The Astrophysical Journal Letters, 709:L49–L52, 2010 January 20 doi:10.1088/2041-8205/709/1/L49

ON THE INTERPRETATION OF MAGNETIC HELICITY SIGNATURES IN THE DISSIPATION RANGE OF SOLAR WIND TURBULENCE

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Received 2009 April 6; accepted 2009 December 4; published 2010 January 4

ABSTRACT

Measurements of small-scale turbulent fluctuations in the solar wind find a non-zero right-handed magnetic helicity. This has been interpreted as evidence for ion cyclotron damping. However, theoretical and empirical evidence suggests that the majority of the energy in solar wind turbulence resides in low-frequency anisotropic kinetic Alfvén wave fluctuations that are not subject to ion cyclotron damping. We demonstrate that a dissipation range comprised of kinetic Alfvén waves also produces a net right-handed fluctuating magnetic helicity signature consistent with observations. Thus, the observed magnetic helicity signature does not necessarily imply that ion cyclotron damping is energetically important in the solar wind.

Key words: solar wind – turbulence

1. INTRODUCTION

The identification of the physical mechanisms responsible for the dissipation of turbulence in the solar wind, and for the resulting heating of the solar wind plasma, remains an important and unsolved problem of heliospheric physics. An important clue to this problem is the observed non-zero fluctuating magnetic helicity signature at scales corresponding to the dissipation range of solar wind turbulence.

Matthaeus et al. (1982) first proposed the "fluctuating" magnetic helicity as a diagnostic of solar wind turbulence, defining the "reduced fluctuating" magnetic helicity spectrum derivable from observational data (see Section 3). A subsequent study, corresponding to scales within the inertial range, found values that fluctuated randomly in sign, and suggested an interpretation that "a substantial degree of helicity or circular polarization exists throughout the wavenumber spectrum, but the sense of polarization or handedness alternates randomly" (Matthaeus & Goldstein 1982). Based on a study of the fluctuating magnetic helicity of solutions to the linear Vlasov–Maxwell dispersion relation, Gary (1986) suggested instead that, at inertial range scales, all eigenmodes have a very small *intrinsic* normalized fluctuating magnetic helicity, eliminating the need to invoke an ensemble of waves with both left- and right-handed helicity to explain the observations.

Subsequent higher time-resolution measurements, corresponding to scales in the dissipation range, exhibited a non-zero net reduced fluctuating magnetic helicity signature, with the sign apparently correlated with the direction of the magnetic sector (Goldstein et al. 1994). Assuming dominantly anti-sunward propagating waves, the study concluded that these fluctuations had right-handed helicity. The proposed interpretation was that left-hand polarized Alfvén/ion cyclotron waves were preferentially damped by cyclotron resonance with the ions, leaving undamped right-hand polarized fast/whistler waves as the dominant wave mode in the dissipation range, producing the measured net reduced fluctuating magnetic helicity. We refer to this as the cyclotron damping interpretation.

A subsequent analysis of more solar wind intervals confirmed these findings for the dissipation range (Leamon et al. 1998b). Leamon et al. (1998a) argued that a comparison of the normalized cross-helicity in the inertial range (as a proxy for the dominant wave propagation direction in the dissipation range) to the measured normalized reduced fluctuating magnetic helicity provides evidence for the importance of ion cyclotron damping, which would selectively remove the left-hand polarized waves from the turbulence; using a simple rate balance calculation, they concluded that the ratio of damping by cyclotron resonant to non-cyclotron resonant dissipation mechanisms was of order unity. A recent study performing the same analysis on a much larger data set concurred with this conclusion (Hamilton et al. 2008).

In this Letter, we demonstrate that a dissipation range comprised of kinetic Alfvén waves produces a reduced fluctuating magnetic helicity signature consistent with observations. A dissipation range of this nature results from an anisotropic cascade to high perpendicular wavenumber with $k_\perp \gg k_\parallel$; such a cascade is consistent with existing theories for low-frequency plasma turbulence (Goldreich & Sridhar 1995; Boldyrev 2006; Howes et al. 2008b; Schekochihin et al. 2009), numerical simulations (Cho & Vishniac 2000; Howes et al. 2008a), and observations in the solar wind (Horbury et al. 2008; Podesta 2009). Our results imply that no conclusions can be drawn about the importance of ion cyclotron damping in the solar wind based on the observed magnetic helicity signature alone.

