

RESIDUAL ENERGY IN MAGNETOHYDRODYNAMIC TURBULENCE

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ABSTRACT

There is mounting evidence in solar wind observations and in numerical simulations that kinetic and magnetic energies are not in equipartition in magnetohydrodynamic (MHD) turbulence. The origin of their mismatch, the residual energy $E_r = E_v - E_b$, is not understood well. In the present work this effect is studied analytically in the regime of weak MHD turbulence. We find that residual energy is spontaneously generated by turbulent dynamics, and it has a negative sign, in good agreement with the observations. We find that the residual energy condenses around $k_{\parallel} = 0$ with its k_{\parallel} -spectrum broadening linearly with k_{\perp} , where k_{\parallel} and k_{\perp} are the wavenumbers parallel and perpendicular to the background magnetic field, and the field-perpendicular spectrum of the residual energy has the scaling $E_r(k_{\perp}) \propto k_{\perp}^{-1}$ in the inertial interval. These results are found to be in agreement with numerical simulations. We propose that residual energy plays a fundamental role in Alfvénic turbulence and it should be taken into account for the correct interpretation of observational and numerical data.

Key words: magnetohydrodynamics (MHD) – methods: statistical – plasmas – turbulence – waves

Online-only material: color figure

1. INTRODUCTION

Turbulence plays an important role in a variety of astrophysical phenomena, including amplification of magnetic fields in planetary and stellar interiors, magnetization and angular momentum transport in accretion disks, scattering of cosmic rays, formation of small-scale density structures in the interstellar medium, and heating of the solar corona and the solar wind. Theoretical description of magnetized plasma turbulence is a complicated task. Valuable guidance for phenomenological modeling has been provided by high-resolution numerical simulations (e.g., Brandenburg & Nordlund 2011; Kritsuk et al. 2011). On the analytical side, significant progress has been made in the limit of weak turbulence, that is, turbulence consisting of weakly interacting linear waves (e.g., Newell et al. 2001; Galtier et al. 2000, 2002), which provides a test bed for fundamental ideas in the theory of turbulence, such as scale invariance, locality of interactions, energy cascades, and anisotropy. Weak magnetohydrodynamic (MHD) turbulence is also interesting on its own as it may play a role in the interstellar medium, in the solar corona and the solar wind, and in planetary and stellar magnetospheres (e.g., Bhattacharjee & Ng 2001; Saur et al. 2002; Luo & Melrose 2006; Rappazzo et al. 2008; Chandran 2010; Chandran et al. 2010).

In the present Letter we address the problem of mismatch between kinetic and magnetic energies recently reported in observations of the solar wind turbulence. Our interest is also motivated by a similar mismatch found in recent numerical simulations of MHD turbulence, suggesting that the phenomenon may, in fact, have a fundamental nature rather than reflect a possible non-universality of solar wind turbulence. By employing the framework of weak MHD turbulence we present the first direct analytical derivation of residual energy generation in MHD turbulence. We demonstrate that kinetic–magnetic equipartition gets spontaneously broken by interacting random Alfvén waves, even if it is present initially. Our results indicate that this effect

plays a fundamental role in the energy cascade and it should be taken into account in phenomenological modeling of MHD turbulence.

For our analysis we write the incompressible MHD equations in terms of the Elsasser variables:

$$\left(\frac{\partial}{\partial t} \mp \mathbf{v}_A \cdot \nabla \right) \mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla) \mathbf{z}^{\pm} = -\nabla P, \quad (1)$$

