

TESTING THE DISTANCE–DUALITY RELATION WITH GALAXY CLUSTERS AND TYPE Ia SUPERNOVAE

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ABSTRACT

In this Letter, we propose a new and model-independent cosmological test for the distance–duality (DD) relation, $\eta = D_L(z)(1+z)^{-2}/D_A(z) = 1$, where D_L and D_A are, respectively, the luminosity and angular diameter distances. For D_L we consider two sub-samples of Type Ia supernovae (SNe Ia) taken from Constitution data whereas D_A distances are provided by two samples of galaxy clusters compiled by De Filippis et al. and Bonamente et al. by combining Sunyaev–Zeldovich effect and X-ray surface brightness. The SNe Ia redshifts of each sub-sample were carefully chosen to coincide with the ones of the associated galaxy cluster sample ($\Delta z < 0.005$), thereby allowing a direct test of the DD relation. Since for very low redshifts, $D_A(z) \cong D_L(z)$, we have tested the DD relation by assuming that η is a function of the redshift parameterized by two different expressions: $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1+z)$, where η_0 is a constant parameter quantifying a possible departure from the strict validity of the reciprocity relation ($\eta_0 = 0$). In the best scenario (linear parameterization), we obtain $\eta_0 = -0.28^{+0.44}_{-0.44}$ (2σ , statistical + systematic errors) for the De Filippis et al. sample (elliptical geometry), a result only marginally compatible with the DD relation. However, for the Bonamente et al. sample (spherical geometry) the constraint is $\eta_0 = -0.42^{+0.34}_{-0.34}$ (3σ , statistical + systematic errors), which is clearly incompatible with the duality–distance relation.

Key words: cosmic background radiation – distance scale – galaxies: clusters: general – supernovae: general – X-rays: galaxies: clusters

Online-only material: color figures

1. INTRODUCTION

Etherington’s reciprocity relation (Etherington 1933) is of fundamental importance in cosmology. Its most useful version in the astronomical context, sometimes referred to as the distance–duality (DD) relation, relates the luminosity distance D_L with the angular diameter distance D_A by means of the following expression:

$$\frac{D_L}{D_A}(1+z)^{-2} = 1. \quad (1)$$

This equation is completely general, valid for *all* cosmological models based on the Riemannian geometry, being dependent neither on Einstein field equations nor on the nature of the matter-energy content. It only requires that source and observer be connected by null geodesics in a Riemannian spacetime and that the number of photons be conserved. Therefore, it is valid for spatially homogeneous and isotropic (anisotropic) cosmologies, as well as for inhomogeneous cosmological models (Ellis 2007).

The DD relation plays an essential role in modern cosmology, ranging from gravitational-lensing studies (Schneider et al. 1999) to analyses from galaxy cluster observations (Cunha et al. 2007; Mantz et al. 2010), as well as the plethora of cosmic consequences from primary and secondary temperature anisotropies of the cosmic microwave blackbody radiation (CMBR) observations (Komatsu et al. 2010). Even the temperature shift equation $T_o = T_e/(1+z)$, where T_o is the observed temperature and T_e is the emitted temperature, a key result for analyzing CMBR observations and the optical theorem that surface brightness of an extended source does not depend on the angular diameter distance (ADD) of the observer from the source (an important result for understanding lensing brightness) are both consequences of Etherington’s reciprocity relation (Ellis 1971, 2007).

The Etherington law, as it is also sometimes called, has so far been taken for granted by virtually all analyses of cosmological observations. Despite this, the DD relation is in principle testable by means of astronomical observations. If one is able to find cosmological sources whose intrinsic luminosities are known (standard candles) as well as their intrinsic sizes (standard rulers), one can determine both D_L and D_A , and after measuring the common redshifts, to test directly Etherington’s result. Note that ideally both quantities must be measured in a way that does not utilize any relationship coming from a cosmological model, that is, they must be determined by means of intrinsic astrophysically measured quantities.

