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Abstract

A fiber optic based source of polarization entangled photon pairs is proposed to connect and entangle a telecom quantum memory to a solid state quantum memory based on single-qubits operating in the near infrared. To span this large wavelength difference while matching the narrow spectral bandwidth of the quantum memories, the photon pairs are generated by four wave mixing in a highly birefringent fiber. Raman noise is expected to be strongly suppressed under these conditions.

1. Need for quantum memories (QM) in long distance information teleportation networks

All-photonic quantum teleportation without memories was first demonstrated in 1997 [1]. However, to achieve long distance communication with general teleportation protocols, quantum repeaters are needed [2]. Briefly, the role of repeaters is to extend the range of entanglement without paying the exponential penalty with distance that is characteristic of photon propagation loss. Quantum repeaters need QM so that entanglement can be verified before the information is sent [3].

The location of memory elements in a long distance quantum communication (QC) system are shown in figure 1. A quantum repeater network would operate by first establishing entanglement between adjacent nodes where the distance between the nodes is short enough to give an acceptable photon loss. This is illustrated in figure 1(b) for an extended repeater network. Entanglement can then be spread to longer distances where the associated losses along the total transmission channel would otherwise be prohibitive by performing an entanglement swap between neighboring nodes as illustrated in figure 1(b). The final result would be entanglement of more distant nodes, as illustrated in figure 1(a).

2. Entangled photon (EP) sources for interfacing optical memories to telecom

Existing quantum repeater architectures require a source of EP pairs. Normally this is accomplished by parametric down conversion (PDC) in a nonlinear crystal [4]. However these photon pairs tend to have wide bandwidths, being determined by phase matching over relatively short lengths. This gives them a large spectral mismatch with most optical memories which tend to operate over narrow bandwidths, in the MHz–GHz range. To address this problem a number of bandwidth-limiting techniques are used, for example incorporating PDC crystals into optical cavities [5]. However even using such techniques, the pair production rate within the memory bandwidth is usually far below that desired for a high-bandwidth QC system.

To address both these problems, the EPs can be generated by four wave mixing (FWM) in fibers [6]. One advantage of fiber-based FWM generation is that the much longer phase matching length can translate to intrinsically narrower bandwidth. Although this option has been around for a long time, it is rarely used because spontaneous Raman generation competes with pair generation resulting in a large quantum noise background. EPs in the telecom and NIR would not only have the potential for entangling hybrid QMs over telecom distances
but also the potential for other applications such as quantum sensing and quantum information processing that can benefit from the diverse wavelengths.

2.1. Highly birefringent optical fibers
To suppress Raman noise and simultaneously address the wavelength offset problem a large frequency difference between the signal/idler and pump is needed [6–8]. These needs can be satisfied by highly-birefringent optical fibers, sometimes known as polarization maintaining (PM) fibers. By generating photon pairs in a polarization orthogonal to the pump, the fiber birefringence can be used to cancel the phase mismatch due to frequency differences between pump and signal/idler [6, 7]. The resulting photon pairs show a good tangle even without spectral filtering. Moreover, their intrinsic spectral width is nearly transform limited (for picosecond pulses) [9] and therefore a better match to solid-state single-qubit memories. A similar design using PM dispersion shifted fiber (DSF) produced polarization EPs in the telecom band [10]. Here, Raman noise was reduced through cooling of the PM DSF. In a related application we calculated the Raman contribution to single and coincidence photon detections and show that if the signal and idler wavelengths are separated enough from the pump wavelength the Raman contributions are highly suppressed even without cooling [8, 11].

2.2. Proposed experimental setup
Figure 2 shows a schematic of our proposed experimental setup with a PM fiber FWM process to generate polarization EPs at telecom and atomic memory wavelengths including increased polarization indistinguishability by using a single pump based on an extension of earlier work [6, 7]. The initial concept was experimentally demonstrated by Fang et al [6, 7] but at 634 and 850 nm, far from telecom wavelengths. A telecom wavelength scheme to create polarization EPs using PM DSF was also demonstrated [10] but does not span the needed telecom to NIR spectral distances and requires cooling to suppress Raman noise.

Our FWM scheme generates polarization entangled signal and idler photons both higher and lower than the pump though a nonlinear FWM process. We adjust the pump and the parameters of the PM fiber and system so that the Stokes photon is suitable for a telecom wavelength QM and the polarization entangled anti-Stokes photon is compatible with an atomic memory. The spectral shift from both photons is large so that Raman generated photons scattered from the pump do not produce noise near the memory wavelengths. As an example for the implementation of our scheme for entangling atomic memories we quantitatively analyze application of this scheme to entangling a telecom QM near 1530 nm such as erbium with an atomic QM near 737 nm such as SiV diamond QM.

In the configuration example a beam of 995 nm photons from a pulsed laser propagate towards a polarizing beam splitter (PBS). The PBS transmits photons with H-polarization and reflects photons with V-polarization. The reflected V-polarized photons can be used for multiple purposes such as generating another FWM process, monitor pump properties, heralding, and synchronization. The transmitted H-photons then interact with a half
wave plate (HWP1) to adjust the polarization to 45°. The photons polarized at 45° then pass through a circulator into a Sagnac loop via a second PBS. The PBS acts to generate counter propagating pump paths in the Sagnac loop. The Sagnac loop has a 90° twist that ensures that the slow axis is oriented in a manner to generate EPs along the fast axis in both propagation directions. The H component of the pump pulse passes through the PBS on a copropagating path into the slow axis of the PM Fiber generating signal and idler |V, V⟩ photons along the fast axis. The twist in the PM fiber causes the H component photons to act like V photons on reentry into the PBS while the generated |V, V⟩ are rotated to |H, H⟩ and propagate through the PBS to exit the Sagnac loop. Similarly, the V component of the pump is reflected by the PBS along a counter propagating path into the slow axis of the PM fiber and generates |H, H⟩ photons along the fast axis. The twist in the PM fiber in this path allows the V pump photons to propagate through the PBS on reentry and the generated |H, H⟩ are rotated into |V, V⟩ to be reflected by the PBS to exit the Sagnac loop. The signal and idler photons generated along the copropagating and counter propagating paths exit the loop and propagate to the right and the residual polarized pump components are reflected to the circulator on the left and directed to an exit port. At the exit of the PBS, the two photon superposition has both |H, H⟩ and |V, V⟩ components where the path along which a signal or idler was generated is not known but the path for the H or V is known. HWP2 after the PM fiber Sagnac loop is added to our setup to create a biphoton polarization entangled state between the signal λs and idler λi photons by creating two alternate but indistinguishable ways to measure an H or a V. The signal λs and idler λi photons can be used to entangle QM λs and QM λi. The reflected pump from the PBS1 can be used for generating another FWM process, monitoring, or heralding.

