Self-propagating miniature device based on shape memory alloy

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Abstract
The development of novel locomotion mechanisms is beneficial for advancing the field of self-propagating devices, which are implemented in various civilian and military applications. In this work, we present a purely mechanic, mm-sized autonomous device capable of linear propagation on a smooth, relatively flat surface. The locomotion mechanism is driven by a shape memory alloy (SMA) wire that is connected to a metallic, ring-shaped, bias spring. Periodic changes in the temperature in the vicinity of the device activate the SMA wire, and result in alternating contraction-elongation deformations of the SMA-bias spring assembly. These deformations are transferred to a linear back and forth motion of small legs that are attached at the bottom of the ring. To generate locomotion, the general conditions for obtaining asymmetric friction between needle shaped legs and a smooth surface were formulated and validated experimentally. The structure and performance of the device are modeled analytically leading to basic design rules that are validated experimentally by real-time optical tracking of the device’s displacements and propagation. In addition, the potential of miniaturization of the presented locomotion concept down to the micro-meter scale is demonstrated.

1. Introduction

Autonomous mobile systems provide solutions for unmanned propagation over various kinds of surfaces, and are employed in a variety of research and engineering fields, including medical applications, planetary exploration missions, emergency and rescue operations as well as intelligence and military applications. Existing autonomous systems vary over a range of sizes, ranging from several meters down to few centimeters. Commonly, locomotion mechanisms employed in autonomous systems are classified into wheeled, legged, tracked, or some combination of these subgroups (see, e.g., [1]). Various actuation methods are employed for realizing the different locomotion mechanisms described above. These include electric motors [2, 3], electromagnetic actuators [4], soft pneumatic actuators [5] and piezoelectric actuation [6]. All of these methods require a dedicated energy source (e.g., a battery) and additional components (e.g., electrical wires, pneumatic tubes). This, along with the mechanical assemblies associated with the actuation and locomotion mechanisms, limit the minimal size of such devices to the cm-range and above, and hinder the possibilities for further miniaturization to smaller scales. Miniaturization is often required for applications in which access is limited, for example in the medical field [7], or for rescue missions after natural disasters [8].

Shape memory alloys (SMA) exhibit large forces and strokes due to the reversible martensitic transformation [9–11]. As such, SMA are often implemented in various actuation mechanisms, which take advantage of the thermally induced martensitic transformation. In addition, SMA are also used in applications that employ their pseudo-elastic characteristics that originate from the stress induced martensitic transformation as in seismic dampers [12, 13]. The use of SMA, in which the material is the actuator, can greatly simplify the actuation mechanism, and thus allow for fewer and smaller mechanical parts [14, 15]. Typically, SMA-based actuators are activated by resistive (Joule) heating [16–19], a method that also requires an external energy source. In some cases, however, by adequate matching of the SMA phase transformation temperatures to the working
environment, the energy required for thermal activation of the SMA can be harvested from changes in the surrounding temperature [20, 21]. This can eliminate the need for a dedicated energy source and the associated electronic assemblies, and allow for unlimited working time of the system.

In this work, we combine the benefits of SMA-based actuation with a unique locomotion mechanism and present a miniature, mm-size autonomous device capable of propagating on an even smooth surface. To the best of our knowledge, there are currently no simple, mm-size autonomous robots that provide solutions for this fundamental engineering challenge. The device is composed of an active SMA wire and a counteracting elastic spring equipped with small needle shaped legs. Cyclic temperature changes applied on the SMA-spring assembly lead to legged-locomotion that is based on directional asymmetric friction with respect to the sliding direction, i.e., the friction forces that act when the leg moves along opposite directions (e.g., $+e_1$ and $-e_1$) are different. The device is free of electronic components, and actuation is driven by changes in the ambient temperature in the vicinity of the device. The actuation and locomotion mechanisms are analyzed analytically, leading to the formulation and validation of design rules. Specifically, we formulate general conditions that are required for obtaining locomotion based on asymmetric friction. The demonstrated design rules predict the feasibility of miniaturization of the proposed mechanism down to the μm-scale.

The paper is organized as follows: first the basic operation concept of the device is presented, and the actuation and locomotion mechanisms are analyzed analytically. This analysis provides the basic design rules for the device, as well as general insights regarding SMA-based actuators. Next, the details of the experimental systems and procedures used to fabricate and test the device are presented, followed by the main results of these experiments. The experimental results are discussed and compared with the analytical predictions. Finally, prospects for miniaturization of the proposed system are given.

2. Design and properties of the device

2.1. Structure and operation principles

The miniature device (see figure 1) is composed of three components: an active SMA wire, a metallic ring that serves as a bias spring and needle shaped legs that enable planar locomotion. The SMA wire is fixed to the metallic ring at two points across the ring’s perimeter (see figure 1(a)). During heating, the martensite to austenite phase transformation in the SMA wire results in contraction of the wire, which in turn compresses the bias spring along the $e_1$ direction. During cooling, the reverse phase transformation takes place, and the elastic force applied by the compressed bias spring leads to detwinning of the martensite and to elongation of the wire and of the entire device along the $e_1$ direction.

