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DETECTING QUANTUM GRAVITATIONAL EFFECTS OF LOOP QUANTUM COSMOLOGY IN THE EARLY UNIVERSE?

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ABSTRACT

We derive the primordial power spectra and spectral indexes of the density fluctuations and gravitational waves in the framework of loop quantum cosmology (LQC) with holonomy and inverse-volume corrections by using the uniform asymptotic approximation method to its third order, at which the upper error bounds are $\lesssim 0.15\%$ and accurate enough for the current and forthcoming cosmological observations. Then, using the Planck, BAO, and supernova data, we obtain the tightest constraints on quantum gravitational effects from LQC corrections and find that such effects could be well within the detection of the current and forthcoming cosmological observations.

Key words: cosmology: observations – cosmology: theory – early universe – inflation

1. INTRODUCTION

Quantization of gravity has been one of the main driving forces in physics in the past decades (Kiefer 2012), and various approaches have been pursued, including string/M-Theory (Becker et al. 2007), loop quantum gravity (Rovelli & Vidotto 2015), and more recently the Horava–Lifshitz theory (Hořava 2009). However, it is fair to say that our understanding of it is still highly limited, and none of the aforementioned approaches is complete. One of the main reasons is the lack of evidence of quantum gravitational effects due to the extreme weakness of gravitational fields.

This situation has been dramatically changed recently, however, with the arrival of the era of precision cosmology (Kiefer & Kramer 2012; Krauss & Wilczek 2014; Woodard 2014). In particular, cosmic inflation (Guth 1981), which is assumed to have taken place during the first moments of time, provides the simplest and most elegant mechanism to produce the primordial density perturbations and gravitational waves. The former is responsible for the formations of the cosmic microwave background (CMB) and the large-scale structure of the universe (Baumann 2009). Current measurements of CMB (Komatsu et al. 2011; BICEP2/Keck & Planck Collaborations 2015; Planck Collaboration 2015) and observations of the large-scale distributions of dark matter and galaxies in the universe (Eisenstein et al. 2005; Tegmark et al. 2006; Beutler et al. 2011; Blake 2011) are in stunning agreement with it. On the other hand, since inflation is extremely sensitive to Planckian physics (Baumann 2009; Brandenberger & Martin 2013; Burgess et al. 2013), it also provides opportunities to get deep insight into the physics at the energy scales that cannot be reached by any man-made terrestrial experiments in the near future. In particular, it provides a unique window to explore quantum gravitational effects from different theories of quantum gravity, whereby one can falsify some of these theories with observational data that have uncomprehended accuracy (Abazajian et al. 2015) and obtain experimental evidence and valuable guidelines for the final construction of the theory of quantum gravity.

In this Letter, we study the quantum gravitational effects of loop quantum cosmology (LQC) in inflation (Bojowald 2005; Ashtekar & Singh 2011; Barrau et al. 2014) and show explicitly that these effects could be well within the detection of the current and forthcoming cosmological experiments (Abazajian et al. 2015). Such effects can be studied by introducing appropriate modifications at the level of the classical Hamiltonian, very similar to those studied in solid state physics (Bojowald 2005; Ashtekar & Singh 2011; Barrau et al. 2014). It was found that there are mainly two kinds of quantum corrections: the holonomy (Mielczarek 2008, 2009, 2014; Grain et al. 2010; Mielczarek et al. 2010, 2012; Li & Zhu 2011; Cailleteau et al. 2012a, 2012b) and inverse-volume corrections (Bojowald & Hossain 2007, 2008a, 2008b; Bojowald et al. 2009, 2010, 2011a, 2011b; Bojowald & Calcagni 2011; Amoros et al. 2014). These corrections modify not only the linear perturbations, but also the space–time background.

In particular, for a scalar field ϕ with its potential $V(\phi)$, the holonomy corrections modify the Friedmann and Klein–Gordon equations to the forms

$$\mathcal{H}^2 = \frac{8\pi G a^2 \rho_\phi}{3}\left(1 - \frac{\rho_\phi}{\rho_c}\right), \tag{1}$$

$$\phi'' + 2\mathcal{H}\phi' + V_{,\phi} = 0, \tag{2}$$

where a denotes the expansion factor, $\mathcal{H} \equiv a'/a$, and a prime denotes the derivative with respect to the conformal time η $(\equiv \int dt/a(t))$. ρ_c is a constant and characterizes the energy scale of the holonomy corrections, with $\rho_\phi = \phi'^2/(2a^2) + V(\phi)$. Clearly, the Big Bang singularity normally appearing at $\rho_\phi = \infty$ is now replaced by a big bounce occurring at $\rho_\phi = \rho_c$. In the infrared, we have $\rho_\phi/\rho_c \ll 1$, and Equation (1) reduces to that of general relativity. The evolutions of the anomaly-free cosmological scalar and tensor perturbations are described by the mode function $\mu_k(\eta)$, satisfying the equation (Cailleteau