2.3. FWM process
In the FWM process, two pump photons of angular frequencies ωp are converted to two daughter photons at angular frequencies ωs and ωi in a third-order nonlinear optical material, such as optical fiber. During this
process, the energy is conserved and the phase-mismatch is zero. These conditions can be written as [14, 15]

$$2 \omega_p = \omega_s + \omega_i,$$

where \( p, s \) and \( i \) stand for pump, signal and idler photons respectively.

The phase mismatch in the FWM process can be written as

$$\Delta k = \Delta k_m + \Delta k_w + \Delta k_{NL} + \Delta k_{bf},$$

where \( \Delta k_m, \Delta k_w, \Delta k_{NL}, \Delta k_{bf} \) and \( L_B \) are the phase-mismatching due to material dispersion, waveguide dispersion, nonlinear effect, birefringence, and the magnitude of the beat length respectively.

$$\Delta k_{NL} = -\frac{2}{3}\gamma P_p,$$

$$\Delta k_{bf} = k_{bf} - k_{bf} = \omega_p \delta n/c,$$

$$L_B = \frac{\lambda}{B},$$

$$B = |\delta n|.$$  

The phase mismatching due to material and waveguide dispersion in the optical fiber are given by

$$\Delta k_m = 2\frac{2\omega_p}{c}n(\omega_p) - \frac{\omega_s}{c}n(\omega_s) - \frac{\omega_i}{c}n(\omega_i)$$

$$= 2\pi \left[ \frac{n(\omega_p)}{\lambda_p} - \frac{n(\lambda_p)}{\lambda_s} - \frac{n(\lambda_i)}{\lambda_i} \right],$$

$$\Delta k_w = 2\frac{2\omega_p}{c}\Delta n(\omega_p) - \frac{\omega_s}{c}\Delta n(\omega_s) - \frac{\omega_i}{c}\Delta n(\omega_i)$$

$$= 2\pi \left[ \frac{\Delta n(\omega_p)}{\lambda_p} - \frac{\Delta n(\lambda_p)}{\lambda_s} - \frac{\Delta n(\lambda_i)}{\lambda_i} \right],$$

where \( n \) is the index of refraction and \( \Delta n \) is the change of the index of refraction due to waveguiding. Waveguide dispersion depends on details of the optical fiber design such as cladding and core, but in single mode fiber \( \Delta k_w \) can be neglected because \( \Delta n \) is nearly the same for all waves [15], although it can be computed using equation (8) when needed.

The birefringence in PM optical fiber is given by

$$\delta n = n_s - n_t,$$

where \( n_s \) and \( n_t \) are refractive indices for polarization modes along the slow and fast axes of the optical fiber which can also depend on wavelength. Figure 3 depicts where photon pairs are generated in a PM-fiber. The vertically polarized pump is aligned along the slow axis and consequently, the generated signal and idler photons are polarized along the fast axis.

The phase mismatching due to the nonlinear effect in optical fiber is

$$\Delta k_{NL} = -\frac{2}{3}\gamma P_p,$$

where \( P_p \) is the pump peak power. The nonlinear coefficient \( \gamma \) is given by \( \gamma = \frac{3\pi}{2\lambda_p A_{eff} n_2 c} \), where \( n_2 \) is nonlinear index of the material and \( \lambda_p \) is the center wavelength of the pump. The terms \( n, A_{eff} \) and \( \chi^{(3)} \) are the average refractive index, the effective mode area and third-order susceptibility of the nonlinear fiber medium respectively. The nonlinear phase term, \( \frac{2}{3}\gamma P_p \), is very sensitive to pump power. This term changes the propagating pulse shape at high pump power due to increased phase-shift [6, 15]. By selecting appropriate wavelengths (colors) of signal and idler photons this phase-shift can be used in the phase matching physics.
2.4. Tuning fibers to match telecom and memory wavelengths
The large wavelength difference between signal and idler photons needed to couple telecom photons to NIR QMs can be achieved in a birefringent fiber with a short ‘beat’ length that must also be single mode for widely separated wavelengths. One way to accomplish both is to design a photonic crystal fiber [16]. The precision to which the birefringence of commercially available fibers can be controlled is sufficient to approximately match particular telecom bands to the narrowband memories.

Fine tuning can be accomplished by taking advantage of the nonlinear refractive index variation with pump power. The tuning range can be estimated from figure 4 which shows the power dependence for three cases.

2.5. Raman noise mitigation
In general, Raman scattering depends on the wavelength values of the EP generation. However, when the EP wavelengths are highly separated from the pump then the Raman noise is virtually zero at those wavelengths. Recently, Lee et al [17] demonstrated the production of nondegenerate EP pairs in the telecom band in optical fiber with reduction of noise to very low levels by cooling the fiber to liquid nitrogen temperature. For silicon fiber Raman scattering causes a pump frequency shift of approximately 40 THz which is equivalent to a 132 nm wavelength shift for a 995 nm pump [18]. Beyond a 40 THz frequency shift, Raman gain \( g_R \) approaches zero.

