PAPER - OPEN ACCESS

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To cite this article: F Y Sun et al 2018 J. Phys. Commun. 2 015008

View the article online for updates and enhancements.
Rabi resonance in Cs atoms and its application to microwave magnetic field measurement

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Keywords: Cs atoms, Rabi resonances, atom-based microwave measurement, phase-modulated field, microwave cavity, Rabi frequency

Abstract
We have recently presented a technique for measuring unknown microwave (MW) fields based on Rabi resonances induced by the interaction of atoms with a phase-modulated MW field. The single-peak feature of the measurement model makes the technique a valuable tool for simple and fast field measurement. Here we demonstrated a detailed investigation of Rabi resonance of Cs atoms inside a vapour cell. For illustration, we used the Rabi resonance-based field detection technique to determine the field strength inside an X-band cavity for applied power in the range of −21 to 20 dBm. The difference between the practical measurement and the simulation was approximately about 3% for an applied power of 20 dBm, and a higher accuracy could be expected for a larger power input.

1. Introduction

To date, atom-based measurements for static and radio-frequency fields have already had remarkable successes [1–15]. Recently, atom-based microwave (MW) measurement has also inspired great interest because of its potential ability to link the MW quantities with SI units. As a result, relying on various physical principles, many atom-based MW sensors have been developed [1], such as the MW power standard [16–19], MW electrometry [20–32], MW electric/magnetic field imaging [22, 33–40], and MW magnetometers [41, 42]. As compared to traditional measurement, atom-based measurement is intrinsically calibrated where field strength is translated into Rabi frequency $\Omega$ via well-known atomic constants. Particularly, the use of an atomic vapour cell serves as a sensing head, making atom-based measurements properly for practical applications [21, 25, 26, 30]. Additionally, a non-conducting dielectric cell containing atomic samples reduces the field perturbation compared with the metal sensing head used in traditional probes [25, 43–45]. For these reasons, atom-based measurement provides the possibility of detecting MW fields with higher accuracy.

In our previous work, as a proof-of-principle demonstration, we presented an approach for continuously frequency-tunable MW magnetic field detection using atomic Rabi resonances of different hyperfine transitions [42]. Here, we demonstrate a detailed investigation of Rabi resonance and its application to MW field measurement. In our technique, a phase-modulated field drives atomic Rabi resonance, which is a function of the modulation frequency $\omega_m$ and reaches its peak when $\omega_m = \Omega/2$. By scanning $\omega_m$ for a given input power, we can obtain the resonance lineshape; then, $\Omega$ is determined by fitting the measured data to the theoretical model of field measurement developed here. For the magnetic dipole transition, $\Omega = \mu_B B/h$, where $\mu_B$ is the Bohr magneton, $h = h/2\pi$ (where $h$ is Planck’s constant), and $B$ represents the MW magnetic field strength which is parallel to quantization axis of the Zeeman sublevels in this case.

For illustration, we performed a series of experiments with our previously proposed atom-based field detection technique to measure a certain range of MW power at approximately 9.2 GHz using our cavity-based Cs Rabi resonances experimental setup. The agreement of the experimental results and theoretical calculations proves our technique is simple and effective. We note that the measurement of MW magnetic fields inside the cavity-cell physical system plays an important role in the development of high-performance, vapour cell-based atomic clocks [38, 39, 46]. There have already been different approaches to this application. MW field strength...
was measured in an Rb clock via adiabatic rapid passage [46], a MW magnetic field distribution was imaged with Rabi oscillations inside a microfabricated Rb vapour cell [38, 39]. The technique here provides an alternate, direct means for accurate measurement of magnetic field strength inside a cell in gas-type atomic clocks, where traditional measurements are unavailable because of the presence of the vapour cell [33, 41].