The method described above for testing the reciprocity law is very difficult to carry out in practice due to limitations in our current understanding of galaxy evolution and, hence, one must still rely on less-than-ideal methods for seeking observational falsification of the reciprocity law. These less-than-ideal methods usually assume a cosmological model suggested by a set of observations, apply this model in the context of some astrophysical effect, and attempt to see if the reciprocity relation remains valid. In this way, Uzan et al. (2004) showed that observations from Sunyaev–Zeldovich effect (SZE) and X-ray surface brightness from galaxy clusters offer a test for the DD relation. It was argued that the SZE+X-ray technique for measuring the ADDs (Sunyaev & Zel’dovich 1972; Cavaliere & Fusco-Fermiano 1978) is strongly dependent on the validity of this relation. When the relation does not hold, the ADD determined from observations is $D_A^{\text{cluster}}(z) = D_A(z)\eta^2$ (actually, multiplied by η^{-2} in their notation). Such a quantity reduces to the ADD only when the reciprocity relation is strictly valid, i.e., when $\eta = 1$. They considered 18 ADD galaxy clusters from the Reese et al. (2002) sample for which a spherically symmetric cluster geometry has been assumed. Their analysis, which is carried out in a Λ CDM model (Spergel et al.

2003), shows that no violation of the DD is only marginally consistent.

Later on, De Bernardis et al. (2006) also searched for deviations from the DD relation by using the ADD from galaxy clusters provided by the sample of Bonamente et al. (2006). They obtained a non-violation of the DD in the framework of the cosmic concordance Λ CDM model. Recently, Avgoustidis et al. (2010) used the distance relation, $d_L = d_A(1+z)^{2+\epsilon}$, in a flat Λ CDM model for constraining the cosmic opacity by combining recent Type Ia supernova (SN Ia) data compilation (Kowalski et al. 2008) with the latest measurements of the Hubble expansion at redshifts on the range $0 < z < 2$ (Stern et al. 2010). They found $\epsilon = -0.04^{+0.08}_{-0.07}$ (2σ). However, what was really being tested in the quoted works was the consistency between the assumed cosmological model and some results provided by a chosen set of astrophysical phenomena.

Following another route, Holanda et al. (2010) discussed the consistency between the strict validity of the DD relation and the assumptions about the geometry, elliptical, and spherical β models, which is used to describe the galaxy clusters. They used the function $\eta(z)$ parameterized in two distinct forms, $\eta = 1 + \eta_0 z$ and $\eta = 1 + \eta_0 z/(1+z)$, thereby recovering the equality between distances only for very low redshifts, in order to test possible deviations. By comparing the De Filippis et al. (2005, elliptical β model) and Bonamente et al. (2006, spherical β model) samples with theoretical D_A^{Th} obtained from Λ CDM (Komatsu et al. 2010), they showed that the elliptical geometry is more consistent ($\eta_0 = 0$ in 1σ) with no violation of the DD relation in the context of Λ CDM (WMAP7).

The possibility of testing new physics based on the validity of DD relation was first discussed by Basset & Kunz (2004). They used current SNe Ia data as measurements of D_L and estimated D_A from FRIIb radio galaxies (Daly & Djorgovski 2003) and ultra compact radio sources (Gurvitz 1994, 1999; Lima & Alcaniz 2000, 2002; Santos & Lima 2008). A moderate violation (2σ) caused by the brightening excess of SNe Ia at $z > 0.5$ was found. In the same vein, De Bernardis et al. (2006) also compared the ADD from galaxy clusters with luminosity distance data from supernovae to obtain a model-independent test. In order to compare the data sets they considered the weighted average of the data in seven bins and found that $\eta = 1$ is consistent in the 68% confidence level (1σ). However, one needs to be careful when using the SZE+X-ray technique for measuring ADDs to test the DD relation because such a technique is also dependent on its validity. In fact, when the relation does not hold, the ADD determined from observations is in general $D_A^{\text{cluster}}(z) = D_A(z)\eta^2$, which reduces to D_A only if $\eta = 1$. So, their work did not test the DD relation, at least not in a consistent way. In addition, both authors binned their data, and, as such, their results may have been influenced by the particular choice of redshift binning.