In order to transform the cyclic contraction-expansion of the device to planar advancement, the metallic ring contains two sets of legs. The legs are located under two anchors, which are small regions in the ring where the cross section is larger compared to the rest of the ring. The anchors’ top surfaces serve as the attachment points for the SMA wire. The legs on both sides of the ring point to the same direction, as shown in figure 1(b).

Under the application of a sufficient vertical load, such a configuration leads to asymmetric friction between the legs and the ground surface. Due to the asymmetric friction, repetitive nearing and distancing of the legs promotes a net advancement of the device along the $e_1$ direction.

2.2. Stress and strain in the SMA wire during temperature cycling

In this section, we formulate an explicit expression for the stress–strain relation in the SMA wire, and correlate it to the properties of the bias elastic spring (i.e., the metallic ring, see figure 1). In addition, stress–strain–temperature dependencies are formulated and provide a general description of the loading paths during slow thermal actuation of an SMA-spring actuator.

The fixed connections between the SMA wire and the metallic ring dictate equal forces and displacements in the two components, i.e.:

$$k \Delta x = \sigma_{\text{wire}} \cdot \pi D^2 / 4$$  \hspace{1cm} (1)

and

$$\Delta x = (e_1 - \varepsilon) L_0$$  \hspace{1cm} (2)

Here, $k$ and $\Delta x$ are the spring’s elastic constant and its contraction, respectively. $\sigma_{\text{wire}}$ is the stress in the SMA wire and $D$ is the wire’s diameter. The contraction $\Delta x$ is measured with respect to the initial distance between the two anchors after attachment of the SMA wire. In equation (2), $e_1$ and $\varepsilon$ are the temporal and initial strains in the SMA wire, with respect to its relaxed length in the austenite phase, $L_0$. Note that the attachment process of the SMA wire to the spring is performed under a tension preload that is applied to the wire, thus creating an initial pre-elongation strain in the martensite phase $e_1$. The temporal strain $\varepsilon$ during the operation of the device is smaller than $e_1$. 


The combination of equations (1) and (2) leads to the following stress–strain relation in the SMA wire:

$$\sigma_{\text{wire}} = E' \cdot (\epsilon_i - \epsilon),$$  \hspace{1cm} (3)

where $E' = 4k \cdot \frac{L_0}{\pi D^2}$ is the effective elastic module of the device. The relation in equation (3) can be superimposed on typical flag-shaped stress–strain curves of SMA NiTi at different temperatures, as demonstrated in figure 2. We note that figure 2 provides a schematic and simplified description of SMA behavior, and does not account, for example, for small residual plastic strains that may appear if the SMA has been subjected to large stress values. The different colors of the curves represent different temperatures. In each of the curves the top stress plateau describes the transformation from austenite to martensite and the bottom stress plateau represents the reverse transformation. At the lowest temperature shown (blue curve in figure 2) the NiTi is in the martensite phase throughout the loading cycle and changes in the strain originate from the reorientation of martensite twin variants.

The gluing of the SMA wire to the ring-shaped spring is performed at room temperature, at which the NiTi is in the martensite phase. The corresponding stress–strain profile at room temperature is denoted by the blue curve in figure 2. During this process, the spring is relaxed and an external tensile force applies a stress $\sigma_i$ on the wire. Under these conditions, equations (1) and (2) are not met and the initial strain in the wire $\epsilon_i$ (point #1 in figure 2) is determined only by the stress–strain curve of the NiTi (the blue curve). When the glue hardens and the initial preload is removed, the wire and the ring reach an equilibrium state (point #2 in figure 2), that is determined by the intersection of the straight black line that represents equation (1) (with a spring constant $k_i$) and the stress–strain curve of the NiTi.

As the temperature of the SMA wire increases above the martensite to austenite transformation value, the stress–strain curves of the SMA change, as shown in figure 2. The equilibrium conditions at each temperature are determined by the intersection of the spring’s stress–strain curve (i.e., the straight black line) and the bottom stress plateau in the relevant stress–strain curve of NiTi. Thus, upon heating, the equilibrium conditions follow the path designated by points #2 → #3 → #4 → #5 in figure 2.

At point #5 heating is stopped and cooling begins. During the reversed path, the transformation from austenite to martensite takes place at the top stress plateaus. Therefore, the strain magnitude at point #5 does not change until the temperature is decreased to a value at which the top stress plateau (not shown in figure 2) coincides with the bottom stress plateau of the red curve. This ‘delay’ represents the hysteretic behavior of NiTi.

Figure 1. (a) Top view of the device showing the metallic ring and the SMA NiTi wire attached at the two anchors. The wire is glued to the ring at small alignment grooves, using cyanoacrylate adhesive. (b) Side view showing the device’s legs located under the anchors.
Next, as the wire’s temperature decreases further, the bias spring stretches the wire and the strain follows the path \( \#5 \rightarrow \#6 \rightarrow \#7 \).