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et al. 2012a, 2012b)

$$\mu_k''(\eta) + \left(\omega_k^2(\eta) - \frac{z''(\eta)}{z(\eta)}\right)\mu_k(\eta) = 0, \tag{3}$$

where $\omega_k^2(\eta) = \Omega(\eta)k^2$ with $\Omega(\eta) \equiv 1 - 2\rho/\rho_c$. The background-dependent function $z(\eta)$ is given by $z_S(\equiv a\phi'/\mathcal{H})$ for the scalar perturbations and z_T $(\equiv a/\sqrt{\Omega})$ for the tensor ones. To the first order of the slow-roll parameters and $\delta_H(\equiv \rho/\rho_c \ll 1)$, the inflationary spectra and spectral indexes with the holonomy corrections have been recently obtained by further assuming that the slow-roll parameters and δ_H are all constants (Mielczarek 2014).

With the inverse-volume corrections, on the other hand, the Friedmann and Klein–Gordon equations are modified to the forms (Bojowald & Calcagni 2011)

$$\mathcal{H}^2 = \frac{8\pi G\alpha}{3}\left(\frac{\phi'^2}{2\vartheta} + pV(\phi)\right), \tag{4}$$

$$\phi'' + 2\mathcal{H}\left(1 - \frac{d\ln\vartheta}{d\ln p}\right)\phi' + \vartheta p V_{,\phi} = 0, \tag{5}$$

in which $p \equiv a^2$, $\alpha \equiv 1 + \alpha_0\delta_{\rm Pl} + \mathcal{O}\left(\delta_{\rm Pl}^2\right)$, $\vartheta \equiv 1 + \vartheta_0\delta_{\rm Pl} + \mathcal{O}\left(\delta_{\rm Pl}^2\right)$, and $\delta_{\rm Pl} \propto a^{-\sigma}$, where α_0, ϑ_0, and σ are constants. (Note that here we use ϑ instead of ν adopted in Bojowald & Calcagni 2011 and reserve ν for other uses.) The values of α_0 and σ are currently subject to quantization ambiguities, while the magnitude of $\delta_{\rm Pl}$ is unknown, as so far we have no control over the details of the underlying full theory of quantum gravity (Bojowald & Calcagni 2011). However, when σ takes values in the range $0 < \sigma \leqslant 6$, the size of $\delta_{\rm Pl}$ does not depend on α_0 and ϑ_0 and can be written in the form $\delta_{\rm Pl} \equiv (a_{\rm Pl}/a)^\sigma$, where $a_{\rm Pl}$ is another arbitrary constant. The constant ϑ_0 is related to α_0 and σ via the consistency relation $\vartheta_0(\sigma - 3)(\sigma + 6) - 3\alpha_0(\sigma - 6) = 0$. However, to make the effective theory viable, we shall assume $\delta_{\rm Pl}(\eta) \ll 1$ at any given moment, so we can safely drop off all the second- and high-order terms of $\delta_{\rm Pl}(\eta)$. This assumption also guarantees that the slow-roll conditions can be imposed, even after the inverse-volume corrections are taken into account.

With the above assumption, Bojowald and Calcagni (BC; Bojowald & Calcagni 2011) studied the scalar and tensor perturbations with the inverse-volume corrections and found that the corresponding mode function $\mu_k(\eta)$ can be also cast in the form (3), but now with

$$\omega_k^2(\eta) = \left(1 + 2\alpha_0\delta_{\rm Pl}(\eta)\right)k^2 \tag{6}$$

