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YOUNG SOLAR SYSTEM'S FIFTH GIANT PLANET?

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ABSTRACT

Studies of solar system formation suggest that the solar system's giant planets formed and migrated in the protoplanetary disk to reach the resonant orbits with all planets inside ∼15 AU from the Sun. After the gas disk's dispersal, Uranus and Neptune were likely scattered by the gas giants, and approached their current orbits while dispersing the transplanetary disk of planetesimals, whose remains survived to this time in the region known as the Kuiper Belt. Here we performed *N*-body integrations of the scattering phase between giant planets in an attempt to determine which initial states are plausible. We found that the dynamical simulations starting with a resonant system of four giant planets have a low success rate in matching the present orbits of giant planets and various other constraints (e.g., survival of the terrestrial planets). The dynamical evolution is typically too violent, if Jupiter and Saturn start in the 3:2 resonance, and leads to final systems with fewer than four planets. Several initial states stand out in that they show a relatively large likelihood of success in matching the constraints. Some of the statistically best results were obtained when assuming that the solar system initially had five giant planets and one ice giant, with the mass comparable to that of Uranus and Neptune, and which was ejected to interstellar space by Jupiter. This possibility appears to be conceivable in view of the recent discovery of a large number of free-floating planets in interstellar space, which indicates that planet ejection should be common.

Key words: planets and satellites: dynamical evolution and stability – planets and satellites: formation

1. INTRODUCTION

Studies of giant planets' interaction with a protoplanetary gas disk show that their orbits radially migrate, and typically achieve a compact configuration, in which the pairs of neighbor planets are locked in orbital resonances (Kley 2000; Masset & Snellgrove 2001). The resonant planetary systems emerging from protoplanetary disks can become dynamically unstable after the gas disappears, leading to a phase when planets scatter off each other. This model can explain the observed resonant exoplanets, commonly large exoplanet eccentricities (Weidenschilling & Marzari 1996), and microlensing data that show evidence for a large number of planets that are free floating in interstellar space (Sumi et al. 2011).

The solar system, with the widely spaced and nearly circular orbits of the giant planets, bears little resemblance to the bulk of known exoplanets. Yet, if our understanding of the physics of planet–gas-disk interaction is correct, it is likely that the young solar system followed the evolutionary path outlined above. Jupiter and Saturn, for example, were most likely trapped in the 3:2 resonance (Masset & Snellgrove 2001; Morbidelli & Crida 2007; Pierens & Nelson 2008), defined as $P_{\rm Saturn}/P_{\rm Jupiter} = 1.5$, where $P_{\rm Jupiter}$ and $P_{\rm Saturn}$ are the orbital periods of Jupiter and Saturn (this ratio is 2.49 today).

To stretch to the present state, the outer solar system most likely underwent a violent phase when planets scattered off each other and acquired eccentric orbits (Thommes et al. 1999; Tsiganis et al. 2005). The system was subsequently stabilized by damping the excess orbital energy into the transplanetary disk, whose remains survived to this time in the Kuiper Belt. Finally, as evidenced by dynamical structures observed in the Kuiper Belt, planets radially migrated to their current orbits by scattering planetesimals (Malhotra 1995; Levison et al. 2008).

2. METHOD

We conducted computer simulations of the early evolution of the solar system to determine the initial states of planetary orbits (Batygin & Brown 2010). First, we performed hydrodynamic and *N*-body simulations to identify the resonant configurations that may have occurred among the young solar system's giant planets. Our hydrodynamic simulations used Fargo (Masset 2000) and followed the method described in Morbidelli et al. (2007). As the Fargo simulations are CPU expensive, we used these results as a guide and generated many additional resonant systems with the *N*-body integrator known as SyMBA (Duncan et al. 1998).

Planets with masses corresponding to those of Jupiter, Saturn, and ice giants were placed in initial orbits with the period ratios slightly larger that those of the selected resonances. The planets were then migrated into resonances with SyMBA modified to include forces that mimic the effects of gas. We considered cases with four and five initial planets, where in the latter case an additional planet was placed into a resonant orbit between Saturn and the ice giants, or beyond the orbit of the outer ice giant. The fifth planet was given a mass between 1/3 and 3 times the mass of Uranus.