2. Field measurement model

The Rabi resonances were observed first by Cappeller and Mueller in 1985 [47], and then intensely studied by Camparo et al including the \( \alpha \) and \( \beta \) Rabi resonances [48–51]. The basic principle of Rabi resonances theory is mentioned here; more details on the theory are available in [49]. When two-level atoms interact with a phase-modulated MW field (i.e., \( \theta = m \omega_m \cos(\omega_m t) \)), where \( m \) is the modulation index, the probability of finding the atoms in the excited-state oscillates at the modulation frequency \( \omega_m \) and \( 2\omega_m \). The oscillation behaviour can be well described with two experimentally distinguished oscillation amplitudes, i.e., \( P_\alpha \) and \( P_B \). The amplitudes of \( P_\alpha \) and \( P_B \) are resonant functions of \( \Omega \) and \( \omega_m \) and exhibit resonant enhancements when \( \Omega = \omega_m \) and \( \Omega = 2\omega_m \); these resonant enhancements of oscillation amplitudes are termed the \( \alpha \) Rabi resonance and the \( \beta \) Rabi resonance, respectively. Under the small-signal approximation, the amplitudes of \( P_\alpha \) and \( P_B \) can be derived from a calculation based on the density-matrix equations (see [49], equation (9)):

\[
P_\alpha = \frac{1}{2} \left[ \frac{m \omega_m \Omega^2 \Delta}{\gamma_1^2 + \Delta^2 + (\gamma_2 / \gamma_1) \Omega^2 \left( \Omega^2 - \omega_m^2 \right)^2 + \gamma_1^2 \omega_m^2} \right] \sin(\omega_m t + \phi_\alpha), \tag{1a}
\]

\[
P_B = \frac{1}{4} \left[ \frac{m \omega_m \Omega^2 \gamma_2}{\gamma_1^2 + \Delta^2 + (\gamma_2 / \gamma_1) \Omega^2 \left( \Omega^2 - 4\omega_m^2 \right)^2 + 4 \gamma_1^2 \omega_m^2} \right] \cos(2\omega_m t + \phi_B), \tag{1b}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are the longitudinal and transverse relaxation rate, respectively, \( \Delta \) is the average MW field-atom detuning, \( \phi_\alpha = \tan^{-1}(\Omega^2 - \omega_m^2) / \gamma_1 \omega_m \), and \( \phi_B = \tan^{-1}(\Omega^2 - 4\omega_m^2) / 2\gamma_1 \omega_m \) [49]. Writing \( P_\alpha(t) = P_\alpha^0 \sin(\omega_m t + \phi_\alpha) \) and \( P_B(t) = P_B^0 \cos(2\omega_m t + \phi_B) \), we could then obtain the amplitudes \( P_\alpha^0 \) and \( P_B^0 \), which are detectable signals in experiments:

\[
P_\alpha^0 = \frac{1}{2} \left[ \frac{m \omega_m \Omega^2 \Delta}{\gamma_1^2 + \Delta^2 + (\gamma_2 / \gamma_1) \Omega^2 \left( \Omega^2 - \omega_m^2 \right)^2 + \gamma_1^2 \omega_m^2} \right], \tag{2a}
\]

\[
P_B^0 = \frac{1}{4} \left[ \frac{m \omega_m \Omega^2 \gamma_2}{\gamma_1^2 + \Delta^2 + (\gamma_2 / \gamma_1) \Omega^2 \left( \Omega^2 - 4\omega_m^2 \right)^2 + 4 \gamma_1^2 \omega_m^2} \right]. \tag{2b}
\]

Observing equation (2), we note that the Rabi resonances have two significant features [49]: (i) the \( \alpha \) Rabi resonance reaches its minimum when the MW-field detuning vanishes (i.e., \( P_\alpha^0 = 0 \) when \( \Delta = 0 \)), meaning the \( \alpha \) Rabi resonance could be used to estimate the atomic transition frequency; and (ii) the \( \beta \) Rabi resonance reaches its maximum when the modulation frequency is one-half of the Rabi frequency (i.e., \( P_B^0 \) peaks when \( \omega_m = \Omega / 2 \)), indicating its potential for measuring the Rabi frequency. With this highly useful feature, several Rabi resonance-based applications, e.g., the strength stabilization of the MW field and/or laser field (so-called atomic candle) [51], measurements of material properties (e.g., the absorption/refractive-index) [52], observations of cavity-mode stability for Rb atomic clocks [53], and generation of the MW power standard [18, 19], have been proposed and/or demonstrated experimentally.