When the temperature rises again, there is a hysteretic ‘delay’ at point \( \#7 \), similar to the behavior at point \( \#5 \). During subsequent heating cycles the contraction of the wire starts from point \( \#7 \), and the stress–strain follows the path \( \#7 \rightarrow \#5 \). We note that only the first heating cycle starts from point \( \#2 \). Therefore, our analysis predicts a larger contraction strain during the first actuation cycle compared to subsequent cycles. Indeed, the experiential results (see section 4) reproduced this behavior.

### 2.3. The locomotion mechanism

In this section, we analyze the legged-locomotion mechanism of the device, and formulate general design rules that predict propagation. We consider a simplified case of two identical legs, one at each side of the metallic ring (see figure 3). We note that in practice, for stability reasons, the device contains two legs at each side. However, this simplification does not alter the validity of the analysis.

A vertical force, \( Q \) (e.g., a gravitational force due to the device’s own mass) and a horizontal force \( F \) (the resultant of the forces in the NiTi wire and in the spring) are applied on each leg, as illustrated by the free body diagrams shown in figure 3. As a result, horizontal friction forces (marked as \( f_1 \) and \( f_2 \) in figure 3) act on the legs when the ring contracts and expands periodically. The force \( f_1 \) resists motion of either legs towards the positive \( e_1 \) direction, while \( f_2 \) resists similar motion along \(-e_1\) (see figure 3). The equilibrium of forces imposes that \( f_1 = f_2 = F \). At the same time, \( f_1 \leq \mu_1 Q \), where \( \mu_1 \) is the friction coefficient at each side of the leg. A leg remains stationary if \( f_1 < \mu_1 Q \) and slides if \( f_1 = \mu_1 Q \).

We define a case of symmetric friction if \( \mu_1 = \mu_2 \), and a case of asymmetric friction if \( \mu_1 < \mu_2 \). The latter case implies that sliding of either legs towards the positive \( e_1 \) direction is associated with a smaller friction coefficient than sliding along the negative \( e_1 \) direction. In the case of symmetric friction, both legs may propagate an equal but opposite displacement during either heating or cooling. Thus, no net motion of the overall device takes place. Contrary, asymmetric friction leads to a situation at which \( F = \mu_1 Q < \mu_2 Q \). As a result, one leg propagates towards the positive \( e_1 \) direction (the left leg in the illustration shown in figure 3) while the other leg remains stationary. When the SMA wire is cooled and the ring expands, the forces on the legs switch directions and the coefficient of friction of the right leg in figure 3 becomes \( \mu_1 \) while that of the left leg in figure 3 becomes \( \mu_2 \). Consequently, the right leg moves along the positive \( e_1 \) direction while the left leg stays in place. Thus, both cases (i.e., heating and cooling) result in a net propagation of the device’s center of mass towards the positive \( e_1 \) direction.

Next, we show that the conditions necessary for asymmetric friction are dictated by the magnitude of the vertical force \( Q \) and the geometry of the legs. The leg’s tip is modeled as part of a sphere with a radius \( R_{\text{tip}} \). The tip extends towards the anchor, at an inclination angle \( \theta_0 \), that is measured between the leg’s right side and the surface (see figure 4(a)). The applied vertical load \( Q \) may result in some penetration of the leg’s tip into the
surface. For small penetration depths, the contact angles at both sides of the leg’s tip with respect to the surface are equal and smaller than $\theta_p$, as shown in figure 4(b). This leads to equal friction coefficients $\mu_1 = \mu_2$, i.e., conditions of symmetric friction. However, above a critical penetration depth $h_c$, the contact angle at the right
side equal to \( \theta_p \), while that at the left side is larger than \( \theta_p \) (figure 4(c)). This leads to different values of the friction forces, i.e., \( \mu_i < \mu_g \).

From geometrical consideration, the critical penetration depth is given by:

\[
h_c = C \cdot R_{\text{tip}} \cdot C = 1 - \frac{1}{\sqrt{1 + \tan^2 \theta_p}},
\]

where \( C \) is a coefficient that depends solely on the angle of the right edge of the leg, \( \theta_p \).

Our next step is to find a relation between the magnitude of the vertical force \( Q \) and the penetration depth \( h_p \). For this purpose, we employ Hertz contact model for a spherical tip that penetrates into an elastic half-space. Using the analysis in [22], the relation between vertical force \( Q \) and the tip’s penetration depth \( h_p \) is given by:

\[
Q = \frac{4E^* \sqrt{R_{\text{tip}}(2h_p)^5}}{3},
\]

Here, \( E^* \) is the combined elastic modulus of the tip and the plane, which is expressed as:

\[
\frac{1}{E^*} = \frac{1 - (\nu_{\text{tip}})^2}{E_{\text{tip}}} + \frac{1 - (\nu_p)^2}{E_p},
\]

where, \( E_{\text{tip}}, \nu_{\text{tip}} \) and \( E_p, \nu_p \) are the Young modulus and Poison’s ratio of the tip and the plane, respectively.