Clearly the idler photon of around 1532 nm is shifted more than 132 nm from the 995 nm pump. A similar analysis is applied to the signal photon of around 737 nm where the anti-Stokes Raman gain is smaller. Since the Raman noise is proportional to the Raman gain \( g_R \), the Raman noise would be virtually zero for this EP source and would operate at room temperature without the need for cooling.

In a similar spontaneous FWM (SFWM) application, ‘Color controllable polarization entanglement generation in optical fiber at telecommunication wavelengths’ [8, 11], we calculated coincidences due to Raman Stokes and anti-Stokes noise for nearly degenerate signal and idler wavelengths. These are depicted in figures 4 and 5 of our reference for a case when Raman is significant for high pump powers and temperatures but is not significant for lower pump powers and temperatures. This shows that at room temperature at relatively low pump power Raman noise is not a problem. The full expressions for computation of Raman noise are shown in the text and equations surrounding the figures in the referenced paper.

2.6. Trade-offs with coincidence rate and pulse distortion
The problems with using pump power for fine tuning of EP wavelengths are two-fold. First, the pair production rate also depends on pump power, and since there is usually an optimal power which gives the best trade-off between pair rates and accidental coincidences, the effective tuning range can be rather small in practice.

The second problem arises because short pulses do not normally have perfectly rectangular pulse shapes although modern pulse carving can achieve nearly rectangular shaping. As a result the pump intensity, and therefore phase mismatch, changes during the pulse. This can lead to both pulse distortion during propagation and possibly the generation of frequency-chirped signal and idler photon pulses. The pulse distortion problem can be simulated as follows. The frequency spectrum of the pump can be written as [18]

\[
F(\omega_p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(t)^{1/2} e^{i\Delta\omega(t)} e^{-i(\omega_0-\omega_p)t} dt,
\]

where pump power \( P(t) \) is given by a Gaussian function \( P(t) = P_p e^{-t^2/\Delta t^2} \), \( P_p \) is pump peak power, \( \Delta t = 1/\sigma_p \) is the pulse duration and \( \sigma_p \) is the pump bandwidth. The introduced phase shift due to the modulation term is
\[ \Delta \phi(t) = -2/3 \gamma P(t) L = \phi_m e^{-t/\Delta \tau}, \]

where \( \phi_m = -2/3 \gamma P_o L \) is the maximum phase shift. The intensity of the pump after phase modulation is proportional to \( |F(\omega_p)|^2 \). The pulse spectral shapes at different maximum phase shifts are shown in figure 5.

In a previously reported FWM in birefringent fibers [6, 7], it was shown that it is possible to generate signal and idler colors at a sufficiently low pump power that the peak phase shift is much smaller than \( \pi/2 \), in which case the pulse shape is not significantly distorted. In our case we show that a peak pump power \(< 1.4 \text{ kW}\) generates a peak phase shift less than \( \pi/2 \). Our graphs to be presented below depict EP generation at a peak pump power of \( 1.0 \text{ kW}\), i.e. below the distortion regime, and therefore serve as conservative estimates of the system capabilities. Since the pump pulse remains unimodal and in addition the generated signal and idler photons power is quite low, higher order signal and idler processes are not significant.

### 2.6.1. Pulse distortions and other fine tuning options

Ideally a phase-match tuning mechanism other than pump intensity is preferred in order to avoid the trade-offs discussed above. One possibility might be heating of the fiber. Another would be an intense auxiliary laser at a different wavelength than the pump or signal/idler. Any additional photon pairs generated by these auxiliary pumps can be removed by filtering.

### 2.7. Theory for the entanglement source

In this section the theory for the entanglement source setup is discussed. In the proposed experimental setup, a PBS separates the pump into orthogonally polarized H- and V-polarization pulses with adjustable relative phase. In the Sagnac loop, the V-component of the pump produces \( |H_s H_i\rangle \), where \( H \) and \( V \) represent horizontally and vertically polarized pulses respectively and \( s \) and \( i \) indicate the signal and idler respectively. Similarly, the H-component produces \( |V_s V_i\rangle \). For equal pump intensities in both polarization modes, the result is a generated output polarization entangled state

\[ |\psi_p\rangle = \frac{1}{\sqrt{2}} [ |H_s H_i\rangle + e^{i\phi_p} |V_s V_i\rangle]. \]  

(12)

Here, \( \phi_p \) is the relative phase between two orthogonally polarized pump pulses. The phase \( \phi_p \) can be set by a quartz crystal or delay line. Appropriate settings of \( \phi_p \) and the fast axis angle of a half-wave plate inserted after the signal and idler photons have been separated can be used to produce all four Bell states in the polarization degree of freedom [19].

In four-wave mixing, the quantum state of the generated signal and idler pair at the output of the fiber can be described as [8, 20]

\[
|\psi\rangle = |0\rangle + gL \sum_{k_s k_i} F(k_s, k_i) a_{k_s}^\dagger a_{k_i}^\dagger |0\rangle \\
+ (gL)^2 \sum_{k_s k_i k'_s k'_i} F(k_s, k_i) a_{k_s}^\dagger a_{k'_s}^\dagger a_{k_i}^\dagger a_{k'_i}^\dagger |0\rangle + ..., \]

(13)

where 1st order term is considered and higher order terms are negligible. The two-photon spectral function is

\[ F(k_s, k_i) = g \int_{-\infty}^{\infty} dz \exp \left\{ i\Delta k z - \frac{(\nu_s + \nu_i)^2}{4\sigma_p^2} \right\}. \]  