It is noteworthy that what these applications have in common is that the \( \beta \) Rabi resonance amplitude \( P_B^0 \) is considered a function of Rabi frequency \( \Omega \), where the modulation frequency \( \omega_m \) is fixed during operation. As a result, only a specific field that corresponds to a known modulation frequency can be detected or stabilized in these cases.

For the MW magnetometer demonstrated here, we rewrote the \( \beta \) Rabi resonance into a function of modulation frequency and used it to measure an unknown magnetic field strength. According to equation (2b),

\[
\frac{m \omega_m \Omega^2 \gamma_2}{\gamma_1^2 + \Delta^2 + (\gamma_2 / \gamma_1) \Omega^2}
\]

can be considered constant under the fixed experiment settings, including laser and MW levels. Thus, the amplitude of the \( \beta \) resonance has the following simple expression [49]:

\[
P_B^0(\omega_m) \propto \frac{\omega_m}{\sqrt{\Omega^2 - 4\omega_m^2} + 4 \gamma_1^2 \omega_m^2}. \tag{3}
\]

From equation (3), the amplitude of the oscillating function, \( P_B^0 \), exhibits resonance when \( \omega_m = \Omega / 2 \) with a peak amplitude of \( 1 / (2\gamma_1) \). To measure an unknown \( \Omega \), we first experimentally measure the resonance lineshape by scanning \( \omega_m \) and then \( \Omega \) (i.e., field strength) can be determined by fitting the theoretical model obtained previously to the measured lineshape.

Similar to the Q definition for the atomic candle [49], we let \( Q = \Omega / 2\Delta \omega_{\text{FWHM}} \) for the field measurement model (equation (3)), where \( \Delta \omega_{\text{FWHM}} \) is the full width at half maximum (FWHM). Knowing that the resonance
peak occurs at \( \omega_m = \Omega/2 \) with an amplitude of \( 1/(2\gamma_1) \), we set \( P^0_\alpha(\Omega - \Delta\omega_{\text{FWHM}})/2 \) = 1/(4\gamma_1). Then, according to equation (3), \( \Delta\omega_{\text{FWHM}} \) can be computed as

\[
\Delta\omega_{\text{FWHM}} = \frac{\sqrt{2\Delta^2 + 3\gamma_1^2 + \gamma_1\sqrt{12\Delta^2 + 9\gamma_1^2}} - \sqrt{2\Delta^2 + 3\gamma_1^2 - \gamma_1\sqrt{12\Delta^2 + 9\gamma_1^2}}}{2\sqrt{2}}.
\]  

Thus, the line Q has the potential to be narrowed greatly by decreasing the longitudinal relaxation rate.

Before measuring the Rabi frequency with the \( \beta \) Rabi resonance, we need to determine the atomic transition frequency precisely because the Rabi frequency is also a function of the detuning from the resonance. Here, we use the \( \alpha \) Rabi resonance to determine the transition frequency [49, 51]. For this purpose, we rewrite the \( \alpha \) Rabi resonance into a function of frequency detuning for given experimental settings and use it to measure an unknown atomic frequency:

\[
P^0_\alpha(\Delta) \propto \frac{\Delta}{\gamma_2^2 + \Delta^2 + (\gamma_2/\gamma_1)\Omega^2},
\]

with a peak amplitude of \( 1/2\gamma_1 + \gamma_2/\gamma_1\Omega^2 \). Then, \( \Delta\omega_{\text{FWHM}} \) of the \( \alpha \) Rabi resonance can be computed as

\[
\Delta\omega_{\text{FWHM}} = (4 - 2\sqrt{3})\sqrt{\gamma_2^2 + (\gamma_2/\gamma_1)\Omega^2}.
\]