In this context, the aim of this Letter is to propose a consistent cosmological-model-independent test for Equation (1) by using two sub-samples of SNe Ia chosen from Constitution data (Hicken et al. 2009) and two ADD samples from galaxy clusters obtained through SZE effect and X-ray measurements with different assumptions concerning the geometry used to describe the clusters: the elliptical β model and the spherical β model. Following Holanda et al. (2010), our analysis here will be based on two parametric representations for a possible redshift

dependence of the DD expression, namely,

$$\frac{D_L}{D_A}(1+z)^{-2} = \eta(z), \quad (2)$$

where

$$(I) \eta(z) = 1 + \eta_0 z,$$

$$(II) \eta(z) = 1 + \eta_0 z/(1+z).$$

For a given pair of data set (SNe Ia, galaxy clusters), one should expect a likelihood of η_0 peaked at $\eta_0 = 0$, in order to satisfy the DD relation. It is also worth noticing that in our approach the data do not need to be binned as assumed in some analyses involving the DD relation. As we shall see, for the Bonamente et al. (2006) sample, where a spherical geometry was assumed, our results show a strong violation ($>3\sigma$) DD relation when the SNe Ia and galaxy clusters data are confronted. However, when the elliptical geometry is assumed (De Filippis et al. 2005), the results are marginally compatible within 2σ with the DD relation.

2. SAMPLES

In order to constrain the possible values of η_0 let us now consider two samples of ADD from galaxy clusters obtained by combining their SZE and X-ray surface brightness observations. The first one is formed by 25 galaxy clusters from the De Filippis et al. (2005) sample. Since *Chandra* and *XMM* observations of clusters in the past few years have shown that in general clusters exhibit elliptical surface brightness maps, De Filippis et al. (2005) studied and corrected, using an isothermal elliptical β model to describe the clusters, the D_A measurements of two samples for which combined X-ray and SZE analysis has already been reported using an isothermal spherical β model. One of the samples, compiled by Reese et al. (2002), is a selection of 18 galaxy clusters distributed over the redshift interval $0.14 < z < 0.8$. The other one, the sample of Mason et al. (2001), has seven clusters from the X-ray-limited flux sample of Ebeling et al. (1996). The second is defined by the 38 ADD galaxy clusters from the Bonamente et al. (2006) sample, where the cluster plasma and dark matter distributions were analyzed assuming the hydrostatic equilibrium model and spherical symmetry, thereby accounting for radial variations in density, temperature, and abundance. This sample consists of clusters that have both X-ray data from the *Chandra Observatory* and SZE data from the BIMA/OVRO SZE imaging project, which uses the Berkeley–Illinois–Maryland Association (BIMA) and Owens Valley radio observatory (OVRO) interferometers to image the SZE. For the luminosity distances, we choose two sub-samples of SNe Ia from Constitution SNe Ia data set whose redshifts coincide with the ones appearing in the galaxy cluster samples. In Figure 1(a), we plot D_A multiplied by $(1+z)^2$ from the galaxy cluster's sample compiled by De Filippis et al. (2005) (the error bars contain statistical and systematic contributions) and D_L from our first SNe Ia sub-sample. In Figure 1(b), we plot the subtraction of redshift between clusters and SNe Ia. We see that the biggest difference is $\Delta z \approx 0.01$ for 3 clusters (open squares) while for the remaining 22 clusters we have $\Delta z < 0.005$. In order to avoid the corresponding bias, the three clusters will be removed from all the analyses presented here so that $\Delta z < 0.005$ for all pairs.

Similarly, in Figure 2(a) we plot D_A multiplied by $(1+z)^2$, but now for the Bonamente et al. (2006) sample (error bars also