for tensor, and

$$\omega_k^2(\eta) = \left(1 + 2\beta_0\delta_{\rm Pl}(\eta)\right)k^2 \tag{7}$$

for scalar, where $\beta_0 \equiv \sigma\vartheta_0(\sigma + 6)/36 + \alpha_0(15 - \sigma)/12$. With such modified dispersion relations, BC calculated the corresponding power spectra and spectral indexes to the first order of the slow-roll parameters, from which, together with Tsujikawa, they found (Bojowald et al. 2011a, 2011b) that the LQC effects are distinguishable from these of the noncommutative geometry or string, as the latter manifest themselves in small scales (Tsujikawa et al. 2003; Calcagni & Tsujikawa 2004; Piao et al. 2004), while the former mainly at large scales. To find explicitly the observational bounds on the inverse-volume quantum corrections, they considered the CMB likelihood for the potentials $V(\phi) = \lambda_n\phi^n$ and $V(\phi) = V_0 e^{-\kappa\lambda\phi}$ by using the *WMAP* 7 year data together with the large-scale structure, the Hubble constant measurement from the *Hubble Space Telescope*, SNe Ia, and Big Bang nucleosynthesis (Burles & Tytler 1998; Kowalski et al. 2008; Riess et al. 2009; Reid et al. 2010; Komatsu et al. 2011), the most accurate data available to them by then, and they obtained various constraints on $\delta(k)$ for different values of σ at the pivots $k_0 = 0.002\ {\rm Mpc}^{-1}$ and $k_0 = 0.05\ {\rm Mpc}^{-1}$, where $\delta(k) = \alpha_0\delta_{\rm Pl}(k)$ for $\sigma \neq 3$ and $\delta(k) = \vartheta_0\delta_{\rm Pl}(k)$ for $\sigma = 3$. An interesting feature is that the constraints are very sensitive to the choice of the pivots k_0, especially when σ is large $(\sigma \geqslant 2)$ but insensitive to the forms of the potential $V(\phi)$.

In this Letter, our goals are two-fold. First, we calculate the scalar and tensor power spectra, spectral indexes, and the ratio r to the second order of the slow-roll parameters for both of the holonomy and inverse-volume corrections so that they are accurate enough to match with the accuracy required by the current and forthcoming experiments (Abazajian et al. 2015). This becomes possible due to the recent development of the powerful uniform asymptotical approximation method (Habib et al. 2002; Zhu et al. 2014a, 2014b, 2014c, 2014d), which is designed specially for the studies of inflationary models after quantum gravitational effects are taken into account. Up to the third-order approximations in terms of the free parameter (λ^{-1}) introduced in the method, which is independent of the slow-roll inflationary parameters mentioned above, the upper error bounds are less than 0.15% (Zhu et al. 2014d). Second, we shall use the most recent observational data to obtain new constraints on $\delta(k_0)$ for the power-law potential $V(\phi) = \lambda_n\phi^n$, where n is chosen so that $r \lesssim 0.1$. With such constraints, we shall prove explicitly that the quantum gravitational effects from the inverse-volume corrections are within the range of the detection of the forthcoming experiments, especially of the Stage IV ones (Abazajian et al. 2015).

2. INFLATIONARY SPECTRA AND SPECTRAL INDEXES

To apply the uniform asymptotic approximation method, we first rewrite Equation (3) as $\frac{d^2\mu_k(y)}{dy^2} = \left[\lambda^2\hat{g}(y) + q(y)\right]\mu_k(y)$, where $y \equiv -k\eta$ and the parameter λ is a large constant to be used to trace the order of approximations. The reason to introduce two functions, $\hat{g}(y)$ and $q(y)$, instead of only one, is to use the extra degree of freedom to minimize the errors (Zhu et al. 2014a). For example, with the holonomy corrections, we have $\lambda^2\hat{g}(y) + q(y) = z''/(k^2z) - \Omega(\eta)$. Then, minimizing the error control function defined explicitly in Zhu et al. (2014a), we find that in this case $q(y)$ must be taken as $q(y) = -1/(4y^2)$. Once $q(y)$ is determined, $\hat{g}(y)$ is in turn uniquely fixed. Then, the corresponding approximate analytical solution will depend on the number and nature (real or complex) of the roots of the equation $\hat{g}(y) = 0$ (Zhu et al. 2014a, 2014b, 2014c). In the quasi–de Sitter background, it can be shown that $\hat{g}(y)$ currently has only one real root. In this case, the general expressions of the mode function, power spectra, and spectral indexes up to the third-order

Table 1
Values of Coefficients $\mathcal{Q}_{-1}^{\star(s)}$, $\mathcal{K}_{-1}^{\star(s)}$, $\mathcal{Q}_{0}^{\star(t)}$, and $\mathcal{K}_{0}^{\star(t)}$ for Different Values of σ