Different starting positions of planets, rates of the semi-major axis and eccentricity evolution, and timescales for the gas disk's dispersal produced different results. For Jupiter and Saturn, we confined the scope of this study to the 3:2 and 2:1 resonances, because the former is strongly preferred from previous hydrodynamic studies (Masset & Snellgrove 2001; Morbidelli et al. 2007; Pierens & Nelson 2008). The 2:1 resonance was included for comparison. The eccentricities, inclinations, and resonant amplitudes obtained here were typically $e < 0.1$, $i < 0\overset{\circ}{.}2$ and $< 60°$. The inner ice giant had $0.05 < e < 0.1$, while the other planets acquired more circular orbits (Jupiter had $e \lesssim 0.01$ in most cases).

The instability of a planetary system can occur after the gas disk's dispersal when the stabilizing effects of gas are removed. Such an instability can be triggered spontaneously (e.g., Weidenschilling & Marzari 1996) or by divergent migration of planets produced by their interaction with planetesimals leaking into the planet-crossing orbits from the transplanetary disk

(Thommes et al. 1999). In the latter case, the instability is produced when Jupiter and Saturn cross a major mean motion resonance (Tsiganis et al. 2005; also known as the *Nice model*).

In the solar system, it is often assumed that the instability occurred at the time of the Late Heavy Bombardment of the Moon some 3.9 Gyr ago, when the lunar basins with known ages formed (Hartmann et al. 2000). If so, the solar system's giant planets would be required to remain on their initial resonant orbits for about 600 Myr. To allow for this possibility, we sifted through the resonant configurations identified above and selected those that were stable over 10^9 yr, if considered in isolation. Only the stable systems were used for the follow-up simulations, in which we tracked the evolution of planetary orbits through and past the instability.

We included the effects of the transplanetary planetesimal disk in these simulations. The disk was represented by 1000 equal-mass bodies that were placed into orbits with low orbital eccentricities and inclinations, and in radial distances between $r_{\rm in} < r < r_{\rm out}$. The surface density was set to be $\Sigma = 1/r$. The outer edge of the disk was placed at $r_{\rm out} = 30$ AU, so that the planetesimal-driven migration is expected to park Neptune near its present semi-major axis (Gomes et al. 2004).

We considered cases with $r_{\rm in} = a_N + \Delta$, where a_N is the semi-major axis of the outer ice giant, N is the number of planets, and $\Delta = 0.5$, 1, and 3.5 AU. The instability was triggered early for $\Delta = 0.5$ AU. To trigger the instability for $\Delta = 1$ and 3.5 AU, we broke the resonant locks by altering the mean anomaly of one of the ice giants. This method was inspired by the recent study of the late instability that showed that planets can slowly exchange orbital energy with a distant planetesimal disk and break from the resonances when the resonant amplitude exceeds certain limits (Levison et al. 2011).

In either case discussed above, the scattering phase between planets starts shortly after the beginning of our simulations, which guarantees a low CPU cost. We considered six different masses of the planetesimal disk, $m_{\rm disk} = 10$, 20, 35, 50, 75, and 100 $M_{\rm Earth}$, and performed 30 simulations in each case, where different evolution histories were generated by randomly seeding the initial orbit distribution of planetesimals. In total, we completed over 6000 scattering simulations. Each system was followed for 100 Myr with standard SyMBA.

3. CONSTRAINTS

We defined several criteria to measure the overall success of simulations. First, the final planetary system must have four giant planets (criterion A) with orbits that resemble the present ones (criterion B). Note that A means that one planet must be ejected in the five-planet runs, while all four planets need to survive in the four-planet runs. As for B, we claim success if the final mean semi-major axis of each planet is within 20% of its present value and if the final mean eccentricities and mean inclinations are no larger than 0.11 and 2°, respectively. These thresholds were obtained by doubling the current mean eccentricity of Saturn ($e_{\rm Saturn} = 0.054$) and mean inclination of Uranus ($i_{\rm Uranus} = 1\overset{\circ}{.}02$).

For the successful runs, as defined above, we also checked on the history of encounters between giant planets, evolution of the secular g_5, g_6, and s_6 modes, and secular structure of the final planetary systems. To explain the observed populations of the irregular moons that are roughly similar at each planet (Jewitt & Haghighipour 2007), all planets—including Jupiter—must participate in encounters (Nesvorný et al. 2007). Encounters of Jupiter and/or Saturn with ice giants may also be needed to excite the g_5 mode in the Jupiter's orbit to its current amplitude ($e_{55} = 0.044$; Morbidelli et al. 2009).