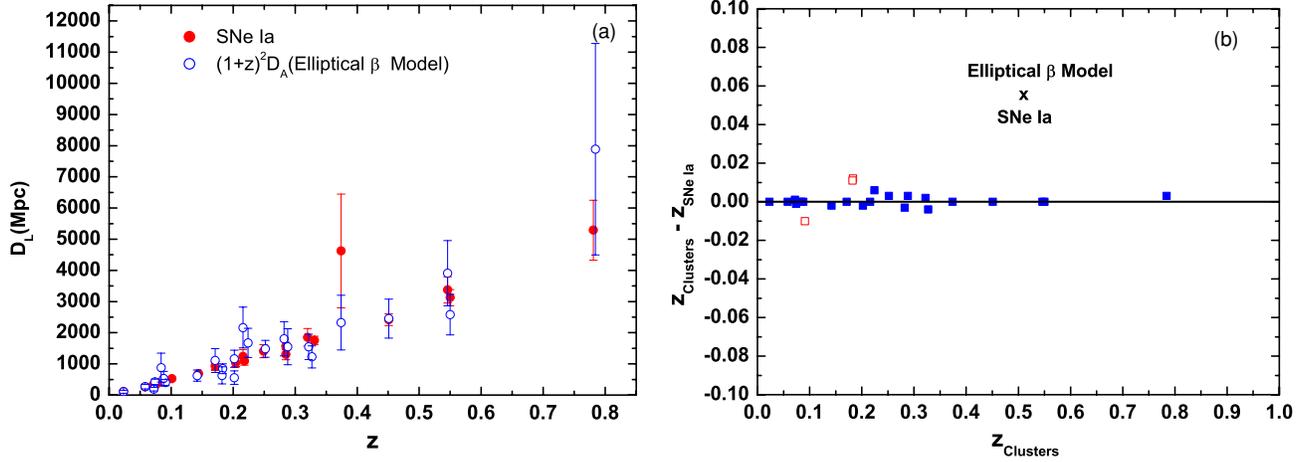


Figure 1. (a) Galaxy clusters and SNe Ia data. The open (blue) and filled (red) circles with the associated error bars stand, respectively, for the De Filippis et al. (2005) and SNe Ia samples. (b) The redshift subtraction for the same pair of cluster-SNe Ia samples. The open squares represent the pairs of points for which $\Delta z \approx 0.01$. (A color version of this figure is available in the online journal.)

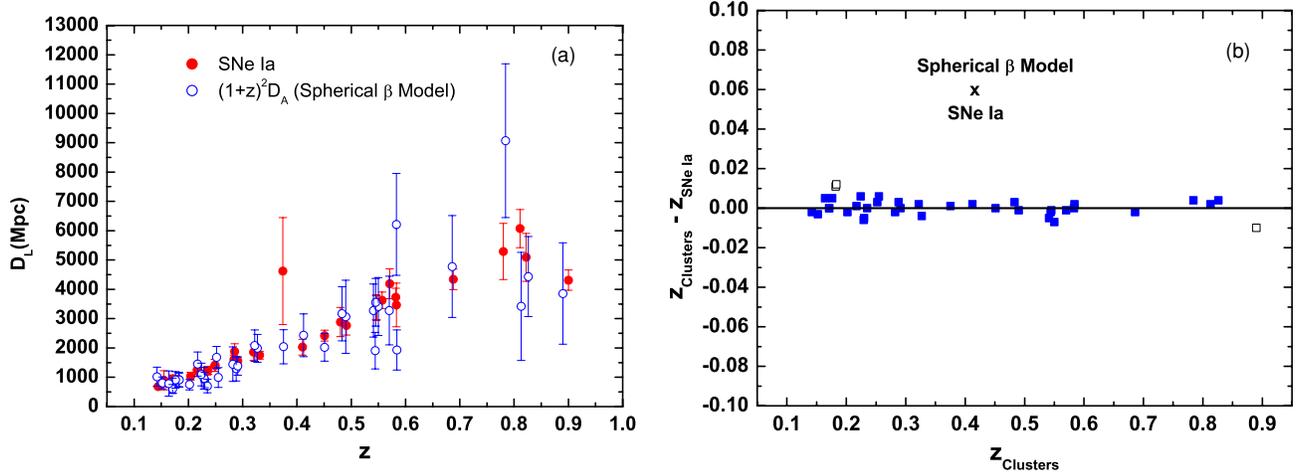


Figure 2. (a) Galaxy clusters and SNe Ia data. The open (blue) and filled (red) circles with the associated error bars stand, respectively, for the Bonamente et al. (2006) and SNe Ia samples. (b) The redshift subtraction for the same pair of cluster-SNe Ia samples. As in Figure 1(b), the open squares represent the pairs of points with the biggest difference in redshifts ($\Delta z \approx 0.01$). (A color version of this figure is available in the online journal.)

include statistical and systematic contributions) and D_L from our second SNe Ia sub-sample. In Figure 2(b), we display the redshift subtraction between clusters and SNe Ia. Again, we see that for 35 clusters $\Delta z < 0.005$. The biggest difference is $\Delta z \approx 0.01$ also for the three clusters, and, for consistency, they will also be removed from our analysis (the next section).