Briefly, in this work, we detailed a Rabi resonance-based MW magnetometer and demonstrated its use in cavity field detection. This is completed via the two main steps, i.e., we first use DR spectra and the \( \alpha \) Rabi resonance (equation (5)) to determine the MW-atom interaction frequency, and then measure the MW magnetic field strength inside the cavity via equation (3).

3. Experiments

Our experimental setup is shown in figure 1(a). We used a Cs vapour cell to perform Rabi resonances between atomic hyperfine states (\( F = 3, m_F = 0 \))—\( F = 4, m_F = 0 \)). Figure 1(b) shows the energy levels of Cs in a static magnetic field \( H_{DC} \). A linearly polarized 4\( \rightarrow \)4’ laser was used to pump and detect the Cs atoms, whose frequency was stabilized to saturated absorption resonances on the D\(_2\) line. For all measurements demonstrated here, a laser power of \( \sim 30 \mu W \) corresponding to density of \( \sim 150 \mu W \ cm^{-2} \) was used. Using braided windings wrapped around the cavity, the temperature of the vapour cell (filled with pure Cs and 10 Torr N\(_2\)) was heated.
and stabilized at \( \sim 36 \, ^\circ C \). The whole physical package was centrally mounted inside three-dimensional Helmholtz coils.

We note that the detection of Rabi resonances allows only for determination of the local Rabi frequency, and hence the local MW field strength. Therefore, uniform field distribution is needed for the determination of unique Rabi frequency. To address this problem, the cell was placed in the centre of a rectangular cavity symmetrically about a magnetic field maximum. The inside dimensions of the rectangular cavity were 22.86 mm \( \times \) 10.16 mm \( \times \) 72 mm, which allowed for the TE\(_{104}\) mode operating near 9.2 GHz. The advantage of this scheme is that the MW magnetic field was approximately uniform over the entire interaction region in the atomic samples. To demonstrate this advantage, we simulated the magnetic field inside the cavity to show how the fields remain uniform in the centre of the cavity, as shown in figure 1(c). MW-couplings were created with two small holes with a diameter of \( \sim 6 \, \text{mm} \). The two waveguide-to-coaxial adapters were connected to the cavity. After passing through the first adapter, the input MW was irradiated into the cavity via a coupling hole to drive Rabi resonances in a thermal Cs atomic vapour. The output port of the cavity was connected to the second adapter, which was terminated by a 50 Ohm matched load. Two cutoff waveguides were attached to the cavity walls with the purpose of suppressing MW leakage and field perturbation during operation. Through the cutoff waveguides, the laser illuminated the cell perpendicular to the direction of the applied MW magnetic field. Transmission of the laser through the vapour was monitored with a 1 cm\(^2\) Si photodiode, and then recorded with an FFT analyser.

To validate our experimental setup, the detection of the optical-MW DR signal is the first step before measuring [34]. We assumed here a static magnetic field of \( H_{\text{DC}} = 700 \, \text{mG} \) in the \( x \) direction; also in this direction was the MW magnetic field. Under these conditions, the seven \( \Delta m_F \equiv 0 \) magnetic dipole transitions were observed as frequency was scanned, and these DR spectra were recorded via the transmission light through the cell, as illustrated in figure 1(d).