It turns out that it is generally easy to have encounters between one of the ice giants and Jupiter if planets start in a compact resonant configuration. The amplitudes of the g_6 and s_6 modes also do not pose a problem. It is much harder to excite e_{55}, however. We therefore opt for a criterion in which we require that $e_{55} > 0.022$ in the final systems, i.e., at least half of its current value (criterion C). The e_{55} amplitude was determined by following the final planetary systems for an additional 10 Myr (without planetesimals) and Fourier-analyzing the results (Šidlichovský & Nesvorný 1996).

The evolution of secular modes, mainly g_5, g_6, and s_6, is constrained from their effects on the terrestrial planets and asteroid belt. As outer planets scatter and migrate, these frequencies change. This may become a problem, if g_5 slowly swipes over the g_1 or g_2 modes, because the strong $g_1 = g_5$ and $g_2 = g_5$ resonances can produce excessive excitation and instabilities in the terrestrial planet system (Brasser et al. 2009; Agnor & Lin 2011). The behavior of the g_6 and s_6 modes, on the other hand, is important for the asteroid belt (Morbidelli et al. 2010; Minton & Malhotra 2011).

As g_5, g_6, and s_6 are mainly a function of the orbital separation between Jupiter and Saturn, the constraints from the terrestrial planets and asteroid belt can be conveniently defined in terms of $P_{\rm Saturn}/P_{\rm Jupiter}$. This ratio needs to evolve from <2.1 to >2.3 in <1 Myr (criterion D), which can be achieved, for example, if planetary encounters with an ice giant scatter Jupiter inward and Saturn outward. The 1 Myr limit is conservative in that a slower evolution of secular frequencies, which fails to satisfy criterion D, would clearly violate the constraints. Note also that in most simulations that satisfy criterion D, $P_{\rm Saturn}/P_{\rm Jupiter}$ evolves from <2.1 to >2.3 in much less than 1 Myr. Most simulations that satisfy D should therefore clearly satisfy the constraints.

In summary, our constraints A and B express the basic requirements of a successful model. Constraint C is more restrictive in that it places a more precise condition on the dynamical structure of the final systems. Constraint D is the least rigorous. It applies only if the instability occurred after the formation of the terrestrial planets, if the amplitudes of the giant planet's secular modes were significant when resonances were crossed, and excluding the low-probability solutions discussed in Brasser et al. (2009).

4. RESULTS

4.1. Four- and Five-planet Results

We start by discussing the results obtained with four planets that were assumed to be initially locked in the (3:2,3:2,4:3) resonances. The inner ice giant has the largest eccentricity ($e_3 = 0.06$). According to Levison et al. (2011), the system should therefore be driven toward instability by the energy and momentum exchange between the inner ice giant and planetesimal disk. We set $r_{\rm in} = 15$ AU, so that the inner disk edge is well beyond the orbits of the two ice giants ($a_3 = 9.6$ AU and $a_4 = 11.6$ AU). This setup should be consistent with the late instability (Levison et al. 2011).

The best results were obtained with $M_{\rm disk} = 35\, M_{\rm Earth}$ and $M_{\rm disk} = 50\, M_{\rm Earth}$. The fraction of simulations producing the final systems with four outer planets is 10% and 13%, respectively (criterion A). Only three of the total of 120 integrations performed for these disk masses satisfied our criterion B. This shows that it is very unlikely that the solar