σ	1	2	3	4	5	6
$\mathcal{Q}_{-1}^{\star(s)}$	$\frac{\pi}{6}\alpha_0$	$\frac{2}{3}\alpha_0$	0	$\frac{-1616}{1629}\alpha_0$	$\frac{475}{2896}\pi\alpha_0$	$\frac{10512}{905}\alpha_0$
$\mathcal{K}_{-1}^{\star(s)}$	$-\frac{\pi}{6}\alpha_0$	$-\frac{4}{3}\alpha_0$	0	$\frac{320\alpha_0}{81}$	$-\frac{125}{144}\pi\alpha_0$	$-\frac{352}{5}\alpha_0$
$\mathcal{Q}_{0}^{\star(t)}$	$-\frac{725\pi}{2172}\alpha_0$	$-\frac{244}{543}\alpha_0$	0	$\frac{11728}{8145}\alpha_0$	$\frac{8165\pi}{5792}\alpha_0$	$\frac{13920}{1267}\alpha_0$
$\mathcal{K}_{0}^{\star(t)}$	$\frac{\pi}{3}\alpha_0$	$\frac{8}{9}\alpha_0$	0	$-\frac{2368}{405}\alpha_0$	$-\frac{1025\pi}{144}\alpha_0$	$-\frac{6976}{105}\alpha_0$

approximations (in terms of λ^{-1}) were given explicitly in Zhu et al. (2014d). Applying them to the case with the holonomy corrections, we find (T. Zhu et al. 2015, in preparation)

$$\begin{aligned}
\Delta_s^2(k) = A_s^\star\Big[& 1 - 2\big(1 + D_p^\star\big)\epsilon_{\star1} - D_p^\star\epsilon_{\star2} + \delta_{\star H} \\
&+ \left(2D_p^{\star2} + 2D_p^\star + \frac{\pi^2}{2} - 5 + \Delta_1^\star\right)\epsilon_{\star1}^2 \\
&+ \left(\frac{1}{2}D_p^{\star2} + \frac{\pi^2}{8} - 1 + \frac{\Delta_1^\star}{4}\right)\epsilon_{\star2}^2 + \frac{3}{2}\delta_{\star H}^2 \\
&- D_p^\star\delta_{\star H}\epsilon_{\star2} + \left(\frac{\pi^2}{24} - \frac{1}{2}D_p^{\star2} + \Delta_2^\star\right)\epsilon_{\star2}\epsilon_{\star3} \\
&+ \left(D_p^{\star2} - D_p^\star + \frac{7\pi^2}{12} - 7 + \Delta_1^\star + 2\Delta_2^\star\right) \\
&\times \epsilon_{\star1}\epsilon_{\star2} - \big(4D_p^\star + 6\big)\delta_{\star H}\epsilon_{\star1}\Big],\\
\Delta_t^2(k) = A_t^\star\Big[& 1 + \delta_{\star H} + \frac{3}{2}\delta_{\star H}^2 - 2\Big(D_p^\star + 1\Big) \\
&\times \epsilon_{\star1} - \big(4D_p^\star + 6\big)\delta_{\star H}\epsilon_{\star1} \\
&+ \left(2D_p^{\star2} + 2D_p^\star + \frac{\pi^2}{2} - 5 + \Delta_1^\star\right)\epsilon_{\star1}^2 \\
&+ \left(-2D_p^{\star2} - D_p^\star + \frac{\pi^2}{12} + 2\Delta_2^\star\right)\epsilon_{\star1}\epsilon_{\star2}\Big],\\
n_s = 1 &- 2\epsilon_{\star1} - \epsilon_{\star2} + 4\delta_{\star H}\epsilon_{\star1} - 2\epsilon_{\star1}^2 \\
&- (3 + 2D_n^\star)\epsilon_{\star1}\epsilon_{\star2} - D_n^\star\epsilon_{\star2}\epsilon_{\star3},\\
n_t = &-2\epsilon_{\star1} + 4\delta_{\star H}\epsilon_{\star1} - 2\epsilon_{\star1}^2 - 2\big(D_n^\star + 1\big)\epsilon_{\star1}\epsilon_{\star2},\\
r = &16\epsilon_{\star1}\big(1 + D_p^\star\epsilon_{\star2}\big),
\end{aligned} \tag{8}$$

where $\delta_H \equiv \rho_\phi/\rho_c \ll 1$, $A_s^\star \equiv 181H_\star^2/(72e^3\pi^2\epsilon_{\star1})$, $A_t^\star \equiv 181H_\star^2/(36e^3\pi^2)$, $D_p^\star = 67/181 - \ln 3$, $D_n^\star = 10/27 - \ln 3$, $\Delta_1^\star = \frac{485296}{98283} - \frac{\pi^2}{2}$, $\Delta_2^\star = \frac{9269}{589698}$, and $\star$ denotes quantities evaluated at horizon crossing $a(\eta_\star)H(\eta_\star) = \sqrt{\Omega(\eta_\star)}k$. ϵ_n denotes the slow-roll parameters, defined as $\epsilon_1 \equiv -\dot{H}/H^2$ and $\epsilon_{n+1} \equiv \dot{\epsilon}_n/(H\epsilon_n)$ $(n \geqslant 1)$. Note that in the above expressions we have ignored terms at the orders higher than $\mathcal{O}(\epsilon^3, \epsilon^2\delta_H)$. To the first order, it can be shown that our results are consistent with those presented in Mielczarek (2014).