3. ANALYSIS AND RESULTS

Let us now estimate the η_0 parameter for each sample in both parameterizations for $\eta(z) = D_L(z)(1+z)^{-2}/D_A(z)$, namely, $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1+z)$. It should be stressed that in general the SZE + X-ray surface brightness observations technique does not give $D_A(z)$, but $D_A^{\text{cluster}}(z) = D_A(z)\eta^2$. So, if one wishes to test Equation (1) with SZE + X-ray observations from galaxy clusters, the angular diameter distance $D_A(z)$ must be replaced by $D_A^{\text{cluster}}(z)\eta^{-2}$ in Equation (2). In this way, we have access to $\eta(z) = D_A^{\text{cluster}}(z)(1+z)^2/D_L(z)$.

Following standard lines, the likelihood estimator is determined by χ^2 statistics

$$\chi^2 = \sum_z \frac{[\eta(z) - \eta_{\text{obs}}(z)]^2}{\sigma_{\eta_{\text{obs}}}^2}, \quad (3)$$

where $\eta_{\text{obs}}(z) = (1+z)^2 D_A^{\text{cluster}}(z)/D_L(z)$ and $\sigma_{\eta_{\text{obs}}}^2$ are the errors associated with the observational techniques. For the galaxy cluster samples the common statistical contributions are SZE point sources $\pm 8\%$, X-ray background $\pm 2\%$, Galactic $N_{\text{H}} \leq \pm 1\%$, $\pm 15\%$ for cluster asphericity, $\pm 8\%$ kinetic SZ, and for CMB anisotropy $\leq \pm 2\%$. Estimates for systematic effects are as follows: SZ calibration $\pm 8\%$, X-ray flux calibration $\pm 5\%$, radio halos $+3\%$, and X-ray temperature calibration $\pm 7.5\%$. We stress that typical statistical errors amount for nearly 20% in agreement with other works (Mason et al. 2001; Reese et al. 2002, Reese 2004), while for systematics we also find typical errors around $+12.4\%$ and -12% (see also Table 3 in Bonamente et al. 2006). In the present analysis, we have combined the statistical and systematic errors in quadrature for the ADD from galaxy clusters (D'Agostini 2004).

On the other hand, after nearly 500 SNe Ia discovered, the constraints on the cosmic parameters from luminosity distance are now limited by systematics rather than by statistical errors. In principle, there are two main sources of systematic uncertainty in supernovae (SNe) cosmology which are closely related to photometry and possible corrections for light-curve shape (Hicken et al. 2009). However, at the moment the method to

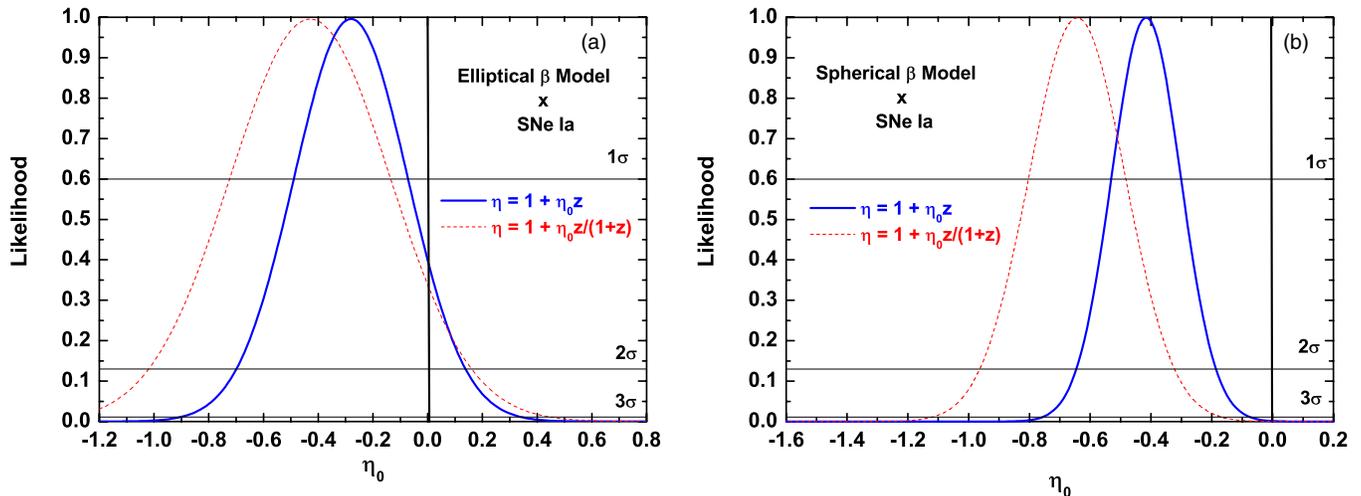


Figure 3. (a) Likelihood distribution functions for the De Filippis et al. sample for both parameterizations. (b) Likelihood distribution functions for the Bonamente et al. sample. Note that the elliptical model is compatible with the Etherington theorem at 2σ while the spherical model is not compatible.