where the Elsasser variables are defined as $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$. \mathbf{v} is the fluctuating plasma velocity, \mathbf{b} is the fluctuating magnetic field normalized by $\sqrt{4\pi\rho_0}$, $\mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity corresponding to the uniform magnetic field \mathbf{B}_0 , $P = (p/\rho_0 + b^2/2)$ includes the plasma pressure p and the magnetic pressure, ρ_0 is the constant mass density, and we neglected the terms describing viscous and resistive dissipation. One observes that when $\mathbf{z}^{\mp}(\mathbf{x}, t) \equiv 0$, an arbitrary function $\mathbf{z}^{\pm}(\mathbf{x}, t) = F^{\pm}(\mathbf{x} \pm \mathbf{v}_A t)$ is an exact solution of (1), which represents a non-dispersive Alfvén wave propagating parallel or anti-parallel to \mathbf{B}_0 with the Alfvén speed. Nonlinear interactions are the result of collisions between counterpropagating Alfvén wave packets. This qualitative picture plays a central role in phenomenological models of MHD turbulence; comprehensive reviews of recent analytical and numerical results can be found in, e.g., Galtier (2009), Sridhar (2010), Mininni (2011), Newell & Rumpf (2011), and Tobias et al. (2011).

The ideal MHD system (1) conserves the two independent Elsasser energies, $E^{\pm} = \langle |\mathbf{z}^{\pm}|^2 \rangle / 4$, related to the total energy and cross-helicity, $E = E^+ + E^-$ and $H_c = E^+ - E^-$, respectively. In a turbulent state, when energy is supplied to the system at large scales, both energies E^{\pm} cascade toward small scales where they are converted into heat by viscosity and resistivity. Theories and numerical simulations of MHD turbulence address the Fourier spectra of the energies $E^{\pm}(k)$ in the inertial range of scales, that is, scales much smaller than the forcing scales and much larger than the dissipation scales.

In the picture of counterpropagating Alfvén modes it is often assumed that energy spectra are the same for both kinetic and

magnetic fluctuations, since such energy equipartition holds for individual Alfvén wave packets. However, recent solar wind observations demonstrate contradictions to this assumption: the Fourier energy spectra of magnetic and velocity fluctuations appear to have different slopes in the inertial interval (Podesta et al. 2007; Salem et al. 2009; Tessein et al. 2009; Chen et al. 2011a, 2011b; Wicks et al. 2011; Li et al. 2011). This is somewhat similar to the mismatch in the spectra found in numerical simulations of strong MHD turbulence (Müller & Grappin 2005; Boldyrev et al. 2011). Most intriguing, however, is the fact that a significant difference between magnetic and kinetic energies was detected in numerical studies of *weak* MHD turbulence (Boldyrev & Perez 2009) consisting of mostly independent Alfvén waves for which equipartition between magnetic and kinetic energies holds exactly.

The mismatch between magnetic and kinetic energies, the so-called residual energy $E_r(\mathbf{k}) = E_v(\mathbf{k}) - E_b(\mathbf{k})$, was noted in previous works (e.g., Grappin et al. 1983; Zank et al. 1996; Müller & Grappin 2005), however, it has not been fully explored in models of MHD turbulence, since such models are often based on the conserved quantities E^\pm , E , H_c , which do not contain information about the residual energy. In this work, we present the analytic derivation of the residual energy in the case of weak MHD turbulence. We demonstrate that residual energy is spontaneously generated by interacting Alfvén waves even if it is absent initially, plays a fundamental role in the turbulent dynamics, and provides a qualitative explanation for the breakdown of magnetic and kinetic equipartition observed in the solar wind.

2. THE ORIGIN OF THE RESIDUAL ENERGY

We concentrate on turbulence of shear-Alfvén waves whose polarizations are normal to the guide field, $\mathbf{z}^\pm \perp B_0$. We assume that the guide field is strong; its strength with respect to the magnetic and velocity fluctuations is characterized by a small parameter $\epsilon \sim z^\pm/v_A \ll 1$. We also assume that the turbulence is *weak*, that is, the wave spectrum in the field-parallel direction is broad enough to ensure that the nonlinear terms are small, $k_\parallel v_A \gg k_\perp z^\pm$, where k_\parallel and k_\perp are typical wavenumbers characterizing the spectral width in the field-parallel and field-perpendicular directions and z^\pm are typical (rms) values of the fluctuations. The Elsasser Fourier energies are given by the correlation functions

$$\langle \mathbf{z}^\pm(\mathbf{k}, t) \cdot \mathbf{z}^\pm(\mathbf{k}', t) \rangle = \epsilon^2 v_A^2 q^\pm(k_\parallel, k_\perp, t) \delta(\mathbf{k} + \mathbf{k}'), \quad (2)$$

while the residual energy is related to

$$\langle \mathbf{z}^+(\mathbf{k}, t) \cdot \mathbf{z}^-(\mathbf{k}', t) \rangle = \epsilon^2 v_A^2 Q^0(k_\parallel, k_\perp; t) \delta(\mathbf{k} + \mathbf{k}'). \quad (3)$$