(14)
where \( g = \frac{\alpha^2 \nu Q}{\nu Q_0} \) is a multiplier which quantifies the two-photon spectral amplitude \( F(k, k') \) \[21\]. For example, \( |g| = 8.653 \times 10^{-7} \text{ m}^{-1} \) for silica fiber at pump \( \lambda_p \) of 995 nm with peak power \( P_p \) of 1 kW and pulse duration of 650 fs. In a 20 cm fiber the value of \( |g| \) is \( 1.731 \times 10^{-7} \). Here, the group velocity dispersion in the fiber is negligible and so has been omitted, \( L \) is the length of the fiber in the Sagnac loop, the wavevector \( k = \frac{\omega}{c} \), \( \Delta k \) is phase-mismatching, \( V_Q \) is the quantization volume, \( \alpha \) is the constant determined by experimental conditions, \( \alpha' \) is a creation operator, \( \sigma_p \) is the bandwidth of the pump and \( \nu \) are the detuning frequencies. The values of higher order perturbation coefficients for equation (13) are proportional to \( (gL)^m \) where \( m = 1, 2, 3, \ldots, M \) represent 1st, 2nd, 3rd, and higher order perturbations. Due to the decreasing values of \( (gL)^m \) higher order powers can be safely neglected.

In each pulse, the signal and idler photon counting probabilities are given by \[8, 20\]

\[
S_c = \int_0^\infty dT \int_0^\infty \langle \psi | E_s^{(-)} E_s^{(+)} | \psi \rangle, \\
I_c = \int_0^\infty dT \int_0^\infty \langle \psi | E_i^{(-)} E_i^{(+)} | \psi \rangle, 
\]

where the electric field operators are

\[
E_s^{(+)} = \sum_{k_s} \sqrt{\frac{\alpha_{\text{eff}}}{4V_Q}} a_{k_s} e^{-i\omega_s t} e^{-\frac{\alpha_s^2}{2\sigma_s^2}}, \\
E_i^{(+)} = \sum_{k_i} \sqrt{\frac{\alpha_{\text{eff}}}{4V_Q}} a_{k_i} e^{-i\omega_i t} e^{-\frac{\alpha_i^2}{2\sigma_i^2}},
\]

where \( \sigma_s \) and \( \sigma_i \) are the bandwidths of the signal and idler pulse.

After simplification equations (15) and (16) at perfect phase-matching condition, \( \Delta k = 0 \), become

\[
S_c = A(\gamma P_p L)^2 \frac{\sigma_f}{\sigma_p} R_s^2(\gamma P_p L), \\
I_c = A(\gamma P_p L)^2 \frac{\sigma_f}{\sigma_p} R_i^2(\gamma P_p L),
\]

where \( A = \frac{\alpha^2 \pi A_0^2}{18\nu Q_0} \), and the self phase modulation (SPM) effects are given by \( R_s^2(\gamma P_p L) = \text{sinc}^2(\gamma P_p L) \). The parameters \( \alpha \) and \( V_Q \) are given by \[20\] \( \alpha = 0.237 \) and \( V_Q = 1.6 \times 10^{-16} \text{ m}^3 \).

The probability of a coincidence from each pulse, assuming 100% detection efficiency, can be written as \[8, 20, 22\]:

\[
CC = \int_0^\infty dT_1 \int_0^\infty dT_2 \langle \psi | E_i^{(-)} E_i^{(+)} E_s^{(-)} E_s^{(+)} | \psi \rangle,
\]

where the fields at the output fiber measured by detector 1 and detector 2 are given by

\[
E_i^{(+)} = \sum_{k_i} \sqrt{\frac{\alpha_{\text{eff}}}{4V_Q}} a_{k_i} e^{-i\omega_i t} e^{-\frac{\alpha_i^2}{2\sigma_i^2}},
\]

and

\[
E_s^{(+)} = \sum_{k_s} \sqrt{\frac{\alpha_{\text{eff}}}{4V_Q}} a_{k_s} e^{-i\omega_s t} e^{-\frac{\alpha_s^2}{2\sigma_s^2}}.
\]

Here, \( t_m = T_m - l_m/c \) is the time at the output tip of the fiber, \( l_m \) is the optical path length from output tip of the fiber to the detectors, \( m = 1, 2 \), \( T_m \) is the registration time at \( m \)th detector, and the sum is over available modes \( k_m \).

The integrand in equation (21) can be rewritten as

\[
\langle \psi | E_i^{(-)} E_i^{(+)} E_s^{(-)} E_s^{(+)} | \psi \rangle = \langle 0 | E_i^{(+)} E_s^{(+)} | 0 \rangle.
\]

Using equations (22), (23) and (13), the term \( \langle 0 | E_i^{(+)} E_s^{(+)} | 0 \rangle \) in equation (24) can be expressed as \( \langle 0 | a_{i_k'} a_{s_k'} | 0 \rangle \times \langle 0 | a_{s_k'} a_{i_k} | 0 \rangle \). Using the commutation relationships of creation \((a)^\dagger\) and annihilation operator \((a)\)

\[
[a_k, a_{k'}] = 0 \\
[a_k^\dagger, a_{k'}^\dagger] = 0 \\
[a_k, a_{k'}^\dagger] = \delta_{kk'}
\]


and the operational properties of creation ($a^\dagger$) and annihilation operator ($a$) on a state

$$a_i |n\rangle = \sqrt{n} |n-1\rangle$$
$$a_i^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle,$$

it is easy to see that the term $\langle 0| E_i^{(+)} E_i^{(+)} |\psi\rangle$ only exists at $k_i = k_s$ and $k_i = k_i$. Now using the two-photon state in equation (13) and the fields defined in equations (22) and (23), the two-photon amplitude, i.e. equation (24) can be written as,

$$\langle 0| E_i^{(+)} E_i^{(+)} |\psi\rangle = \frac{\mathcal{A}_{\text{eff}}}{4V_Q} \sum_{k_i} F(k_i) e^{-i\omega_{k_i} t_{\text{ep}}} e^{-\frac{\gamma_i^2 + \gamma_i^2}{2}},$$

(26)

where $\gamma_i = \sigma_i$ is considered for this case.