(A color version of this figure is available in the online journal.)

estimate the overall systematic effects for this kind of standard candle is not very neat (Komatsu et al. 2010), and, therefore, we will neglect them in the following analysis. The basic reason is that systematic effects from galaxy clusters seem to be larger than the ones of SNe observations, but their inclusion does not appreciably affect the results concerning the validity of the DD relation.

In Figures 3(a) and (b), we plot the likelihood distribution function for each sample. For De Filippis et al., we obtain $\eta_0 = -0.28^{+0.44}_{-0.44}$ ($\chi^2_{\text{dof}} = 1.02$) and $\eta_0 = -0.43^{+0.6}_{-0.6}$ ($\chi^2_{\text{dof}} = 1.03$) in 2σ (statistical + systematic errors). For Bonamente et al., we obtain $\eta_0 = -0.42^{+0.34}_{-0.34}$ ($\chi^2_{\text{dof}} = 0.88$) and $\eta_0 = -0.66^{+0.5}_{-0.5}$ ($\chi^2_{\text{dof}} = 0.86$) in 3σ (statistical + systematic errors). We can see that the confrontation between the ADD from the former sample with SNe Ia data, points to a moderate violation of the reciprocity relation (the DD relation is marginally satisfied in 2σ). This result remains valid even when only clusters with $z > 0.1$ are considered. In this case, we obtain $\eta_0 = -0.29^{+0.34}_{-0.34}$ ($\chi^2_{\text{dof}} = 0.91$) within 2σ (statistical + systematic errors). However, for the Bonamente et al. sample, where a spherical β model was assumed to describe the clusters, we see that the DD relation is not obeyed even at 3σ .

4. CONCLUSIONS

In this Letter, we have discussed a new and model-independent cosmological test for the DD relation, $\eta(z) = D_L(1+z)^{-2}/D_A$. The basic idea of our statistical test is very simple. We consider the angular diameter distances from galaxy clusters (two samples) which are obtained by using SZE and X-ray surface brightness together with the luminosity distances given by two sub-samples of SNe Ia taken from the Constitution data. The key aspect is that the SNe Ia sub-samples were carefully chosen in order to have the same redshifts of the galaxy clusters ($\Delta z < 0.005$). For the sake of generality, the $\eta(z)$ parameter was also parameterized in two distinct forms, namely, $\eta = 1 + \eta_0 z$ and $\eta = 1 + \eta_0 z/(1+z)$, thereby recovering the equality between distances only for very low redshifts. It should be noticed that in our method the data do not need to be binned. Interestingly, although independent of any cosmological scenario, our analysis depends on the starting physical hypotheses describing the galaxy clusters.

By comparing the De Filippis et al. (2005, elliptical β model) and Bonamente et al. (2006, spherical β model) samples with two sub-samples of SNe Ia, we show that the elliptical geometry is more consistent with no violation of the DD relation. In the case of the De Filippis et al. (2005) sample (see Figure 3(a)), we find $\eta_0 = -0.28^{+0.44}_{-0.44}$ and $\eta_0 = -0.43^{+0.6}_{-0.6}$ for linear and nonlinear parameterizations in 2σ (statistical + systematic errors), respectively. On the other hand, the spherical β model (see Figure 3(b)) is not compatible with the validity of the DD relation. For this case, we obtain $\eta_0 = -0.42^{+0.34}_{-0.34}$ and $\eta_0 = -0.66^{+0.5}_{-0.5}$ for linear and nonlinear parameterizations in 3σ (statistical + systematic errors), respectively.

Finally, it is also interesting to compare the present results with the ones of Holanda et al. (2010). Their analysis revealed that the isothermal elliptical β model is compatible with the Etherington theorem at 1σ moduli of the Λ CDM model while the non-isothermal spherical model is only marginally compatible at 3σ . Here as there, the sphericity assumption for the cluster geometry resulted in a larger incompatibility with the validity of the duality relation in comparison with an isothermal non-spherical cluster geometry.

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