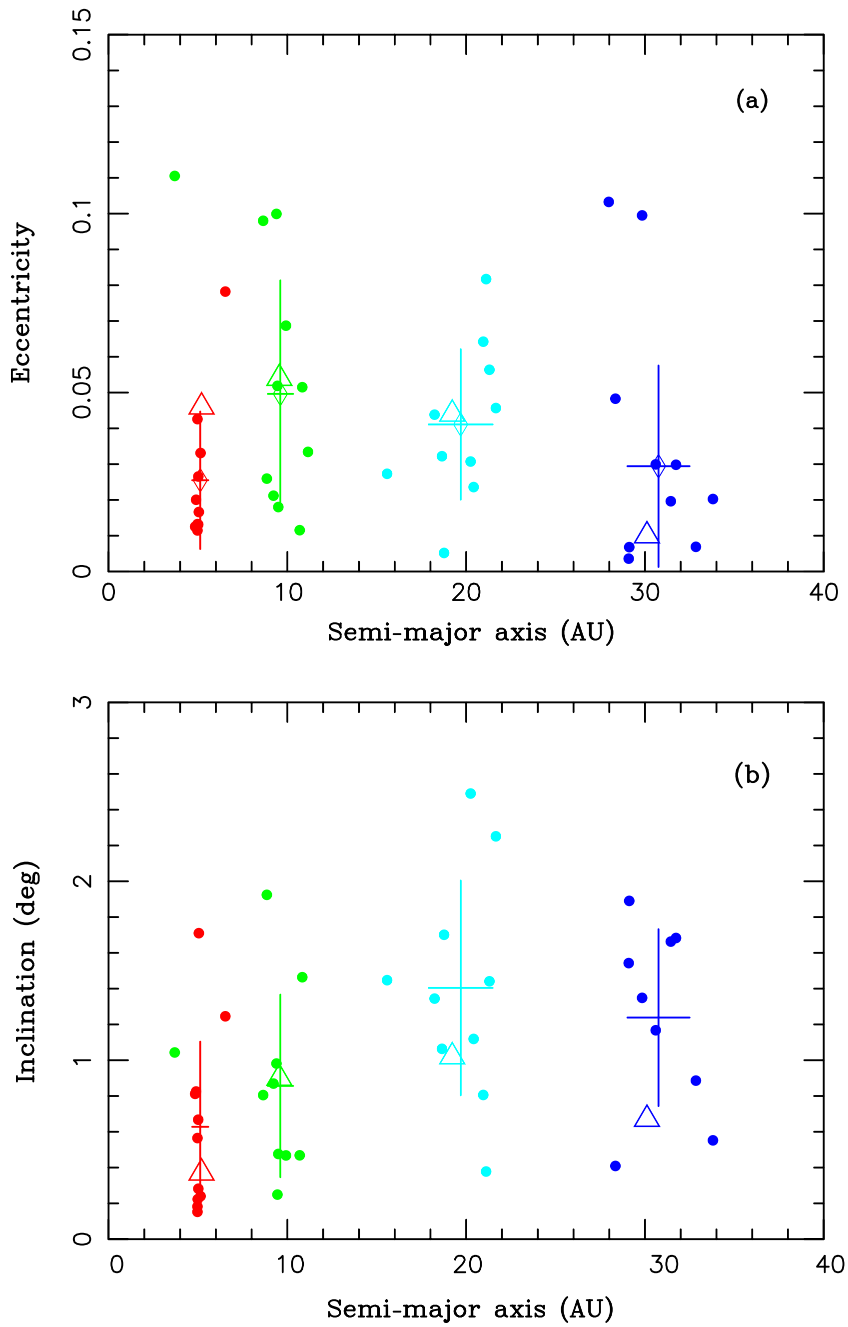

Figure 1. Final orbits obtained with five planets starting in the (3:2,3:2,4:3,5:4) resonances, $M_{\rm disk} = 50\,M_{\rm Earth}$, and $r_{\rm in} = 15$ AU. The systems ending with four planets are plotted (dots). The error bars show the average and rms of the orbital elements. The mean orbits of Jupiter, Saturn, Uranus, and Neptune are shown by red, green, turquoise, and blue triangles.

system evolved from these initial states. The results do not improve when $M_{\rm disk}$ is varied (Section 4.2).

We compare these results with those obtained for the five-planet systems. To start with, the five planets were placed into the (3:2,3:2,4:3,5:4) resonances and fifth planet was given a mass equal to that of Uranus. As in the four-planet case, we fix $r_{\rm in} = 15$ AU so that the most eccentric inner ice giant ($e_3 = 0.07$) has about the same radial distance from the inner edge of the disk.

The best five-planet results were obtained for $M_{\rm disk} = 50\,M_{\rm Earth}$ with the fraction of systems matching criteria A and B raising to 37% and 23%, respectively (Figure 1). This shows that it is roughly 10 times more likely to obtain a good solar system analog starting from five planets than from four planets, at least in the case discussed above. This result is not unexpected because systems starting from a compact resonant configuration typically suffer a rather violent instability and tend to lose planets (Figure 2).