Substituting the expression for the critical penetration depth \( h_c \) (equation (4)) in (3) results:

\[
Q(h_p = h_c) = \frac{4E^*R_{\text{tip}}^2 \sqrt{(2C)^5}}{3}.
\]

We note that the Hertz model presented here is valid only for elastic behavior of both the tip and the surface. Alternatively, if the load applied on the surface is larger than its yield stress, plastic deformation will take place in the surface plane. Here we assume that the yield strength of the surface is much smaller than that of the tip. In this case, the relation between the load \( Q \) and the critical penetration depth \( h_c \) is different than the one given by equation (7). In [23] it is demonstrated that when the plane is deformed plastically, the following relation is approximately obtained:

\[
\sigma_{\gamma,p} = \frac{Q}{2.8\pi a^2};
\]

where, \( \sigma_{\gamma,p} \) is the yield stress of the plane and \( a \) is the radius of contact between the tip and the plane. While equation (8) merely represents a rough empirical law, it has been validated for different materials by numerous experimental and computational studies, which obtained similar empirical relations, changing only the coefficient 2.8 by up to 10% of its value (see [24] and references therein). The contact radius \( a \) is related to the tip’s radius \( R_{\text{tip}} \) and the penetration depth \( h_p \) by [22]:

\[
h_p = \frac{a^2}{2R_{\text{tip}}},
\]

The combination of equations (4), (8) and (9) provides:

\[
Q(h_p = h_c) = 5.6\pi R_{\text{tip}}^2 C\sigma_{\gamma,p}.
\]

Equation (10) is used for determining the values of \( Q, R_{\text{tip}} \) and \( \theta_p \) in the devices that were manufactured and tested in this work.

3. Experimental system and data analysis

3.1. Materials and physical properties of the devices

The main geometrical and physical parameters of the devices that were manufactured and tested in this work are summarized in Table 1. The SMA wire used for all the devices is a 0.2 mm diameter Flexinol 70 °C NiTi wire from Dynalloy Inc. [25] with austenite start and finish temperatures of \( A_s = 34 \, ^\circ \text{C} \) and \( A_f = 61 \, ^\circ \text{C} \), respectively [26]. The metallic ring is a single-piece rapidly manufactured Ti-6Al-4V alloy. Titanium alloy obtained using this process has a typical yield stress larger than 800 MPa [27]. The stiffness of the ring \( k \) is designed such that the stress in the NiTi wire (equation (3)) will not exceed \( \sigma_{\text{wire}} \leq 200 \, \text{MPa} \) for a strain of \( \varepsilon_i = \varepsilon \leq 3.5\% \), as recommended by the NiTi manufacturer. A detailed mechanical analysis showing the relations between the ring’s geometry and stiffness is given in the appendix.

The integration of the SMA wire to the metallic ring is performed at \( RT \approx 23 \, ^\circ \text{C} \) under a tension stress of 175 MPa. This load leads to an initial pre-strain of approximately \( \varepsilon_i = 3.5\% \). This large strain is associated with detwinning of the martensite phase, during which favorable twins grow at the expense of less-favorable twins.
through the motion of twin boundaries. This process typically takes place at a nearly constant stress value [14]. While preloaded, the wire is attached to the anchors of the metallic ring using a special high temperature cyanoacrylate adhesive (PERMABOND® 922) that can withstand the applied loads. The typical weight of the metallic ring is 0.4 g, and the yield stress of the aluminum surface is estimated as $\sigma_{y,p} \approx 30$ MPa. According to equation (4), for practical inclination angles of $\theta_p \leq 45^\circ$, the geometric constant $C$ is smaller than $\sim 0.3$. Under these conditions, equation (10) predicts that planar propagation is possible only if the leg’s tip radius $R_{\text{tip}}$ is on the order of few microns. Such values are smaller than the typical resolution of rapid manufacturing processes. Thus, the device’s legs are produced from commercially available tungsten needles, that are glued to the metallic ring under both anchors. We used two types of needle geometries: one with a tip radius of $R_{\text{tip}} \approx 3 \mu m$, and the second with a larger radius that varies within a range of $R_{\text{tip}} \geq 60 \cdots 200 \mu m$. In the latter case, an additional magnetic force was added in order to increase the vertical force $Q$ to a value that will enable planar propagation (see equation (10)). A magnetic force of $Q \approx 25 \cdots 110$ g is achieved by attaching two mm-sized external magnets to the upper part of the metallic ring (one on top of each anchor) and placing a thin magnetic Mu-metal sheet under the ground surface.

### 3.2. Experimental setup

The experimental setup provides two main features: first, it enables cyclic heating and cooling of the plane above which the device propagates, and second, it provides simultaneous optical tracking of the device’s shape and location. A schematic description showing the different parts of the experimental system is given in figure 5.