After simplification, the expected number of a coincidence is calculated from each pulse (without any Raman noise), i.e. Equation (21), at the conditions of $\Delta k = 0$ becomes

$$\text{CC} = B (\gamma P_L L)^2 \frac{\sigma_i^2}{\sigma_p \sqrt{\frac{\gamma_p^2}{\gamma_p^2} + \frac{\sigma_i^2}{\sigma_i^2}}} R_{\text{CC}}^2(\gamma P_L L)$$
$$\text{CC} = B (\gamma P_L L)^2 \frac{1}{\sigma_p \sqrt{\left(\frac{\gamma_p}{\sigma_p}\right)^2 + 1}} R_{\text{CC}}^2(\gamma P_L L),$$

(27)

where $B = \frac{\sigma_p^2 A_{\text{eff}}}{4V_Q}$, and $R_{\text{CC}}^2(\gamma P_L L) = \sin^2(\gamma P_L L)$ is the SPM effect on the coincidence counts.

Here the introduced SPM term $\phi_m = \frac{2}{3} \gamma P_L L$ is $\ll \pi$ and the bandwidth of the pulse varies as a function of wavelength. The ratios of bandwidths of idler or signal to that of the pump remain the same as that of the pulse without broadening. To include the SPM, we derived the expression for the SPM effect $R_{\text{CC}}^2$ using the two-photon spectral amplitude and the Glauber probability of coincidence expressions above [8, 20, 22].

Since both single counts and coincidences depend on the ratio of idler or signal bandwidth to pump bandwidth, single counts and coincidences are unaffected by pulse broadening. The CC, $S_i$ and $I_L$ can be used to calculate $g^{(2)}$, the normalized second order correlation, as a measure of nonclassicality and entanglement.

In the reported FWM, (see figure 4) the signal and idler colors are chosen in a narrow bandwidth and this bandwidth provides a very small phase shift $\sum_{i} \Delta\phi_{m}(t_i)$ which is much smaller than $\pi/2$ for the range of pump intensities to be considered later. When the phase shift is much smaller than $\pi/2$, the pulse shape does not significantly distort as is shown in figure 5.

Varying the pump power while maintaining phase matching conditions results in a nonlinear variation of the intensity of single photon counts, coincidence photon counts, $g^{(2)}$ and entanglement generation. Below we discuss (a) single and coincidence photon generation and (b) the expected values of $g^{(2)}$ which are an indicator of entanglement generation [23–25].

2.7.1 Single and coincidence photon generation as a function of pump power

Now we discuss the generation of EP in regimes that can be useful for QC. For secure QC one or less EP pairs per pump pulse is normally required. Consider a 995 nm laser with a pulse duration of 650 fs and repetition rate of 80 MHz used as a pump to generate idler and signal photons in the regimes of 1530 nm and the 737 nm respectively. The 1530 nm idler photon can propagate through an optical fiber with minimum loss and can also be stored in a cryogenically cooled erbium doped fiber [26] while the 737 nm signal photon can be detected by a single photon detector very efficiently and can be stored in silicon (Si) in diamond as a long-lived memory [27]. A 20 cm long polarization maintained silica fiber with beat length ($L_b$) of 1.2 mm, [28–30] was chosen to match the beat wavelength between pump and signal–idler photons for use in the Sagnac loop. A pump pulse polarized along one of the principal axes of a birefringent fiber (slow ($j$) axis) generates signal and idler photons polarized along the opposite fiber axis (fast ($f$) axis), respectively. Here the nonlinear coefficient of refractive index $n_2 = 3.4 \times 10^{-20}$ m$^2$ W$^{-1}$ and $A_{\text{eff}} = 20 \mu$m$^2$.

Using equations (1)–(10) and the Sellmeier equations, [31] the phase matching is shown in figure 4 for two different peak pump powers of 1 kW (52 mW average) and 15 kW (1700 mW average) respectively. From figure 4, one can say that the phase matching (i.e. phase-mismatching would be zero) occurs at idler wavelengths of 1531.5 nm and 1527.3 nm at peak power 1 kW and 15 kW respectively. The corresponding signal wavelengths would be 736.9 nm and 737.8 nm respectively. It is clear that polarization entanglement is generated between the telecommunication wavelength ($\sim 1530$ nm) and the atomic wavelength of ($\sim 737$ nm).

Using equations (19), (20), and (27), figure 6 shows the variations of both single counts and coincidence counts as a function of peak pump power when equal bandwidth of the signal and idler beams is considered. The
slight drop in single photon and coincident photon counts with increasing pump power is due to the SPM of the pump. The expected number of single photons from each pump pulse at average power of 5.2 mW (i.e. peak 100 W) and average power of 52 mW (i.e. peak 1 kW) are $9.5 \times 10^{-5}$ and $1.5 \times 10^{-3}$ respectively. In erbium doped fiber, the absorption and emission cross sections are similar in the wavelength range of 1525–1535 nm [32] and the pump itself has a bandwidth of 5 nm. Consequently, a value of 1527.3 may be selected as the center wavelength of the idler since it interfaces to an erbium memory.