4.2. *Effects of M_{disk}*

The results for the four-planet case do not improve when the mass of the planetesimal disk is varied. The light disks with $M_{\rm disk} \leqslant 20\,M_{\rm Earth}$ lead to final systems with fewer than four planets. With $M_{\rm disk} = 100\,M_{\rm Earth}$, on the other hand, Neptune migrates too far and/or the divergent migration of Jupiter and Saturn moves these planets too far apart so that $2.8 < P_{\rm Saturn}/P_{\rm Jupiter} < 3.2$. The former problem can be resolved if the disk was truncated at $\simeq$25 AU, but we do not

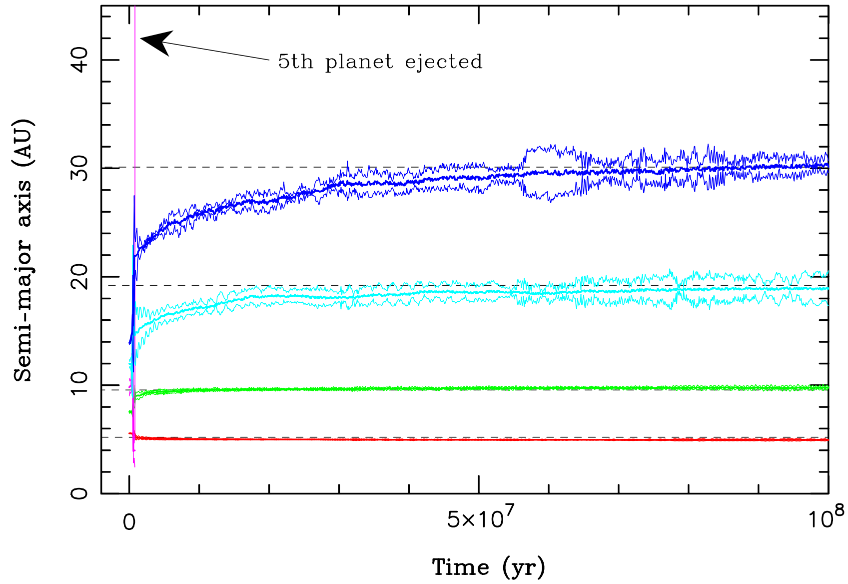

Figure 2. Orbit histories of giant planets in a simulation with five initial planets. The five planets were started in the (3:2,3:2,4:3,5:4) resonances, $M_{\rm disk} = 50\,M_{\rm Earth}$, and $r_{\rm in} = 15$ AU. After a series of encounters with Jupiter the inner ice giant was ejected from the solar system at 8.2×10^5 yr (purple path). The remaining planets were stabilized by the planetesimal disk and migrated to orbits that very closely match those of the outer planets (dashed lines).

see any obvious cure to the second problem. This is because the final radial spacing of Jupiter's and Saturn's orbits depends on the mass of material processed through <10 AU, which is simply too large for $M_{\rm disk} = 100\,M_{\rm Earth}$. In addition, Saturn and Jupiter tend to end up on unrealistically circular orbits for large $M_{\rm disk}$ because their eccentricities are damped by the large mass that these planets interact with.

4.3. Effects of r_{in}

The success rate of simulations depends on $r_{\rm in}$ with the larger $r_{\rm in}$ values leading to a lower success rate. For example, the best four-planet case discussed above matched criterion B in 10% of cases for $r_{\rm in} = 12.6$ AU and $M_{\rm disk} = 50\,M_{\rm Earth}$, which is only slightly lower that the success rate obtained for the five-planet case with $r_{\rm in} = 17.5$ AU and $M_{\rm disk} = 50\,M_{\rm Earth}$. On the other hand, the success rate of the four-planet case drops to $\lesssim$3% if $r_{\rm in} = 17.5$ AU.

The sensitivity of the success rate to $r_{\rm in}$ stems from the following. With the small $r_{\rm in}$ values, planetesimals can rapidly leak from the inner part of the transplanetary disk into the planetary region and stabilize planets very soon after the onset of the instability. With large $r_{\rm in}$, on the other hand, the system lacks the stabilizing effect of planetesimals on planet-crossing orbits, planetary orbits become increasingly excited, and planets can be ejected. This highlights the importance of planet-crossing population of planetesimals on the results.