In the case with the inverse-volume corrections, we have $\lambda^2\hat{g}(y) + q(y) = k^{-2}(z''/z - \omega_k^2(\eta))$, where $\omega_k^2(\eta)$ is given by Equation (6), with $z_s(\eta) \equiv a\dot{\varphi}[1 + \frac{1}{2}(\alpha_0 - 2\vartheta_0)\delta_{\rm Pl}]$ and $z_t(\eta) \equiv a(1 - \alpha_0\delta_{\rm Pl}/2)$, respectively. To minimize the errors, $q(y)$ must also be chosen as in the last case, and then it can be shown that $\hat{g}(y) = 0$ has only one real root, and as a result, the general expressions of the mode function, power spectra, and spectral indexes given in Zhu et al. (2014d) are also applicable to this case, which yield (T. Zhu et al. 2015, in preparation)

$$\begin{aligned}
\Delta_s^2(k) = A_s\Big[& 1 - 2\big(1 + D_p^\star\big)\epsilon_{\star1} - D_p^\star\epsilon_{\star2} + \epsilon_{\rm Pl}\left(\frac{3}{2}\mathcal{H}_\star\right)^\sigma \\
&\times \left(\mathcal{Q}_{-1}^{\star(s)}\epsilon_{\star1}^{-1} + \mathcal{Q}_{0}^{\star(s)} + \mathcal{Q}_{1}^{\star(s)}\epsilon_{\star2}\epsilon_{\star1}^{-1}\right)\Big],\\
\Delta_t^2(k) = A_t\Big[& 1 - 2\big(D_p^\star + 1\big)\epsilon_{\star1} + \epsilon_{\rm Pl}\left(\frac{3}{2}\mathcal{H}_\star\right)^\sigma\mathcal{Q}_0^{\star(t)}\Big],\\
n_s = 1 &- 2\epsilon_{\star1} - \epsilon_{\star2} - 2\epsilon_{\star1}^2 \\
&- \big(3 + 2D_n^\star\big)\epsilon_{\star1}\epsilon_{\star2} - D_n^\star\epsilon_{\star2}\epsilon_{\star3} \\
&+ \epsilon_{\rm Pl}\left(\frac{3}{2}\mathcal{H}_\star\right)^\sigma\left(\mathcal{K}_{-1}^{\star(s)}\epsilon_{\star1}^{-1} + \mathcal{K}_{0}^{\star(s)} + \mathcal{K}_{1}^{\star(s)}\epsilon_{\star2}\epsilon_{\star1}^{-1}\right),\\
n_t = &-2\epsilon_{\star1} - 2\epsilon_{\star1}^2 - 2\big(D_n^\star + 1\big) \\
&\times \epsilon_{\star1}\epsilon_{\star2} + \epsilon_{\rm Pl}\left(\frac{3}{2}\mathcal{H}_\star\right)^\sigma\mathcal{K}_0^{\star(t)},\\
r = 16&\epsilon_{\star1}\Big[1 + D_p\epsilon_{\star2} - \epsilon_{\rm Pl}\left(\frac{3}{2}\mathcal{H}_\star\right)^\sigma\mathcal{Q}_{-1}^{\star(s)}\epsilon_{\star1}^{-1}\Big].
\end{aligned} \tag{9}$$