In these formulas we assume spatial homogeneity and average over a statistical ensemble. The Elsasser energies are real, while the residual correlation function satisfies $Q^0(k_\parallel, k_\perp)^* = Q^0(-k_\parallel, k_\perp)$. Its real part is then the residual energy $e_r(k) = \text{Re}Q^0$. In practical applications one also uses the phase-volume-compensated residual energy defined as $E_r(k) = \epsilon^2 v_A^2 e_r(k) 2\pi k_\perp$. In previous treatments (Galtier et al. 2000, 2002), the residual energy was assumed to be zero as it would be the case for independent z^+ and z^- waves. In the present study we do not make such an assumption; rather, we *derive* the equation for the residual energy.

The derivation is performed perturbatively in ϵ , following the standard procedure (e.g., Galtier et al. 2002). When the

nonlinear terms in Equation (1) are neglected, both $\mathbf{z}^\pm(\mathbf{k})$ oscillate in time and so does the residual energy, $Q^0(k, t) = q^0(k) \exp(2ik_\parallel v_A t)$. When the nonlinear interaction is taken into account, q^0 acquires small corrections that one finds, to the first non-vanishing order, by integrating the equation

$$\begin{aligned} \partial_t q^0(k, t) = & -e^{-2ik_\parallel v_A t} \epsilon^2 \int R_{k,pq} \{q^+(\mathbf{q})[q^-(\mathbf{p}) - q^-(\mathbf{k})] \\ & + q^-(\mathbf{q})[q^+(\mathbf{p}) - q^+(\mathbf{k})]\} \tilde{\delta}(q_\parallel) \delta(\mathbf{k} - \mathbf{p} - \mathbf{q}) d^3 p d^3 q, \quad (4) \end{aligned}$$

where $R_{k,pq} = (\pi v_A/2)(\mathbf{k}_\perp \times \mathbf{q}_\perp)^2 (\mathbf{k}_\perp \cdot \mathbf{p}_\perp) (\mathbf{k}_\perp \cdot \mathbf{q}_\perp) / (k_\perp^2 p_\perp^2 q_\perp^2)$, and we introduce a ‘‘smeared’’ delta-function $\tilde{\delta}(k_\parallel) = (1 - \exp(-2ik_\parallel v_A t)) / (i\pi k_\parallel)$, which approaches $\delta(k_\parallel) + (i/\pi) \mathcal{P}(1/k_\parallel)$ at large t . Since we are interested in spontaneous generation of the residual energy, we assume that it is absent initially; therefore, in the right-hand side of Equation (4) we neglected the terms proportional to q^0 . The derivation of Equation (4) is standard although lengthy, and it will be presented elsewhere. In what follows we discuss the main consequences of the obtained result.

3. THE SPECTRUM OF THE RESIDUAL ENERGY

First, we notice that the integral in the right-hand side of Equation (4) is essentially nonzero. It therefore describes the *generation* of the residual energy by Alfvén waves. The important conclusion is that the residual energy is generated by nonlinear interactions even if it is zero initially. Moreover, one can directly verify that for all the spectra of interest, the integral in the right-hand side of Equation (4) is positive. As we shall see below, this means that the generated residual energy is negative, that is, the magnetic energy exceeds the kinetic one.⁴