4.4. Mass of the Fifth Planet

We tested cases with different masses of the fifth planet. Following the mass gradient, we placed the fifth planet with either a mass intermediate between those of Saturn and Uranus into an exterior resonance with Saturn or a planet with a mass lower than that of Neptune and beyond the orbit of Neptune. The former case did not lead to any improvement of the results discussed above, because the system became too violent and frequently ejected the less massive planets from the system. In the latter case, the statistics also did not improve because the inner ice giant got ejected, thus leaving the final system with incorrect masses.

4.5. Initial Resonant Configuration

We found that the behavior of planetary systems, and whether or not the final systems end up resembling the outer solar system, does not depend in detail on the initial resonant sequence between Saturn and ice giants. Instead, the results are mainly sensitive to the initial resonance between Jupiter and Saturn, with the 3:2 or 2:1 cases considered here, and the overall initial spread of ice giants' orbits.

As for the 3:2 Jupiter–Saturn resonance, the case with (3:2,3:2,4:3,5:4) discussed above was one of the most compact resonant systems studied here. Other compact systems show similar success rates. The most radially extended system studied here was (3:2,3:2,3:2,3:2) = $(3:2)^4$. With $r_{\rm in} = 17.5$ AU ($a_5 = 16.1$ AU in this case) and $M_{\rm disk} = 35\,M_{\rm Earth}$, the success rate was higher than the one obtained for a similar setup with (3:2,3:2,4:3,5:4). For example, criterion A was satisfied in 33% of simulations with $(3:2)^4$, while it was satisfied in 13% of simulations in the former case, and criterion B was satisfied in 17% of simulations. This shows that a radially extended initial resonant configuration of ice giants can improve the statistics (mainly for the lower mass disks; $M_{\rm disk} \lesssim 35\,M_{\rm Earth}$).

The four-planet case with the 2:1 Jupiter–Saturn resonance shows a relatively large success rate in matching our criteria A and B. The instability produced in these simulations tends to be weaker than in the 3:2 case, which seems to have a positive impact on the results. There are fewer cases of Jupiter encounters with one of the ice giants, however, leading to orbit histories that violate constraints C and D. We did not find any case, for (2:1,3:2,3:2) and $M_{\rm disk} = 35\,M_{\rm Earth}$, where these criteria would be satisfied.

The five-planet case with the 2:1 Jupiter–Saturn resonance is interesting. Better results were obtained in this case when the inner ice giant had lower mass, because when Jupiter and Saturn start in the 2:1 resonance, their period ratio needs to change by ∼0.5 only (from 2 to 2.49), which requires a smaller perturbation. Assuming a planet with half the mass of Uranus, $20 \leqslant M_{\rm disk} \leqslant 35\,M_{\rm Earth}$ and $\Delta = 1$ AU, criterion A was satisfied in ∼50% of cases and criterion B was satisfied in 20%–30% of cases. These results are only weakly dependent on