### 2.7.2. $g^{(2)}$ as a function of pump power

The term $g^{(2)}$ is the ratio of the coincidences $CC$ to the accidental coincidences of a Poissonian process which is equivalent to $S/I$, which is the product of the singles. The second order correlation, $g^{(2)} = \frac{CC}{S/I}$ as a function of pump power is shown on the left axis of figure 7. The value of $g^{(2)}$ is 66 at a pump power of 1 kW, which is equivalent to 52 mW average power. At this and lower pump powers, $g^{(2)} \gg 2$ and is indicative of the entangled nature of the photon pairs generated in the optical fiber [23–25]. Thus, figure 7 identifies the pump power regimes for applications requiring entanglement and for those only requiring high coincidence rates.
were computed as a four way integral and performed with the pump bandwidth set to 100 GHz, peak pump power of 1 kW as shown in figure 8. The calculation was performed with the pump bandwidth set to 100 GHz, peak pump power of 1 kW (52 mW), and the pulse rate set to 1 GHz. The plots in figure 8 show that for a 1 GHz bandwidth \( g^{(2)} = 12 \) and there are 270 coincidence photons/s. The signal or idler bandwidth was varied from 1 to 100 GHz and the corresponding wavelength bandwidths were 0.0018–0.18 nm for the signal photons and 0.0078–0.7818 nm for the idler photons. For a 100 GHz bandwidth \( g^{(2)} = 56 \) and the coincident photon pair rate is nearly \( 2.9 \times 10^5 \) s\(^{-1}\). Values of \( g^{(2)} \gg 2.0 \) imply quantum non-classicality and entanglement [23–25]. Currently Erbium memories have been experimentally demonstrated as feasible in the 1–100 GHz ranges or more though with reduced efficiency beyond \( \sim 10 \) GHz [33]. As will be discussed below most diamond memories demonstrated have been in the MHz bandwidth range except for the recent GHz bandwidth demonstration for SiV [34]. The SiV bandwidth can be up to the 60 GHz ground state hyperfine splitting, in principle. The germanium-vacancy (GeV) bandwidth can be up to 150 GHz and tin-vacancy (SnV) beyond 0.5 THz [35].

3. Connecting telecom to QM

To show the feasibility of entangling QMs, the expected single and coincidence counts for signal and idler photons for typical pump parameters were calculated (Case (a) below) as a function of prospective QM bandwidths. In general, as the signal and idler bandwidth filters decrease, the single photon and coincidence counts also decrease. For case (a) the \( g^{(2)} \) and coincidence rates for the specific wavelengths of \( \lambda_s = 736.9 \) nm and \( \lambda_i = 1531.5 \) nm as a function of the bandwidths of the EP pairs are shown in figure 8. The calculation was performed with the pump bandwidth set to 100 GHz, peak pump power of 1 kW (52 mW), and the pulse rate set to 1 GHz. The plots in figure 8 show that for a 1 GHz bandwidth \( g^{(2)} = 12 \) and there are 270 coincidence photons/s. The signal or idler bandwidth was varied from 1 to 100 GHz and the corresponding wavelength bandwidths were 0.0018–0.18 nm for the signal photons and 0.0078–0.7818 nm for the idler photons. For a 100 GHz bandwidth \( g^{(2)} = 56 \) and the coincident photon pair rate is nearly \( 2.9 \times 10^5 \) s\(^{-1}\). Values of \( g^{(2)} \gg 2.0 \) imply quantum non-classicality and entanglement [23–25]. Currently Erbium memories have been experimentally demonstrated as feasible in the 1–100 GHz ranges or more though with reduced efficiency beyond \( \sim 10 \) GHz [33]. As will be discussed below most diamond memories demonstrated have been in the MHz bandwidth range except for the recent GHz bandwidth demonstration for SiV [34]. The SiV bandwidth can be up to the 60 GHz ground state hyperfine splitting, in principle. The germanium-vacancy (GeV) bandwidth can be up to 150 GHz and tin-vacancy (SnV) beyond 0.5 THz [35].

3.1. Joint spectral amplitude (JSA) and factorability

Spectral entanglement and factorability properties were characterized in terms of the spectral purity \( (P) \) of the filtered JSA. The generated photon pair properties were computed from the JSA as a function of signal and idler frequency filter bandwidths.

The purity \( P \) involving the trace can be calculated from the JSA \( f(\omega_i, \omega_j) \) as a four way integral and normalization, [36]

\[
P = \frac{\int \int \int \int f(\omega_i, \omega_j) f^*(\omega'_i, \omega'_j) f(\omega_i, \omega_j) f^*(\omega_i, \omega_j) d\omega_i d\omega_i d\omega_j d\omega_j}{\left( \int \int f(\omega_i, \omega_j)^2 d\omega_i d\omega_j \right)^2}.
\]

The computations used the wavefunction probability amplitude JSA functions as a function of frequency with the signal and idler bandwidth filters, and were efficiently evaluated using Singular Value Decomposition. For a fixed 100 GHz pump bandwidth four frequency filter bandwidths of the signal and idler were computed with values of 500, 100, 50 and 10 GHz as shown in figure 9. In the normalized figures dark red represents the highest value and blue represents the lowest value. In general, purity approaching \( P = 0 \) indicates entanglement and \( P = 1 \) indicates factorable states. The joint spectral intensity and \( P \) were computed from JSA for signal–idler frequency filters. The purity as a function of the frequency filter were computed to be (a) \( P = 0.2575 \) (500 GHz), (b) \( P = 0.7071 \) (100 GHz), (c) \( P = 0.8944 \) (50 GHz) and (d) \( P = 0.9950 \) (10 GHz).
As expected, the 500 GHz signal and idler filters with the 100 GHz pump indicates spectral entanglement and therefore not factorable. However, as the signal and idler filters reduce bandwidth the energy entanglement decreases. The 10 GHz signal and idler bandwidth filters with the 100 GHz pump having $\mathcal{P} = 0.9950$ approaching $\mathcal{P} = 1$ indicates a highly factorable JSA. Clearly filtering, although imposing a reduction in photon counting rates, can come arbitrarily close to exact factorability. Adjustment of process parameters by fiber optics and pump engineering or adding another pump can make further improvements in purity without excess filtering.

### Figure 9
Joint spectral intensity (JSI) and $P$ computed from JSA for signal–idler frequency filters at (a) 500 GHz ($P = 0.2575$), (b) 100 GHz ($P = 0.7071$), (c) 50 GHz ($P = 0.8944$) and (d) 10 GHz ($P = 0.9950$).