A simple evaluation of the integral in Equation (4) can be performed for the case of balanced MHD turbulence, that is, for $q^+ = q^-$. In this case, the Elsasser energy spectra have the well-known scaling (e.g., Ng & Bhattacharjee 1996; Galtier et al. 2000) $q^+ = q^- = f(k_\parallel) k_\perp^{-3}$, where $f(k_\parallel)$ is arbitrary, and we assume that it is smooth at least initially. For the reasons that will be clear momentarily, we estimate the integral in the right-hand side of Equation (4) at $k_\parallel = 0$:

$$\int R_{k,pq} \{ \dots \} \tilde{\delta}(q_\parallel) \delta(\mathbf{k} - \mathbf{p} - \mathbf{q}) d^3 p d^3 q = \alpha k_\perp^{-2} v_A, \quad (5)$$

where the numerical factor is $\alpha \sim f^2(0)$.⁵ Non-vanishing of this term at $k_\parallel = 0$ implies that Equation (4) has a solution diverging in time, which is unphysical. The reason is that when q^0 becomes large one cannot neglect the nonlinear terms containing q^0 in Equation (4). As we shall see, in order to obtain the convergent result, it is enough to retain the terms linear in q^0 , which are proportional to $q^\pm q^0$. Those terms describe relaxation of the residual energy due to its interaction with the Elsasser energies. Their structure can be found from a simple estimate, if one notices that these terms should have the same dimensional structure as the right-hand side term in Equation (4) if one replaces one of q^\pm by q^0 . The corresponding relaxation term can thus be added to the right-hand side of Equation (4) in the

⁴ This result is a statistical effect of many independent collisions of Alfvén waves. It does not imply that a particular collision of two Alfvén wave packets necessarily produces negative residual energy.

⁵ A more precise numerical integration gives $\alpha \approx 0.065\pi f^2(0)$; however, our discussion of scaling properties of the residual energy does not require the knowledge of numerical coefficients with such precision.

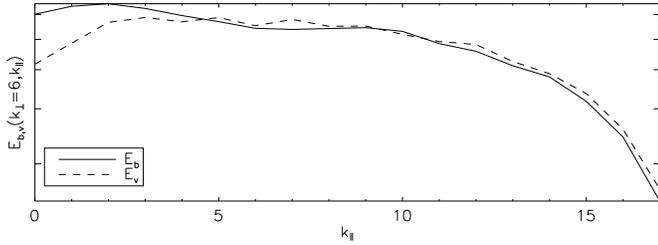


Figure 1. Field-parallel spectra of magnetic (solid line) and kinetic (dashed line) energies at $k_{\perp} = 6$ for weak balanced MHD turbulence. The magnetic energy exceeds the kinetic one at small k_{\parallel} . Note the logarithmic scale on the vertical axis.

form γq^0 , with the relaxation rate $\gamma \sim -\epsilon^2 v_A k_{\perp}^4 q^{\pm}(k)$. The estimate of the relaxation rate at $k_{\parallel} = 0$ gives

$$\gamma(0, k_{\perp}) = -\beta \epsilon^2 k_{\perp} v_A, \quad (6)$$

where $\beta \sim f(0)$.⁶

Collecting results (5) and (6), we rewrite Equation (4) in the form

$$\partial_t q^0(k, t) = -\epsilon^2 \beta k_{\perp} v_A q^0(k, t) - \epsilon^2 \alpha k_{\perp}^{-2} v_A e^{-2ik_{\parallel} v_A t}, \quad (7)$$

which has to be solved together with the initial condition $q^0 = 0$. The solution is given by

$$q^0(k, t) = -\frac{\alpha \epsilon^2 k_{\perp}^{-2}}{\beta \epsilon^2 k_{\perp} - 2ik_{\parallel}} e^{-2ik_{\parallel} v_A t}, \quad (8)$$

and the residual energy is found as

$$e_r(k_{\parallel}, k_{\perp}) = \text{Re} Q^0 = -\frac{\alpha \beta \epsilon^4 k_{\perp}^{-1}}{\beta^2 \epsilon^4 k_{\perp}^2 + 4k_{\parallel}^2}. \quad (9)$$