2. FLUCTUATING MAGNETIC HELICITY

The magnetic helicity is defined as the integral over the plasma volume $H_m \equiv \int d^3\mathbf{r}\mathbf{A}\cdot\mathbf{B}$, where $\mathbf{A}$ is the vector potential which defines the magnetic field via $\mathbf{B} = \nabla \times \mathbf{A}$. This integral is an invariant of ideal magnetohydrodynamics (MHD) in the absence of a mean magnetic field (Woltjer 1958a, 1958b). Matthaeus & Goldstein (1982) chose to set aside the complications associated with the presence of a mean magnetic field, defining the fluctuating magnetic helicity by $H'_m = \int d\mathbf{r}\delta\mathbf{A}\cdot\delta\mathbf{B}$, where the fluctuating quantities denoted by δ do not include contributions from the mean field.

Modeling the turbulent magnetic field[3] by

$$\mathbf{B}(\mathbf{r},t) = B_0\hat{\mathbf{z}} + \sum_{\mathbf{k}} \mathbf{B}(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \quad (1)$$

[3] We assume that turbulent fluctuations are reasonably modeled as a collection of linear wave modes. Nonlinear interactions, neglected here, will serve to replenish energy lost from wave modes via damping, so we neglect the linear wave damping and take only the real frequency.

in a periodic cube of plasma with volume L^3, we obtain $H'_m = L^3 \sum_{\mathbf{k}} H'_m(\mathbf{k})$, where the fluctuating magnetic helicity density for each wavevector $\mathbf{k}$ is defined by $H'_m(\mathbf{k}) \equiv \mathbf{A}(\mathbf{k}) \cdot \mathbf{B}^*(\mathbf{k})$. Here $\mathbf{B}(-\mathbf{k}) = \mathbf{B}^*(\mathbf{k})$ and $\omega(-\mathbf{k}) = -\omega^*(\mathbf{k})$ are reality conditions and $\mathbf{B}^*(\mathbf{k})$ is the complex conjugate of the Fourier coefficient. Specifying the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, we obtain

$$H'_m(\mathbf{k}) = i\frac{B_y B_z^* - B_y^* B_z}{k_x} = i\frac{B_z B_x^* - B_z^* B_x}{k_y} = i\frac{B_x B_y^* - B_x^* B_y}{k_z}, \quad (2)$$

where the components $B_j(\mathbf{k})$ arise from the eigenfunctions of the linear wave mode. It is easily shown that this result is invariant to rotation of the wavevector $\mathbf{k}$, along with its corresponding linear eigenfunction, about the direction of the mean magnetic field. The normalized fluctuating magnetic helicity density is defined by

$$\sigma_m(\mathbf{k}) \equiv k H'_m(\mathbf{k})/|\mathbf{B}(\mathbf{k})|^2, \quad (3)$$

where $k = |\mathbf{k}|$. This normalized measure has values within the range $[-1, +1]$, where negative values denote left-handed helicity and positive values denote right-handed helicity.

We numerically calculate $\sigma_m(\mathbf{k})$ over the $k_\perp$–$k_\parallel$ plane for the eigenmodes of the linear Vlasov–Maxwell dispersion relation (Stix 1992) for a proton and electron plasma with an isotropic Maxwellian equilibrium distribution function for each species and no drift velocities (see Howes et al. 2006 for a description of the code). The dispersion relation depends on five parameters, $\omega = \omega_{VM}(k_\perp \rho_i, k_\parallel \rho_i, \beta_i, T_i/T_e, v_{\mathrm{th}_i}/c)$, for ion Larmor radius ρ_i, ion plasma beta β_i, ion to electron temperature ratio T_i/T_e, and ion thermal velocity to the speed of light v_{th_i}/c.