Note that we parameterize $\delta_{\rm Pl}(\eta) = (a_{\rm Pl}/k)^\sigma(-a\eta)^{-\sigma}y^\sigma$ with $\epsilon_{\rm Pl} \equiv (a_{\rm Pl}/k)^\sigma, k \equiv (-a\eta)^{-\sigma}$. In Table 1, we list the values of the coefficients $\mathcal{Q}_{-1}^{\star(s)}, \mathcal{K}_{-1}^{\star(s)}, \mathcal{Q}_{0}^{\star(t)}$, and $\mathcal{K}_{0}^{\star(t)}$ for different values of σ, as they represent the dominant contributions. The rest of the terms appearing in the above expressions are subdominant and will not be given here, but they are given explicitly in T. Zhu et al. (2015, in preparation). When $\sigma = 3$, $\mathcal{Q}_{-1}^{\star(s)}$ and $\mathcal{K}_{-1}^{\star(s)}$ vanish, so one has to consider contributions from $\mathcal{Q}_{0}^{\star(s)}$ and $\mathcal{K}_{0}^{\star(s)}$, which are given by $\mathcal{Q}_0^{\star(s)} = \frac{513\pi}{11584}\vartheta_0$ and $\mathcal{K}_0^{\star(s)} = -\frac{9\pi}{64}\vartheta_0$. We emphasize that the modified power spectra and also the spectral indices are now explicitly scale dependent because of $\epsilon_{\rm Pl} \sim k^{-\sigma}$.

Before considering the observational constraints, let us first note that in Bojowald & Calcagni (2011) and Bojowald et al. (2011a, 2011b) the observables n_s, n_t, and r were calculated up to the first order of the slow-roll parameters. Comparing their results with ours, after writing all expressions in terms of the same set of parameters, say, $\epsilon_V\,[\equiv M_{\rm Pl}^2(V_{,\phi}/V)^2/2]$, $\eta_V\,[\equiv M_{\rm Pl}^2V_{,\phi\phi}/V]$, and $\xi_V^2 = M_{\rm Pl}^4V_{,\phi}V_{,\phi\phi\phi}/V^2]$, we find that our results are different from theirs. A closer examination shows that this is mainly due to the following. (a) In Bojowald

& Calcagni (2011), the horizon crossing was taken as $k = \mathcal{H}$. However, due to the quantum gravitational effects, the dispersion relation is modified to the form (6), so the horizon crossing should be at $\omega_k = \mathcal{H}$. (b) In Bojowald & Calcagni (2011), the mode function was first obtained at two limits, $k \gg \mathcal{H}$ and $k \ll \mathcal{H}$, and then matched together at the horizon crossing where $k \simeq \mathcal{H}$. This may lead to huge errors (Joras & Marozzi 2009; Ashoorioon et al. 2011), as neither $\mu_{k\gg\mathcal{H}}$ nor $\mu_{k\ll\mathcal{H}}$ is a good approximation of the mode function μ_k at the horizon crossing. The above arguments can be seen further by considering the exact solution of μ_k:

$$\mu_k(\eta)\big|_{\sigma=2} = \frac{c_1}{\sqrt{-\eta}}\mathrm{WW}\left(-\frac{ia_1}{4\sqrt{a_2}}, \frac{\nu}{2}, -i\sqrt{a_2}k^2\eta^2\right), \quad (10)$$

for the case $\sigma = 2$, where $\mathrm{WW}(b_1, b_2, z)$ denotes the WhittakerW function, $a_1 \equiv 1 - m\epsilon_{\mathrm{Pl}}\kappa$, $a_2 \equiv 2\beta_0\epsilon_{\mathrm{Pl}}\kappa$, $m(\eta)$ is the coefficient of $\delta_{\mathrm{Pl}}(\eta)$ in the definition $-\frac{1}{k^2}\frac{z''}{z} = \frac{\nu^2 - 1/4}{y^2} + \frac{m}{y^2}\delta_{\mathrm{Pl}}$, and $\nu = 3/2 + \epsilon_1 + \epsilon_2/2$ for the scalar perturbations, and $\nu = 3/2 + \epsilon_1$ for the tensor. Matching it to the Bunch–Davies vacuum solution at $k \gg \mathcal{H}$, we find that $c_1 = e^{-\frac{a_1\pi}{8\sqrt{a_2}}}\big/\left(\sqrt{2}ka_2^{1/4}\right)$. With the above mode function, the power spectra and spectral indexes can be calculated and are found to be the same as those given here, but are different from those of Bojowald & Calcagni (2011) and Bojowald et al. (2011a, 2011b). For more details, see T. Zhu et al. (2015, in preparation).

3. DETECTION OF QUANTUM GRAVITATIONAL EFFECTS

The contributions to the inflationary spectra and spectral indices from the holonomy corrections are introduced through the parameter $\delta_{\star H}$, which are of the order of 10^{-12} for typical values of the parameters (Mielczarek 2014). Then, with the current and forthcoming observations (Abazajian et al. 2015), it is very difficult to detect such effects.