The residual energy can be rewritten as

$$e_r(k_{\parallel}, k_{\perp}) = -\alpha \pi \epsilon^2 k_{\perp}^{-2} \Delta(2k_{\parallel}), \quad (10)$$

where the Δ function has support inside a narrow wedge-shaped domain:

$$k_{\parallel} \leq \beta \epsilon^2 k_{\perp} / 2. \quad (11)$$

According to the result $\lim_{\epsilon \rightarrow 0} \epsilon / (\epsilon^2 + \epsilon^2) = \pi \delta(x)$, the function Δ would turn into the δ -function in the limit $\epsilon \rightarrow 0$. This explains why the region around $k_{\parallel} = 0$ was relevant in our estimates (5) and (6). We will refer to such a spectrum as ‘‘condensate.’’ It is also convenient to define the field-perpendicular spectrum of the residual energy,

$$E_r(k_{\perp}) = \int \epsilon^2 v_A^2 e_r(k_{\parallel}, k_{\perp}) 2\pi k_{\perp} dk_{\parallel} = -\alpha \pi^2 \epsilon^4 v_A^2 k_{\perp}^{-1}. \quad (12)$$

Expressions (9)–(12) are the main results of the present work.

4. NUMERICAL RESULTS

In this section, we test our analytic predictions in numerical simulations. In the presence of a strong guide field, the universal properties of MHD turbulence can be studied in the framework of reduced MHD, which can be effectively used in numerical

⁶ It is easy to see that γ coincides with the inverse time of nonlinear spectral energy transfer in weak MHD turbulence (e.g., Ng & Bhattacharjee 1996; Galtier et al. 2000).

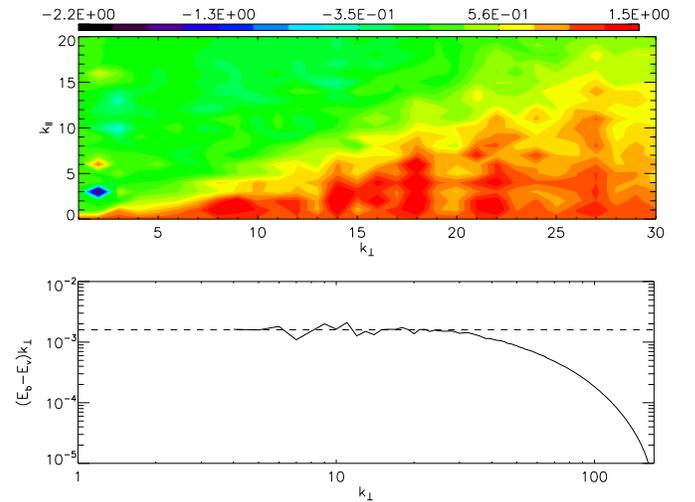


Figure 2. Upper panel: the (normalized) residual energy $E_r(k_{\parallel}, k_{\perp})/E_r(0, k_{\perp})$ plotted as a function of k_{\parallel} and k_{\perp} for the weak balanced MHD turbulence. Lower panel: the field-perpendicular spectrum of the residual energy for weak balanced MHD turbulence compensated by k_{\perp} . At small k_{\perp} the deviations are strong due to proximity to the forcing region (not shown).

(A color version of this figure is available in the online journal.)

simulations of both strong and weak turbulence (e.g., Dmitruk et al. 2003; Oughton et al. 2004; Galtier & Chandran 2006; Perez & Boldyrev 2008; Rappazzo et al. 2010). In our numerical setup, the \mathbf{z}^+ and \mathbf{z}^- fields are *independently* driven by Gaussian-distributed random, short-time correlated forces with zero mean and prescribed variances, so that no residual energy is supplied by the driving routine. The forces are applied at $k_{\parallel} = 1, \dots, 16$ and $k_{\perp} = 1, 2$ in the Fourier space. The box is elongated in the field-parallel direction according to $L_{\parallel}/L_{\perp} = B_0/b_{\text{rms}}$. We employ a fully dealiased pseudo-spectral method; the run has a resolution of 512^3 , the Reynolds number $\text{Re} = 2400$, and it is averaged over 41 large-scale eddy turnover times. We choose viscosity equal to magnetic diffusivity. The details on the code and the numerical setup can be found in Perez & Boldyrev (2010).