### 4. Results

Prior to the excitation by the modulated field, we carry out the typical DR operation (see figure 1(d)) and use the obtained DR spectra to estimate the transition frequency. After that, we connect the PD output to an FFT spectra analyser and turn on the phase-modulation function of the MW signal source. Finally, we tune the applied MW frequency finely (around the obtained estimation atomic frequency via DR spectra) until \( P_m^0 = 0 \), while keeping \( P_p^0 \neq 0 \) during the tuning process. Figure 2 shows examples of \( P_p^0 \) and \( P_m^0 \) resonating at 10 kHz and 20 kHz, respectively, with different applied MW frequencies, where a 10 kHz phase modulation signal is applied to the MW field. Specifically, figure 2(a) shows the on-resonance state (\( P_m^0 = 0 \) when \( \Delta = 0 \)) on which the oscillation amplitude depends only on the \( \beta \) Rabi resonance. From figures 2(b) and (c), which represent the near-resonance cases, we find that a slight frequency detuning could also induce MW-atoms interaction. As expected, no fields-atoms interaction occurs at MW frequencies far from resonance (see figure 2(d)). Obviously, the atomic transition frequency is slightly shifted from 9.19263177 GHz due to the presence of \( \text{N}_2 \). Thus, the MW frequency, which is required for MW-atom interaction is precisely confirmed through the procedures described previously.

Next, to validate our theoretical model, we show how the Rabi frequency can be determined by this technique. As an illustrative example, figure 3 presents a comparison of theoretical and experimentally measured lineshapes of the \( \beta \) Rabi resonances at two different power levels. The experimental data were obtained by measuring \( P_p^0 \) oscillations occurring at \( 2\omega_{\text{om}} \) on the FFT spectra analyser as a function of \( \omega_{\text{om}} \). The solid curves in this figure represent the theoretical fit to the experimental data using equation (3); their peaks yield information about the Rabi frequencies. As can be seen in figure 3, the measured data closely follow the theoretical curves.

For completeness and quantitative understanding of our method, figure 4 presents the power-dependent Rabi resonance signals as a function of modulation frequency at six different power levels (\( P = -12 \, \text{dBm}, -6 \, \text{dBm}, 0 \, \text{dBm}, 6 \, \text{dBm}, 12 \, \text{dBm} \) and \( 18 \, \text{dBm} \)) without normalizing the resonance amplitudes, which correspond to Rabi frequencies of \( \Omega \left/ \omega_{\text{om}} \right. = 0.02 \, \text{kHz}, 3.54 \, \text{kHz}, 6.79 \, \text{kHz}, 13.49 \, \text{kHz}, 27.30 \, \text{kHz}, \) and \( 54.46 \, \text{kHz} \), respectively. From figure 4, we find that for every 6 dB increase in \( P \), the Rabi frequency approximately doubles. This relationship is valid especially for a relatively large power input. It is worth noting that the validity of the small-signal assumption (\( \Omega \gg \gamma_p \)) gradually weakens as the applied power decreases, and this causes the decline of measurement accuracy. Despite this, our method still exhibits a reasonable approximation. Additionally, signal amplitude varies rapidly with the variation of applied power. However, of greatest concern is the frequency corresponding to the resonance peak rather than the amplitude in these measurements. This a major advantage of atom-based measurement. Clearly, the Rabi resonance lineshape has a unique peak at \( \omega_m = \Omega / 2 \) for any given \( \Omega \), as predicted by equation (3).
Following the technique described previously, we repeated the measurements while varying the incident power level $P$ in the range from $-21$ dBm ($7.94 \mu W$) to $20$ dBm ($100$ mW). We then plotted the Rabi frequencies as a function of power and fit the linear function of our measurements, as shown in figure 5. Because the magnetic field sensed by atoms is proportional to the square root of the incident MW power, $\sqrt{P}$, in the case of the cavity structure, the measured Rabi frequency exhibits a linear relationship with the square root of the incident MW power, as one would expect.
5. Comparison and discussion