The temperature of the device is cycled by heating and cooling the horizontal surface on which the device is placed. This is achieved by a combination of an electrical heating plate and a cooling plate that is connected to a chiller that is set to a temperature of $0 \, ^\circ\text{C}$. Cooling liquid flows through a sealed aluminum plate whose top cover serves as the surface on which the device is placed. During heating periods, the heating plate is turned on and the flow of the cooling liquid is stopped. During cooling the flow is activated and the heating plate is turned off. The

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**Table 1. Selected properties of the tested devices.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value/Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of NiTi wire</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Initial length of the NiTi wire, $L_0$</td>
<td>16 mm</td>
</tr>
<tr>
<td>Radius of leg’s tip, $R_{\text{tip}}$</td>
<td>3, 60 – 200 $\mu$m</td>
</tr>
<tr>
<td>Inclination angle of the legs, $\theta_p$</td>
<td>$10^\circ$ – $45^\circ$</td>
</tr>
<tr>
<td>Stiffness of the metallic ring along $e_1$, $k$</td>
<td>$10$ – $25$ N $\cdot$ mm$^{-1}$</td>
</tr>
<tr>
<td>Device’s own weight</td>
<td>0.25 – 0.4 g</td>
</tr>
<tr>
<td>Vertical force, $Q$</td>
<td>0.25 – 0.4 g; 25 – 110 g$^\star$</td>
</tr>
</tbody>
</table>

$^\star$ Forces achieved by the addition of external magnets.
temperature is constantly monitored by a thermocouple (with measurement error of ±1 °C) that measures the surface's temperature close to the device and is connected to a computer. The experimental setup allows varying the temperature of the surface within a range of 40 °C–95 °C. Heat is transferred from the upper plate surface to the NiTi wire by convection through the air and conduction through the legs and the metallic ring. Each temperature cycle (i.e., heating to 95 °C and subsequent cooling to 40 °C) lasts about 25 min. Due to heat losses, the temperature of the SMA element is lower than that measured by the thermocouple. According to our results presented in section 5, we estimate that this temperature difference is given by

$$\Delta T = \alpha (T_{\text{surface}} - T_{\text{room}}),$$

where $$\alpha \cong 0.24.$$

The shape and planar location of the device are continuously tracked using an optical camera that is placed above the device, and takes individual images every 5 s. A typical top view image of the device is shown in figure 5. The camera is connected to a computer and the obtained images are synchronized with the thermocouple readings.

3.3. Data analysis

Quantitative description of the contraction and planar propagation data of the device is obtained from the optical images that are continuously collected during thermal cycling. The images are analyzed using a Labview code based on Feature Tracking approach [28]. The algorithm allows tracking the coordinates of predefined marks in consecutive images. We use three marks, two on the device’s anchors that are located above the legs and one on the surface (see inset in figure 5). The marks are assigned with planar coordinates ($x$, $y$) using a coordinate system of the image. Mark (1) is located on the right anchor ($x_{\text{right}}$, $y_{\text{right}}$), mark (2) is located on the left anchor ($x_{\text{left}}$, $y_{\text{left}}$), and mark (3) is the reference point ($x_{\text{ref}}$, $y_{\text{ref}}$) located on a ruler that is rigidly attached to the surface. The device’s SMA wire is aligned along the $x$ direction of the image coordinates (see figure 5), and thus the temporary length of the wire between the two anchor points, is calculated according to:

$$L = x_{\text{right}} - x_{\text{left}}. \quad (11)$$

The contraction is calculated as the difference between the temporary length $L$ in equation (11) and the initial length $L(t = 0)$. Similarly, the location of the device’s center of mass along the $x$ direction is calculated according to:

$$u = \frac{x_{\text{right}} - x_{\text{left}}}{2} - x_{\text{ref}}. \quad (12)$$

The propagation is calculated according to the difference between the temporary location $u$ and its initial location at $t = 0$. The $y$ coordinates allow tracking rotations, if exist. The noise level of the contraction and propagation measurements was evaluated by taking a series of images at a constant temperature, and was found to be about 0.02 mm.

4. Results

In this section we present contraction and propagation data for a representative device (see figure 6). A video showing typical propagation behavior of the device over several temperature cycles is provided as supplementary material is available online at stacks.iop.org/JPCO/2/015015/mmedia. The main geometrical and physical parameters of the device are summarized in table 2. We note that other devices with different geometrical parameters (see table 1) were produced and tested, and their performance is mentioned in section 5. The rotational displacements were found to be negligible, and thus the values of the contraction and propagation were calculated according to equations (11) and (12).

The measurements shown in figure 6(a) reveal that the first heating cycle resulted in a larger contraction compared to subsequent cycles. This behavior is in agreement with our theoretical analysis shown in figure 2. The typical contraction amplitude during a single temperature cycle is approximately 0.4 mm, which is equivalent to a longitudinal strain in the NiTi wire of 2.4%. In this experiment, the first five temperature cycles ranged between 43 °C and 87 °C, and the following cycles ranged between 43 °C and 95 °C. In accordance, it is observed that both the contraction and the propagation during the first cycles are smaller compared to their values during the following cycles.

The temporary location of the device’s center of mass is presented in figure 6(b). The average propagation distance per cycle (calculated over sixteen temperature cycles shown in figure 6(b)) is approximately 0.22 mm. Thus, the ratio of average propagation over the average contraction, which is a measure for the effectiveness of the locomotion mechanism, is ~0.35. We note that the measured propagation shown in figure 6(b) is not monotonous, and exhibits small ‘backwards’ motion during every temperature cycle. This phenomenon is attributed to the fact that both legs do not move simultaneously. Since we calculate the location of the center of
mass based on the locations of the two legs, such non-synchronized motion results in an apparent backwards motion.