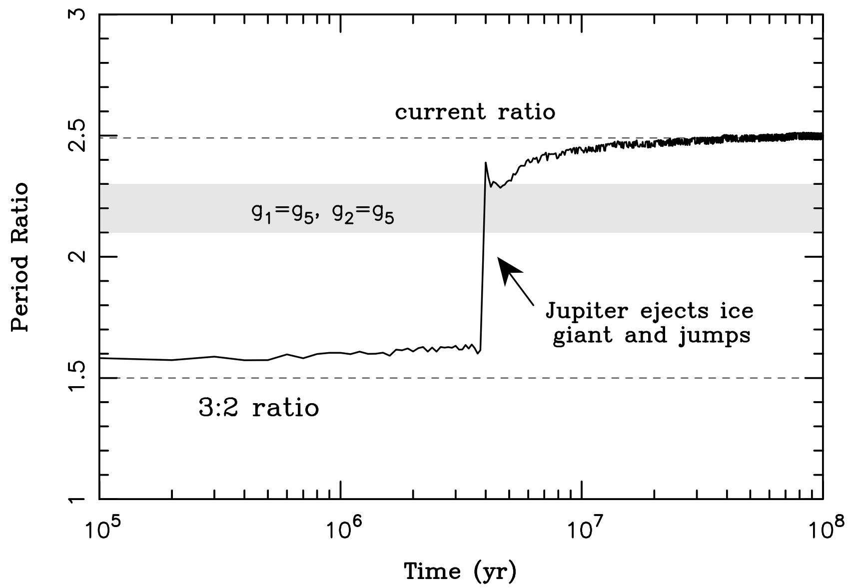

Figure 3. Period ratio, $P_{\rm Saturn}/P_{\rm Jupiter}$, for a selected five-planet case with (3:2,3:2,4:3,5:4) and $M_{\rm disk} = 35\ M_{\rm Earth}$. The fifth planet was ejected by Jupiter at 3.5 Myr. This produced a jump of $P_{\rm Saturn}/P_{\rm Jupiter}$ from ∼1.5 to 2.4. The kind of evolution shown here, known as jumping Jupiter (Morbidelli et al. 2010), may be needed to avoid the region at 2.1–2.3, where the $g_1 = g_5$ and $g_2 = g_5$ resonances occur (Brasser et al. 2009; Agnor & Lin 2011).

the initial resonant sequence between Saturn and the ice giants. All successful jobs show Jupiter encounters and have ∼10% chance of simultaneously matching criteria C and D as well.

4.6. Planetary Encounters

Only ∼3% of simulations performed for the four-planet case with (3:2,3:2,4:3), $r_{\rm in} = 12.6$ AU, and $M_{\rm disk} = 50\ M_{\rm Earth}$, satisfy criteria A and B, *and* show encounters of one of the ice giants with Jupiter. The statistics for other resonances is similarly low. Consequently, in absence of Jupiter encounters, Jupiter and Saturn end up too close to each other if $M_{\rm disk} < 50\ M_{\rm Earth}$, and their orbits are too circular.

Larger disk masses produce more plausible final period ratios, as Jupiter and Saturn can slowly separate from each other by scattering planetesimals, but these evolution paths do not satisfy criterion D. Overall, criterion D was satisfied only in ∼1% of the four-planet cases, which is troubling. For a comparison, practically in all five-planet simulations reported here, all planets, including Jupiter, participate in planetary encounters. For example, criterion D was satisfied in 50% of cases that also satisfied A and B for (3:2,3:2,4:3,5:4), $M_{\rm disk} = 50\ M_{\rm Earth}$, and $r_{\rm in} = 15$ AU (Figure 3), and in over 60% of good cases for (2:1,3:2,3:2,4:3), $M_{\rm disk} = 20\ M_{\rm Earth}$, and $r_{\rm in} = 19$ AU.

4.7. Secular Modes

We analyzed our simulations to determine the fraction of cases in which the secular structure of the final systems was similar to that of the present solar system. The overall success rate for criterion C was disappointingly low, both for the four- and five-planet systems. In most cases, the initial eccentricity of Jupiter, or the one excited by planetary encounters, was damped by planetesimals passing through <10 AU, so that $e_{55} \lesssim 0.01$ in the end.

This problem could most easily be resolved for low initial masses of the planetesimal disk, because in this case, the e_{55} amplitude—initial or produced by the planetary encounters—will most likely survive. As planets can easily be ejected for low $M_{\rm disk}$, more than four initial planets will probably be required. Still, to make things work for the promising five-planet case, the success in matching criteria A and B should be improved for low $M_{\rm disk}$, because the systems studied here with $M_{\rm disk} = 20\ M_{\rm Earth}$ were generally too violent. This could be achieved, for example, by including some sort of dissipation in the planetesimal disk, possibly produced by the collisions between planetesimals.

5. CONCLUSIONS

The formation of Uranus and Neptune is a long-standing problem in planetary science because their accretion at ∼20 and ∼30 AU would require implausibly long timescales (Safronov 1969, Levison & Stewart 2001). The ice giants can form more easily at <15 AU, where densities are higher and dynamical timescales are shorter (e.g., Robinson & Bodenheimer 2010). Our five-planet resonant systems start with all ice giant at <15 AU, if Jupiter and Saturn are in the 3:2 resonance. If Jupiter and Saturn start in the 2:1 resonance, on the other hand, the whole planetary system is more spread out, and the outer ice giant is at ∼18–20 AU, where its formation can be problematic. Future work will need to address these issues.

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