To further verify the validity of this measurement technique, we used HFSS (finite element simulation tool) to calculate the field strength inside the interaction region, and compared it to the measured results. It is challenging to determine the wall thickness and the dielectric characteristic of the vapour cell accurately because of manufacturing limitations [26, 27, 30]. In our simulation, we adjusted the two parameters slightly to match real measurement conditions, which mainly included the measurement frequency and cavity resonant frequency. This is particularly important for accurate simulation, because field distribution is frequency-dependent according to the cavity response. Figure 6 shows the simulated MW magnetic field along three directions of the cavity with a power injection of 20 dBm, indicating that the field distribution is quite uniform over the entire interaction region (i.e., $-2.5 \text{ mm} \leq x, z \leq 2.5 \text{ mm}$, and $-3.3 \text{ mm} \leq y \leq 3.3 \text{ mm}$), where the centre of the cell is defined as the origin of coordinates (0, 0, 0). A field variation of only $\sim 7.5\%$ is observed in the $y$-direction, but there is almost no variation in the $x$- and $z$-directions because of the narrow laser diameter ($\sim 5 \text{ mm}$). A significant field perturbation due to the presence of the cell and optical holes is observed. Figure 6 also indicates how to determine the simulated field. Figure 7 shows a comparison between the simulation results and those obtained from the practical measurements at different power levels, where the experimental values were calculated from the Rabi frequencies shown in figure 5. From figure 7, similar linear behaviours are observed from both methods, as expected, and it is clear that our setup served as a field/power sensor in these measurements. The difference between the practical measurement and the simulation is approximately $3\%$ for an applied power of 20 dBm, and a higher accuracy could be expected for a larger power input.
Note that no attempt has been made to optimize this setup to obtain the ultimate ability to detect MW field. Clearly, this ability can be easily extended to cover not only the broader scope of under-test field/power, but also other frequency bands by using different atomic species and/or energy level transitions (especially for the use of magnetic-sensitive hyperfine transition). It is worth noting that the technique discussed here demonstrates how to measure unknown field strength inside a MW cavity via Rabi resonances. Furthermore, this technique is also available for near/far-field cases.

We note also that a MW magnetic field can be measured by means of Rabi oscillations [35–41]. Rabi oscillations present a multi-peak feature in measurement, in which the Rabi frequency for a given power level is determined by fitting the data to a cosine function. However, the demonstrated field detection technique here based on the Rabi resonance presents a single-peak feature, making it easy to perform field measurements. Similar to the field detection based on the Rabi oscillations, our technique also allows frequency-tuneable field detection by adjusting the applied static magnetic field. Additionally, we performed this experiment with Cs, because of the many practical requirements for MW measurements often occurring in X-band. To the best of our knowledge, this is the first time that Rabi resonances have been used to measure unknown field strength inside a cavity.

![Figure 6. Simulated magnetic field strength along the quantization axis as a function of position. (a) Field strength along the x direction (y = z = 0). (b) Field strength along the y direction (x = z = 0). (c) Field strength along the z direction (x = y = 0); clearly, the cavity is operated in TE_{104} mode. The yellow circle indicates the field point under measurements.](image-url)
6. Conclusions

In summary, we demonstrated a technique for measuring any unknown MW magnetic field via atomic dynamic Rabi resonances. To detail the Rabi resonance-based detection technique, we performed a demonstrating experiment, and we compared the power levels measured from the experiments to those computed from the simulations. Both results show that, in principle, Rabi resonance can be used to measure field strength. Finally, we emphasize that the Rabi resonances technique, because of its simplicity and single-peak character, may offer a highly useful technique to measure MW field strength over a broad dynamic range. Using this atom-based, SI-traceable measurement technique, under test MW power can be linked directly to the modulation frequency, which could be referenced to a primary frequency standard.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No. 61601084), and by the State Key Laboratory of Advanced Optical Communication Systems and Networks, China. The authors especially acknowledge Da-Nian Zhang and Ting-Ting Zhao for valuable discussions.

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Figure 7. Comparison of experimental and simulated magnetic field strength as a function of applied MW power. The black solid line and the red solid line are linear fits to the measured values and simulated values, respectively.