An optical image of the scratch mark left by one of the device’s legs during propagation is shown in figure 6(c). The existence of a scratch verifies our prediction that the gravitational stress imposed by the legs on

**Figure 6.** Experimental measurements showing the temperature, the device’s contraction (a) and its position (b) as a function of time. In (c), the scratch left by one of the legs on the surface is shown (left) along with an image of the leg produced from a commercial tungsten needle (right).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of leg’s tip, $R_{tip}$</td>
<td>3 μm</td>
</tr>
<tr>
<td>Inclination angle of the legs, $\theta_p$</td>
<td>$10^6$</td>
</tr>
<tr>
<td>Stiffness of the metallic ring along $e_1$, $k$</td>
<td>10.1 N mm$^{-1}$</td>
</tr>
<tr>
<td>Initial length of the NiTi wire, $L_0$</td>
<td>16 mm</td>
</tr>
<tr>
<td>Vertical force, $Q$</td>
<td>0.38 g</td>
</tr>
</tbody>
</table>
the aluminum surface is larger than the surface’s yield stress, and results in plastic deformation in the surface. In this case the relation between the required vertical force for propagation and the geometrical parameters of the device’s legs can be predicted based on equation (10).

5. Discussion

5.1. Validation of design rules for the locomotion

In section 2.3 a relation between the minimal vertical force required for planar propagation and the geometrical parameters of the device’s legs was suggested (see equation (10)). By manufacturing and testing several devices with different geometrical and physical parameters (i.e., different values of $k$, $R_{tip}$, $Q$), the validity of this prediction is evaluated.

Figure 7 presents the propagation behavior of different devices, plotted on a plane defined by the vertical force $Q$ and the leg’s tip radius, $R_{tip}$. The linear blue and green lines represent the analytical conditions for propagation given by equation (10) for $\theta_p = 10^\circ$ and $\theta_p = 37^\circ$, respectively. The lines represent the borders between the predicted propagation range (above the line) and non-propagation range (below the line).

The blue and green cross marks represent devices with $\theta_p = 10^\circ$ and $\theta_p = 37^\circ$, respectively, that did propagate. Green circles represent devices with $\theta_p = 37^\circ$ that did not exhibit a detectable propagation. It can be observed that the experimental results agree well with the prediction of the model. Specifically, all the devices that propagated experimentally are located within the ‘propagation region’ defined by equation (10), i.e. the blue and green cross marks are above the blue and green lines, respectively. At the same time, the marks that represent devices that did not propagate experimentally (green circles in figure 7) are located below the green line, i.e., within the ‘non-propagation region’. The good agreement between the analytical condition for propagation (equation (10)) and measured results (figure 7) imply that the penetration of the device’s legs into the surface is accompanied by plastic deformation. The scratch marks left on the surface (see figure 6(c)) support this observation.

5.2. Loading paths in an SMA-linear spring actuator

The measurements of the device’s contraction (similar to those shown in figure 6) provide experimental evidence for the general analytical relations that characterize the loading paths of the NiTi element in an SMA-spring actuator. For example, the relations between the temperature and the plateau stresses $\sigma_p^{M\rightarrow A}$ and $\sigma_p^{A\rightarrow M}$, for the forward and reverse martensitic phase transformations, can be expressed via the classical Clausius–Clapeyron equation adjusted to uniaxial stress [26, 29].
Here, $\rho$ is the mass density of NiTi, $\Delta s$ and $\Delta \varepsilon$ are the specific entropy change and strain change due to the phase transition, respectively. The term $-\rho_H/\Delta s$ is always positive and $\Delta s = H/\Delta \varepsilon$, where $H$ is the latent heat of the transformation and $\Delta s$ is the temperature of the wire in kelvin. $H$ is identical for the forward and reverse transformations, while the differences in $\Delta s$ are not larger than ~10%. Thus, we can assume that values of the term $-\rho_H/\Delta s$ in the two expressions in equation (13) are identical [29]. The combination of equations (3) and (13) provides the strain–temperature relation in the NiTi wire:

$$T = -\frac{\Delta \varepsilon}{\rho\Delta s} E' \cdot \varepsilon' + T_0.$$  

Here, $E'$ is an effective elastic constant (see equation (3)) and $\varepsilon' = \varepsilon_1 - \varepsilon$ is the strain change. Equation (14) is plotted graphically in figure 8(a), showing the strain–temperature relation in the wire for two different values of the spring’s stiffness $E'$. The intersection points of the curves with the $\varepsilon' = 0$ axis, marked as $T_{0A}$ and $T_{0M}$, represent the austenite and martensite transformation temperatures, respectively, under zero load/deformation, which are independent of the spring stiffness. Note that the transformation temperatures are usually characterized by a range of values between the ‘start’ and ‘finish’ of the phase transformation. Here, $T_{0A}$ and $T_{0M}$ represent the temperatures during the plateau of the phase transformation, i.e., the temperatures at the peaks of differential scanning calorimetry (DSC) measurements.