On the other hand, for the inverse-volume corrections, let us consider the power-law potential $V(\phi) = \lambda_n\phi^n$, for which we find that $\eta_V = 2(n-1)\epsilon_V/n$, $\xi_V^2 = 4(n-1)(n-2)\epsilon_V^2/n^2$, where $\epsilon_V = M_{\mathrm{Pl}}^2 n^2/(2\phi^2)$. Thus, without the inverse-volume corrections ($\delta_{\mathrm{Pl}} = 0$), we have $n_s = n_s(\epsilon_V)$ and $r = r(\epsilon_V)$, and up to the second order of ϵ_V, the relation (Creminelli et al. 2014)

$$\begin{aligned}\Gamma_n(n_s, r) \equiv & \left(n_s - 1\right) + \frac{(2+n)r}{8n} \\ & + \frac{\left(3n^2 + 18n - 4\right)\left(n_s - 1\right)^2}{6(n+2)^2} = 0\end{aligned} \quad (11)$$

holds precisely. The results from Planck 2015 are $n_s = 0.968 \pm 0.006$ and $r_{0.002} < 0.11$ (95% CL; Komatsu et al. 2011; BICEP2/Keck & Planck Collaborations 2015; Planck Collaboration 2015), which yields $n \lesssim 1$. In the forthcoming experiments, especially the Stage IV ones, the errors of the measurements on both n_s and r are $\leqslant 10^{-3}$ (Abazajian et al. 2015), which implies $\sigma(\Gamma_n) \leqslant 10^{-3}$. On the other hand, when the inverse-volume corrections are taken into account ($\delta_{\mathrm{Pl}} \neq 0$), we have $n_s = n_s(\epsilon_V, \epsilon_{\mathrm{Pl}})$ and $r = r(\epsilon_V, \epsilon_{\mathrm{Pl}})$, and Equation (11) is modified to

$$\Gamma_n(n_s, r) = \mathcal{F}(\sigma)\frac{\delta(k)}{\epsilon_V}, \quad (12)$$

where $\delta(k) \equiv \alpha_0\epsilon_{\mathrm{Pl}}H^\sigma$ and $\mathcal{F}(\sigma) \simeq \mathcal{O}(1)$ (T. Zhu et al. 2015, in preparation). Clearly, the right-hand side of the above equation represents the quantum gravitational effects from the inverse-volume corrections. If it is equal to or greater than $\mathcal{O}(10^{-3})$, these effects shall be well within the detection of the current or forthcoming experiments. It is interesting to note that the quantum gravitational effects are enhanced by an order ϵ_V^{-1}, which is absent in Bojowald & Calcagni (2011).

In the following, we run the Cosmological Monte Carlo code (Gong et al. 2008) with the Planck (Ade 2014), BAO (Anderson et al. 2012), and Supernova Legacy Survey (Conley et al. 2011) data for the power-law potential with $n = 1$, which can be naturally realized in the axion monodromy inflation motivated by string/M theory (Silverstein & Westphal 2008; McAllister et al. 2010). To compare our results with those acquired in Bojowald et al. (2011b), we shall carry out our CMB likelihood analysis as close to theirs as possible. In particular, we assume the flat cold dark matter model with the effective number of neutrinos $N_{\mathrm{eff}} = 3.046$ and fix the total neutrino mass $\Sigma m_\nu = 0.06$ eV. We vary the seven parameters: (i) baryon density parameter, $\Omega_b h^2$; (ii) dark matter density parameter, $\Omega_c h^2$; (iii) the ratio of the sound horizon to the angular diameter, θ; (iv) the reionization optical depth τ; (v) $\delta(k_0)/\epsilon_V$; (vi) ϵ_V; and (vii) $\Delta_s^2(k_0)$. We take the pivot wavenumber $k_0 = 0.05\ \mathrm{Mpc}^{-1}$ used in Planck to constrain $\delta(k_0)$ and ϵ_V. In Figure 1, the constraints on δ/ϵ_V and ϵ_V are given, respectively, for $\sigma = 1$ and $\sigma = 2$. In particular, we find that $\delta(k_0) \lesssim 6.8 \times 10^{-5}$ (68% CL) for $\sigma = 1$ and $\delta(k_0) \lesssim 1.9 \times 10^{-8}$ (68% CL) for $\sigma = 2$, which are much tighter than those given in Bojowald et al. (2011b). The upper bound for $\delta(k_0)$ decreases dramatically as σ increases (Bojowald et al. 2011b; T. Zhu et al. 2015, in preparation). However, for any given σ, the best-fitting value of ϵ_V is about 10^{-2}, which is rather robust when comparing with the case without the gravitational quantum effects (Ade 2014). It is remarkable to note that despite the tight constraints on $\delta(k_0)$, because of the ϵ_V^{-1} enhancement of Equation (12), such effects can be well within the range of the detection of the current and forthcoming cosmological experiments (Abazajian et al. 2015) for $\sigma \lesssim 1$. Note that small values of σ are also favorable theoretically (Bojowald & Calcagni 2011).