#### 3.2. Spectral hole burning memories including Er
As mentioned earlier, a number of solid-state quantum optical memories have been demonstrated in the visible to NIR spectral ranges. Some of these operate with high efficiencies (up to 70% for Pr:YSO [37]). Additionally, long storage times are possible by transfer to spin waves [38], in principle up to hours for Eu:YSO [39]. However in the telecom band near 1.5 μm wavelength Er is the only choice. In crystals like Er:YSO storage times up to 20 ms [40] are possible but the bandwidth is limited to 100s of MHz [41]. To partially overcome the bandwidth limitation Er doped fibers have been explored [33]. They have demonstrated broad bandwidths on the order of 8 GHz (full inhomogeneous bandwidth >1 THz) which are a better match for the short pulses desired in future QC systems. They can also have a large time-bandwidth product, in excess of 100, and so can store many quantum optical modes. Current weaknesses are short storage times (∼5 ns) and low efficiencies (∼1%) [33].

#### 3.3. Ensemble versus single atom memories
Quantum optical memories based on single-emitter systems, like single atoms, ions, quantum dots or color centers, are preferred. For single emitter QMs, the stored qubit can in principle be used in quantum logic...
operations with other qubits in the node [42] to perform operations like deterministic Bell state measurements for entanglement swapping [43] or active correction of arbitrary errors [44].

The problem with single emitters is that the capture probability for photons tends to be very small, especially for solid-state emitters. Ensembles in contrast have large capture probabilities owing to their large numbers even for relatively weak optical transitions as in QMs based on rare earth doped materials [42].

3.4. SiV, GeV and SnV diamond memories
Nonetheless, significant recent progress has been made in the strong coupling of single optical emitters in diamond to single photons. In particular, SiV centers in diamond nano-photonic structures have demonstrated promise for scalable quantum photonic devices [45] due to low spectral diffusion. Significantly, single SiV centers have been shown to interact coherently with 12 picosecond laser pulses despite the bandwidth mismatch, [34] and even to store the coherence for 0.5 ns giving a time bandwidth product near 50. Recently, the spin coherence time for SiV was recently extended to 13 ms [27].

More recently, the GeV centers in diamond have shown even stronger coupling between emitters and photons [46] than SiV. In fact, the coupling is so strong that fluorescent emission from single GeV can be comparable to the light scattered by the diamond surface [46]. Thus, it seems efficient coupling of single photons to single solid-state emitters will soon be a reality. In addition to the higher quantum efficiency, the ground state hyperfine splitting of GeV is also larger (~150 GHz compared to 60 GHz for SiV) which translates to even higher bandwidth potential. SnV has bandwidth potential approaching 1 THz [35] and optical coupling similar to GeV.

4. FWM sources and spectral properties
This section discusses FWM sources of polarization EPs that can be used in QC and compares them to the properties of our fiber photon pair source connecting telecom to QMs. We show that our setup has favorable properties such as low Raman noise, ability to connect to memories of widely separated wavelengths without the need for frequency conversion, and can operate at ambient temperatures. In addition, we show that our design properties of our EPs are effectively spectrally decoupled from each other and due to the large separation of both components of the polarization state of each photon of the entangled pair to be stored in separate QM with one at short wavelength \( \lambda_s \) and the other at long wavelength \( \lambda_l \), Smith et al [48] in ‘Photon pair generation in birefringent optical fibers,’ theoretically and experimentally studied SFWM in birefringent optical fibers. Smith figure 6(a) shows experimental data where in single mode PM fiber the signal and idler correlated photons are generated far from the residual pump and the signal, idler, and residual pump all have single peaks without the apparent contamination of Raman noise peaks. In 2013 Zhou et al [10] demonstrated generation of polarization EP with high fidelity in the telecom band using PM DSF. These experiments also demonstrate the feasibility of our FWM scheme using DSF or non DSF type of PM fibers. Adjustable dispersion and nonlinear properties in nonlinear optical fibers allow phase matching in a large wavelength region [49].
In 2017 Kultavevuti et al. [50] demonstrated a SFWM polarization EP source implemented in an AlGaAs waveguide. This source reported CHSH violations >2.6 and a maximum fidelity of 0.92 ± 0.01. This source used pumping at \( \lambda_p = 1554.9 \) nm that generated polarization EPs of \( \lambda_s = 1533.7 \) nm and \( \lambda_i = 1577.03 \) nm. The estimated bandwidths were 60 nm for the waveguide and in the experimental demonstration the signal and idler were filtered using a DWDM at 100 GHz.

In 2011 Rarity et al. [51] demonstrated intrinsically narrowband polarization EP pairs generated in birefringent photonic crystal fiber which followed on from earlier work [52]. This source used a pump with \( \lambda_p = 705 \) nm with \( \lambda_s = 597 \) nm and \( \lambda_i = 860 \) nm. The signal and idler wavelengths are widely separated from the pump to lower the Raman background. Polarization EP visibilities were measured to be 90%–98% where a visibility >71% indicates that the photons are entangled enough to violate a Bell inequality. Tomography experiments indicated that the fidelity of the polarization EP state was 83.9% ± 0.078%. The polarization EPs generated in this system are intrinsically pure without the need for spectral filtering.

In 2010, Bennink [53] derived the spectral purity for spontaneous parametric downconversion (SPDC) assuming Gaussian states. In particular he notes that the spectral purity is directly related to the factorability of the two-photon state \( \Psi(\omega_s, \omega_i) \). When the two-photon state is completely factorable in terms of \( \omega_s \) and \( \omega_i \) the spectral purity of each photon of the entangled pair can approach 100%. Difficulties of spectral purity arise when the photon pairs are entangled in energy.

Silberhorn et al. performed an analysis for spectrally entangled (energy entangled) SPDC and FWM sources with Gaussian filters to show that an increase in purity has a corresponding drop in heralding efficiency [54] including analysis of the spectral purity in the supplemental information. They performed a verification experiment using a SPDC source and made recommendations for improving heralding efficiency.