We find that the residual-energy condensate is indeed generated in weak balanced MHD turbulence, as shown in Figure (1). The condensate is negative and its broadening with k_{\perp} shown in Figure (2) is consistent with the linear law predicted in Equation (11). The scaling behavior of the field-perpendicular spectrum of the residual energy is also consistent with the analytic prediction in Equation (12); see Figure (2). When we repeated the computation for the imbalanced case, that is, the case when $q^+ \neq q^-$ (not presented here), we did not detect essential changes in the structure and scaling of the condensate.

5. DISCUSSION

In this Letter, we demonstrated that residual energy is spontaneously generated by nonlinearly interacting Alfvén waves and it accumulates in a narrow region of phase space (11). This explains the phenomenon of ‘‘condensate’’ of the residual energy numerically observed in Boldyrev & Perez (2009). Although the total (phase-space integrated) residual energy is small, its presence is crucial for turbulent dynamics for the following reason. Inside region (11), that is, for $k_{\parallel} \approx 0$, the residual energy is comparable to the Elsasser energies and it follows the same scaling, $e_r(0, k_{\perp}) \sim q^{\pm}(0, k_{\perp}) \propto k_{\perp}^{-3}$. Since the turbulent cascade of Alfvén waves crucially depends on the modes with $k_{\parallel} \approx 0$

(e.g., Montgomery & Turner 1981; Sridhar & Goldreich 1994; Ng & Bhattacharjee 1996; Galtier et al. 2000, 2002), the residual energy crucially affects the dynamics of the Elsasser modes.

More formally, this is expressed in the fact that one of the Elsasser energies enters the kinetic equations in the combination $q^\pm(k_\parallel, k_\perp)\tilde{\delta}(k_\parallel)$, where $\tilde{\delta}(k_\parallel)$ are concentrated in region (11), see, e.g., Equation (4). In previous treatments of weak MHD turbulence (Galtier et al. 2000, 2002), it was assumed that $q^\pm(k_\parallel, k_\perp)$ are smooth functions of k_\parallel , that is, their dynamically important components with $k_\parallel = 0$ have the same k_\perp -scaling as the components with $k_\parallel \neq 0$. Such an assumption would be self-consistent if the residual energy were absent. However, as we have demonstrated, the residual energy is spontaneously generated by interacting Alfvén waves. One can argue that the presence of the residual energy modifies the spectra of the Elsasser energies that, together with their smooth parts, $q^\pm = f^\pm(k_\parallel)k_\perp^{-3}$, now acquire their own singular parts $\delta q^\pm = \alpha^\pm \epsilon^2 \Delta^\pm(k_\parallel)k_\perp^{-2}$, where $\Delta^\pm(k_\parallel)$ are concentrated in region (11). When multiplied by $\tilde{\delta}(k_\parallel)$ and integrated over k_\parallel , both the smooth and the singular parts provide comparable contributions to the integrals.

We established that the spontaneously generated residual energy is always negative. Although this result is obtained in the framework of weak MHD turbulence, it provides the first analytic explanation for the observational and numerical findings that magnetic energy exceeds kinetic energy in the inertial interval of MHD turbulence (e.g., Müller & Grappin 2005; Podesta et al. 2007; Boldyrev & Perez 2009).

We derived that the residual energy has the field-perpendicular spectrum $E_r(k_\perp) \propto k_\perp^{-1}$ (12). This relatively shallow spectrum holds in the inertial interval and breaks down at sufficiently large k_\perp when the nonlinear broadening of the residual energy spectrum in the field-parallel direction (11) becomes comparable to the width of the field-parallel energy spectra of the Alfvén waves, $q^\pm(k_\parallel)$. It is easy to see, however, that this is precisely the scale beyond which the weak interaction approximation breaks down, and the turbulence becomes strong. In our future work we will extend our analysis of residual energy to the case of strong MHD turbulence.