4. CONCLUSIONS

Using the uniform asymptotic approximation method developed recently in Zhu et al. (2014a, 2014b, 2014c, 2014d), we have accurately computed the power spectra, the spectral indices, and the ratio r of the scalar and tensor perturbations of inflation in LQC to the second order of the slow-roll parameters after the corrections of the holonomy (Mielczarek 2008, 2009, 2014; Grain et al. 2010; Mielczarek et al. 2010; Cailleteau et al. 2012a, 2012b) and inverse-volume (Bojowald & Hossain 2007, 2008a, 2008b; Bojowald et al. 2009, 2010, 2011b; Bojowald & Calcagni 2011; Li & Zhu 2011; Mielczarek et al. 2012; Amoros et al. 2014) are taken into account. The upper error bounds are $\lesssim 0.15\%$, which is accurate enough for the current and forthcoming experiments

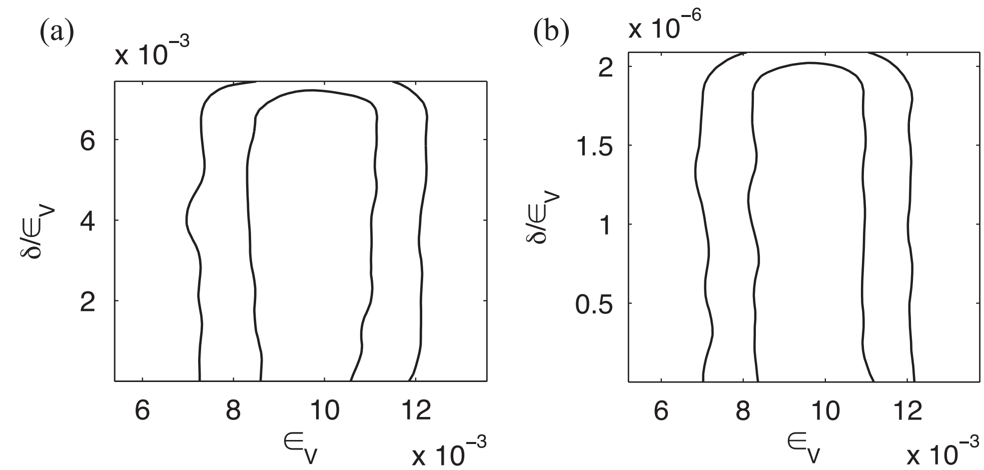

Figure 1. Two-dimensional marginalized distribution for the parameters δ/ϵ_V and ϵ_V at the pivot $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ for the power-law potential with $n = 1$. (a) The left panel is for $\sigma = 1$, and (b) the right panel is for $\sigma = 2$. The internal and external lines correspond to the confidence levels of 68% and 95%, respectively.

(Abazajian et al. 2015). Utilizing the most accurate CMB, BAO, and supernova data currently available publicly (Conley et al. 2011; Ade 2014; Anderson et al. 2012), we have carried out the CMB likelihood analysis and found constraints on $(\delta(k_0), \epsilon_V)$, the tightest ones obtained so far in the literature. Even with such tight constraints, the quantum gravitational effects due to the inverse-volume corrections of LQC can be well within the range of the detection of the current and forthcoming cosmological experiments (Abazajian et al. 2015), provided that $\sigma \lesssim 1$.

It should be noted that in our studies of the holonomy corrections, the effects of bouncing of the universe are insignificant by implicitly assuming that inflation occurred long after the bouncing. This is the same as those considered in Mielczarek (2008, 2009, 2014), Grain et al. (2010), Mielczarek et al. (2010, 2012), Li & Zhu (2011; Mielczarek et al. 2012), and Cailleteau et al. (2012a, 2012b). Thus, it is expected that quantum gravitational effects from these corrections are negligible. However, when the whole process of the bouncing is properly taken into account, such effects may not be small at all (Grain & Barrau 2009; Barrau & Grain 2014). It would be very interesting to reconsider the observational aspects of these effects, although cautions must be taken, as Equation (1) was derived only for small potentials. Without this condition, there would be additional quantum corrections that are neither of holonomy nor of inverse-volume type.

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