Experimental temperature–strain relations measured for two devices with different spring stiffnesses are presented in figure 8(b). Note that although the temperature presented in figure 8(b) was measured on the surface, figure 8(b) exhibits a similar shape to the paths predicted by equation (14) and displayed in figure 8(a). In addition, while the two curves shown in figure 8(b) follow different slopes ($9.8 \degree C$, $25.3 \degree C$ respectively) due to differences in the springs’ stiffness, the extensions of these slopes coincide at the intersection with the $\varepsilon' = 0$ axis, as predicted by equation (14) and figure 8(a). Dividing the different slopes of the curves by the corresponding effective modulus $E'$ results in a value for the Clausius–Clapeyron coefficient (equation (13)): 

$$\frac{d\sigma_p^{M \rightarrow A}}{dT} = -\frac{\rho_H}{\Delta s} \Delta \varepsilon; \quad \frac{d\sigma_p^{A \rightarrow M}}{dT} = -\frac{\rho_H}{\Delta \varepsilon}.$$  

(13)

![Figure 8](image-url)
\[ \frac{d\sigma_p}{dT_{\text{surface}}} = 5.0 \pm 0.5 \text{ (MPa } ^{\circ}\text{C}^{-1}) \]  

This value is slightly lower compared to previous results reported for similar Flexinol wires, ranging between 6 and 8 MPa °C^{-1} [26, 30]. This difference can be attributed to the fact that the actual temperature \( T \) in the NiTi wire is lower than the measured temperature \( T_{\text{surface}} \) on the surface, resulting in a lower value for the measured slope, i.e. \( \frac{d\sigma_p}{dT_{\text{surface}}} \). Assuming that this temperature difference is described by the relation 
\[ \Delta T = \alpha (T_{\text{surface}} - T_{\text{room}}) \], a slope of \( \frac{d\sigma_p}{dT} = 7 \text{ MPa } ^{\circ}\text{C}^{-1} \) is obtained for a value of \( \alpha \simeq 0.24 \). In addition, the data in figure 8(b) allows extracting the zero-strain transformation temperatures of the NiTi wire, which yields \( T_{\text{AM}} \simeq 45 \text{ °C} \) and \( T_{\text{PM}} \simeq 24 \text{ °C} \). Note that for these temperature values, \( \Delta T \) is smaller than 5 °C. Indeed, these values are consistent with DSC measurements for a similar NiTi wire [26].

### 5.3. Downscaling of the device

The analytical analysis presented in this work can be applied to predict the properties of a similar device at much smaller scales, and the feasibility of its manufacturing. For example, consider scaling down all the dimensions of the presented device by a factor of \( \sim 50 \), which means a device with a total ring diameter of \( \sim 400 \mu\text{m} \) and feature sizes (e.g. the ring thickness) smaller than 100 \( \mu\text{m} \). The stiffness of the ring-shaped bias spring scales down at the same factor (see the analysis in Appendix). Consequently, maintaining the required stress level in the SMA NiTi wire implies that its diameter also scales down by the same factor, resulting \( D \approx 4 \mu\text{m} \) (see equation (3)).

The manufacturing of a scaled-down \( \mu\text{m} \)-scale device should be feasible using MEMS-based technologies. Recently, for example, we demonstrated the successful production and testing of an SMA-based MEMS device with a free standing active NiTi film with thickness of 3 \( \mu\text{m} \) [31, 32]. This value is similar to the required dimension of the SMA element in the \( \mu\text{m} \)-scale device.

Apparently, the most challenging part in manufacturing a \( \mu\text{m} \)-scale device is associated with the legs’ tip radius, whose value is critical for obtaining locomotion. The total weight of the scaled down Si-based MEMS device is calculated to be of few micro-grams. Consequently, our analysis (see equation (10) and related discussion) predicts that the required radius of the leg’s tip should be on the order of few nm. Si-based needles with such nano-scale tip radius (e.g., atomic force microscopy tips) can be manufactured using existing MEMS technologies, e.g., deep reactive ion etching and focused ion beam [33–36], thus enabling the miniaturization of the proposed concept.

### 6. Conclusions

An autonomous legged-locomotion miniature device with a total volume smaller than 1 cm³ is presented. The actuation and propagation mechanisms are analyzed analytically and the necessary conditions for asymmetric friction, which enable legged-locomotion, are identified and formulated. Actuation is driven by cyclic thermal response of an SMA wire that is connected to a metallic ring that serves as a bias elastic spring. We show that periodic changes in the temperature of the underlying surface provide the necessary driving force for actuating the SMA wire, and thus for propagation of the device. Experimental measurements performed on a variety of fabricated devices with different parameters validate the analytical analysis, providing a solid theoretical baseline for the open challenge of obtaining asymmetric friction for legged locomotion. While in this work the temperature changes were induced by controlled external sources (i.e., heater and chiller), our results imply that appropriate adjustment of the transformation temperatures of the SMA element can allow the proposed locomotion mechanism to propagate by harvesting changes in the environment temperature. Finally, the combination of analytical analysis and experimental results suggest that miniaturization of the proposed locomotive mechanism down to the micron scale is possible.