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REFERENCES

- Bhattacharjee, A., & Ng, C. S. 2001, *ApJ*, **548**, 318
 Boldyrev, S., & Perez, J. C. 2009, *Phys. Rev. Lett.*, **103**, 225001
 Boldyrev, S., Perez, J. C., Borovsky, J. E., & Podesta, J. J. 2011, arXiv:1106.0700
 Brandenburg, A., & Nordlund, Å. 2011, *Rep. Prog. Phys.*, **74**, 046901
 Chandran, B. D. G. 2010, *ApJ*, **720**, 548
 Chandran, B. D. G., Li, B., Rogers, B. N., Quataert, E., & Germaschewski, K. 2010, *ApJ*, **720**, 503
 Chen, C. H. K., Bale, S. D., Salem, C., & Mozer, F. S. 2011a, *ApJ*, **737**, L41
 Chen, C. H. K., Mallet, A., Yousef, T. A., Schekochihin, A. A., & Horbury, T. S. 2011b, *MNRAS*, **415**, 3219
 Dmitruk, P., Gómez, D. O., & Matthaeus, W. H. 2003, *Phys. Plasmas*, **10**, 3584
 Galtier, S. 2009, *Nonlinear Process. Geophys.*, **16**, 83
 Galtier, S., & Chandran, B. D. G. 2006, *Phys. Plasmas*, **13**, 114505
 Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2000, *J. Plasma Phys.*, **63**, 447
 Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2002, *ApJ*, **564**, L49
 Grappin, R., Pouquet, A., & Leorat, J. 1983, *A&A*, **126**, 51
 Kritsuk, A. G., Nordlund, Å., Collins, D., et al. 2011, *ApJ*, **737**, 13
 Li, G., Miao, B., Hu, Q., & Qin, G. 2011, *Phys. Rev. Lett.*, **106**, 125001
 Luo, Q., & Melrose, D. 2006, *MNRAS*, **368**, 1151
 Mininni, P. D. 2011, *Annu. Rev. Fluid Mech.*, **43**, 377
 Montgomery, D., & Turner, L. 1981, *Phys. Fluids*, **24**, 825
 Müller, W., & Grappin, R. 2005, *Phys. Rev. Lett.*, **95**, 114502
 Newell, A. C., Nazarenko, S., & Biven, L. 2001, *Physica D: Nonlinear Phenom.*, **152**, 520
 Newell, A. C., & Rumpf, B. 2011, *Annu. Rev. Fluid Mech.*, **43**, 59
 Ng, C. S., & Bhattacharjee, A. 1996, *ApJ*, **465**, 845
 Oughton, S., Dmitruk, P., & Matthaeus, W. H. 2004, *Phys. Plasmas*, **11**, 2214
 Perez, J. C., & Boldyrev, S. 2008, *ApJ*, **672**, L61
 Perez, J. C., & Boldyrev, S. 2010, *ApJ*, **710**, L63
 Podesta, J. J., Roberts, D. A., & Goldstein, M. L. 2007, *ApJ*, **664**, 543
 Rappazzo, A. F., Velli, M., & Einaudi, G. 2010, *ApJ*, **722**, 65
 Rappazzo, A. F., Velli, M., Einaudi, G., & Dahlburg, R. B. 2008, *ApJ*, **677**, 1348
 Salem, C., Mangeney, A., Bale, S. D., & Veltri, P. 2009, *ApJ*, **702**, 537
 Saur, J., Politano, H., Pouquet, A., & Matthaeus, W. H. 2002, *A&A*, **386**, 699
 Sridhar, S. 2010, *Astron. Nachr.*, **331**, 93
 Sridhar, S., & Goldreich, P. 1994, *ApJ*, **432**, 612
 Tessein, J. A., Smith, C. W., MacBride, B. T., et al. 2009, *ApJ*, **692**, 684
 Tobias, S. M., Cattaneo, F., & Boldyrev, S. 2011, arXiv:1103.3138
 Wicks, R. T., Horbury, T. S., Chen, C. H. K., & Schekochihin, A. A. 2011, *Phys. Rev. Lett.*, **106**, 045001
 Zank, G. P., Matthaeus, W. H., & Smith, C. W. 1996, *J. Geophys. Res.*, **101**, 17093