In our case, when we measure within a signal bandwidth we will get a corresponding measurement in the idler bandwidth with a nonlinear shift of the central wavelength of the signal depending on the beat length and pump power. Figure 3 shows the phase matching conditions near erbium and SiV memory wavelengths as a function of pump power and beat length. For a beat length 1.2 mm at 52 mW pump power \( \lambda_s = 736.9 \) nm and \( \lambda_i = 1531.5 \) nm and at 780 mW pump power \( \lambda_s = 737.8 \) nm and \( \lambda_i = 1527.35 \) nm. For a beat length of 1.18 mm and a pump power of 1700 mW \( \lambda_s = 737.1 \) nm and \( \lambda_i = 1530.3 \) nm. Since in our case the polarizations are entangled but the energies have been filtered by the spectrally selective dichroic mirror in the setup and Gaussian frequency bandwidth filters in the signal and idler detection paths, the Bennink analysis above suggests high spectral purity. Our computation of the spectral purity above shows high spectral purity for bandwidth filtered signal and idler.

In 2007 Chen et al. performed experiments and analysis on fiber based FWM EP generation [55]. They noted that the primary source of error is spontaneous Raman scattering and that this can be mitigated by cooling the nonlinear fiber which improves the two photon visibility and entanglement purity. Our setup avoids spontaneous Raman scattering by moving the signal and idler wavelengths far from the pump generated Raman scattering. The use of a Sagnac loop in these fiber based EP generation systems also inherently provides 30 dB filtering of the pump. Our system still allows cooling of the nonlinear fiber should that be desired. In 2005 Chen et al. described using the coincidence to accidental ratio (CAR) as a figure of merit [56]. In this article they reported that values of this ratio >10 have been achieved indicating that 90% of the coincidences are correlated as a result of the source properties and are not accidental. Later in 2006 Chen and Lee et al. [57] in the article ‘High-Purity Telecom-Band EP-Pairs via Four-Wave Mixing in Dispersion-Shifted Fiber,’ reported that by cooling the fiber to 77 K a CAR of 111 was measured with a polarization two-photon interference (TPI) visibility >98% at room temperatures (300 K) the CAR was 28 and the reported polarization TPI visibility was 91%. The CAR ratio reported by Chen et al. [55–57] is the ratio of coincidence measurements to a background Poissonian process is equivalent to the \( g^{(2)} \) shown in figures 7–8. Our calculated \( g^{(2)} \) values in figure 8 range from 8.49 at 100 GHz bandwidth of the signal–idler wavelengths to 12.01 at 1 GHz signal–idler bandwidth which also indicates that approximately 90% of the coincidences are a result of the entanglement generated by our photon source and are not accidental.

The importance of the generation of polarization EPs and multipartite photon correlations by FWM as a function of the parameter space was highlighted by Petrov et al. in 2017 [58]. They showed that controlling parameters such as the nonlinearity, pump power, and bandwidth measurement filtering creates different regimes that can satisfy important quantum optical applications.

For the generation of polarization EPs, such as in our system, high values of two-photon polarization interference visibility (V) is an indicator of polarization entanglement and was described in 2012 for SPDC by Arihira et al. [59] and in 2007 for FWM by Takesue [60]. The visibility (V) is given as
\[
V = \frac{\mu_c \alpha_c}{2} + 2 \left( \frac{\mu_c \alpha_i}{2} + d_i \right) \left( \frac{\mu_c \alpha_i}{2} + d_i \right),
\]

where \( \mu_c \) is the expected number of coincident photons per pulse, \( \alpha_i \) and \( d_i \) are the transmittance and dark count probabilities for the signal (s) and idler (i) channels. Arihira et al also related the visibility to the Clauser-Horne-Shimony-Holt (CHSH) \( S \) parameter by

\[
S = 2 \sqrt{2} V,
\]

where a value of \( S \) greater than 2 is deemed non-classical and entangled. Correspondingly, a value of \( V \) greater than 70\% is also indicative of nonclassicality and entanglement.

For our system the polarization TPI visibility \( V \) and CHSH \( S \) parameter as a function of signal–idler bandwidth are shown in figure 10. Using equations (29) and (30) \( V \) and \( S \) curves are shown for cases (a) when the probability of a dark count per pulse is \( 1.669 \times 10^{-4} \) and (b) when the probability of a dark count per pulse is \( 2.295 \times 10^{-5} \). These low dark count probabilities are readily achieved by current detector measurement capabilities. Shibata et al [61] in 2015 describe a superconducting nanowire single photon detector (SNSPD) operating in the telecom band with a dark count rate (DCR) of \( 10^{-4} \) Hz, Rath in his 2017 thesis describes DCRs of \( 1 \) Hz [62], while Schuck et al [63] in 2013 describe a SNSPD operating over wavelengths from 768 to 1542 nm having DCRs as low as \( 1 \) mHz. In our system, pulsed at 1 GHz, the probability of the dark count per pulse ranges from \( 10^{-13} \) to \( 10^{-9} \) which are significantly smaller that the values used for the calculations shown figure 10. Note that the visibility and CHSH parameter remain relatively flat over a wide range of signal and idler bandwidths. This clearly demonstrates the feasibility of measuring high-purity EPs from the proposed source in the signal and idler bandwidth range of 1–100 GHz commensurate with current memories.

5. Summary

A fiber optic based polarization EP source is proposed that directly connects telecom processes, including a telecom memory, to a single qubit consisting of a SiV center in diamond (737 nm). Direct connection is achieved by generating the necessary photon pairs with a birefringent optical fiber, where the large wavelength difference allows operation in a regime safe from Raman scattering. The generated EPs are shown to be able to connect to an erbium QM and a QM in diamond having the potential to operate with single optical emitters thus enabling error correction and entanglement purification to entangle both QMs over telecom distances. Applications of the fiber optic based polarization EP source to future quantum repeaters and information teleportation networks are explored.
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