### Appendix. The stiffness of the metallic ring

In this section, an analytical expression for the elastic constant \( k \) of the ring, which acts as a bias spring, is developed. As we showed in section 2.2, the stiffness of the ring directly determines the actuation strain and the propagation performance of the device and has to be designed based on the expected stresses developed in the SMA NiTi. We note that thermal strains developed in the metallic ring as it is heated and cooled within the tested temperature range (i.e., smaller than 100 °C) are two orders of magnitude smaller than those associated with its expansion/contraction due to the actuation of the NiTi wire. Thus, we neglect this effect in this analysis.

Recalling the symmetry of the device, a free body diagram of half of the ring is considered (see figure A1). The structure is composed of a curved beam with radius \( R \) and two anchors with length \( l \) (which is half of the full anchors’ length in the full device) both having rectangular cross sections. The appropriate boundary conditions...
imply that the points located at the lower edge of the anchors can move linearly along the $e_1$ direction but cannot rotate to form an angle with respect to the $e_1$ axis. These boundary conditions are represented by two mobile fixed supports and an internal torque $M_0$ (see figure A1(a)). The force $P$ equals to half of the total force that is applied by the NiTi wire. $\theta_1$ and $\theta_2$ are the angles between the $e_1$ axis and the two points that connect the anchors to the circular beam (figure A1(a)). The cross section dimensions of the circular beam and the anchors are given by $b$, $b_1$, and $h_1$, respectively (figures A1(b) and (c)).

The stiffness of the entire device $k$ equals to the ratio between the total force $F_{\text{NiTi}} = 2P$ and the resulting displacement along $e_1$, $q_p$:

$$k = \frac{2P}{q_p}. \tag{16}$$

Next, we seek an explicit expression for $q_p$. According to Castigliano’s second theorem, the displacement $q_p$ is the derivative of the elastic energy $U$ with respect to the generalized force $P$ that acts in the direction of this displacement, i.e., $q_p = \frac{\partial U}{\partial P}$. The overall elastic energy $U$ is given by:

$$U = \frac{R}{2EI} \int_0^{\theta_1} [M(\theta)]^2 \, d\theta + \int_0^l [M(y)]^2 \, dy. \tag{17}$$

$E$ is the Young’s modulus of the metallic material, and $I$, $I_1$ are the second moments of area of the circular beam and the anchors, respectively. The coordinate $y$ is measured along the anchor in the $e_2$ direction. $M(\theta, y)$ is the torque in an arbitrary cross section along the beam, and its value is obtained from equilibrium of moments:

$$M(\theta, y) = \begin{cases} 
M_0 - y P & 0 < y < l \\
M_0 - P(l - R \sin \theta_1) - PR \sin \theta_1 & \theta_1 < \theta < \theta_2 
\end{cases}. \tag{18}$$

The resulting expression for $q_p$ becomes:

$$q_p = \frac{\partial U}{\partial P} = -R \frac{I}{IE} \left[ M_0(l - R \sin \theta_1) - P(l - R \sin \theta_1)^2 - \frac{PR^2}{2} \right] (\theta_2 - \theta_1) - R (M_0 - 2P(l - R \sin \theta_2)) (\cos (\theta_2) - \cos (\theta_1)) + \frac{PR^2}{4} (\sin (2\theta_2) - \sin (2\theta_1)) - \frac{2}{I_1 E} \left[ \frac{M_0 l^2}{2} - \frac{P l^3}{3} \right]. \tag{19}$$

Equation (19) can be written as:

$$q_p = \frac{P}{E} f(R, \theta_1, \theta_2, l, I, I_1) + \frac{M_0}{E} g(R, \theta_1, \theta_2, l, I, I_1). \tag{20}$$

Here, $f$ and $g$ are functions that depend only on the geometrical parameters of the device. Since the boundary conditions dictate only planar displacement at the supports (i.e., the internal torque $M_0$ cannot cause
bending), the generalized displacement $q_{M_0}$ due to the torque $M_0$ is zero:

$$q_{M_0} = \frac{\partial U}{\partial M_0} = 0.$$

(21)

Inserting the expressions for the elastic energy and the torque along the beam (equations (17), (18), respectively) into equation (21) leads to an expression for the torque $M_0$:

$$M_0 = \frac{P}{l} \left[ \frac{R(\theta_2 - \theta_1)}{l} + \frac{2I_1}{I} \right],$$

(22)

Equation (22) can be written in a simplified form:

$$M_0 = P \cdot z(R, \theta_1, \theta_2, l, I, l),$$

(23)

where $z$ is a function that depends only on the geometrical parameters of the device. Finally, inserting equations (20) and (23) into (16) results in the desired expression for the stiffness $k$:

$$k = \frac{2P}{q_p} = \frac{2E}{f + g \cdot z},$$

(24)

where the functions $f$, $g$ and $z$ are purely geometrical.

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