We specify plasma parameters characteristic of the solar wind at 1 AU: $\beta_i = 1$, $T_i/T_e = 1$, and $v_{\mathrm{th}_i}/c = 10^{-4}$. Figure 1 is a contour plot of $\sigma_m(\mathbf{k})$ obtained by solving for the Alfvén wave root over the $k_\perp$–$k_\parallel$ plane, then using the complex eigenfunctions to determine $\sigma_m(\mathbf{k})$. The MHD regime corresponds to the lower left corner of the plot, $k_\parallel \rho_i \ll 1$ and $k_\perp \rho_i \ll 1$; here, the Alfvén wave with $k_\perp \sim k_\parallel$ is linearly polarized with $\sigma_m \simeq 0$. As one moves up vertically on the plot to the regime $k_\parallel \gg k_\perp$, the solution becomes left handed with values of $\sigma_m \to -1$. In this regime of nearly parallel wavevectors, the solution represents Alfvén waves in the limit $k_\parallel \rho_i \ll \sqrt{\beta_i}$ and ion cyclotron waves in the limit $k_\parallel \rho_i \gtrsim \sqrt{\beta_i}$. The linear wave mode becomes strongly damped via the ion cyclotron resonance at a value of $k_\parallel \rho_i \gtrsim \sqrt{\beta_i}$ (Gary & Borovsky 2004). This is precisely the behavior supporting the cyclotron damping interpretation of the measured magnetic helicity in the solar wind.

But the Alfvén wave solution does not always produce left-handed magnetic helicity. If one moves instead from the MHD regime horizontally to the right, the solution becomes right handed with $\sigma_m \to +1$ as $k_\perp \rho_i \to 1$, a behavior previously found by Gary (1986). In this regime of nearly perpendicular wavevectors with $k_\perp \gg k_\parallel$, the solution represents Alfvén waves in the limit $k_\perp \rho_i \ll 1$ and kinetic Alfvén waves in the limit $k_\perp \rho_i \gtrsim 1$. Thus, if the dissipation range is comprised of kinetic Alfvén waves, as suggested by theories for critically balanced, low-frequency plasma turbulence (Schekochihin et al. 2009; Howes et al. 2008a), one would expect to observe a positive normalized fluctuating magnetic helicity signature in that regime.

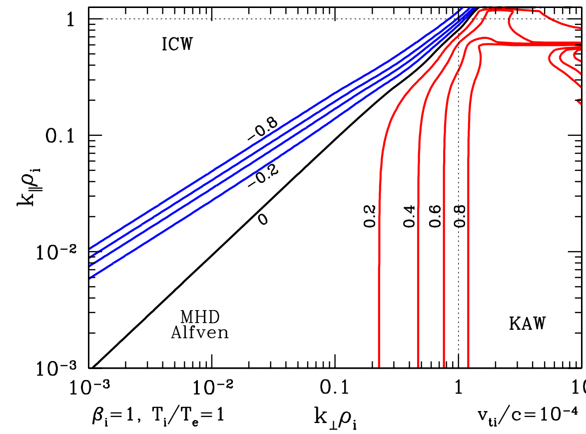

Figure 1. Normalized fluctuating magnetic helicity density $\sigma_m(\mathbf{k})$ (Equation (3)) for linear Alfvén waves over the $k_\perp$–$k_\parallel$ plane. The MHD Alfvén wave (MHD Alfvén), ion cyclotron wave (ICW), and kinetic Alfvén wave (KAW) regimes are identified. Plasma parameters are representative of the near-Earth solar wind.

3. REDUCED FLUCTUATING MAGNETIC HELICITY

Unfortunately, due to the limitations of single-point satellite measurements, Equations (2) and (3) cannot be used directly to calculate the fluctuating magnetic helicity from observations; approximations must be introduced to define a related measurable quantity. In this section, we calculate the reduced fluctuating magnetic helicity density, as defined by Matthaeus et al. (1982) and used by subsequent authors, for the magnetic field defined by Equation (1), but without assuming the Taylor hypothesis.

The two-point, two-time magnetic field correlation function is

$$R_{ij}(\mathbf{r}, t) = \left\langle \delta B_i(\mathbf{x}, \tau) \delta B_j(\mathbf{x} + \mathbf{r}, \tau + t) \right\rangle, \quad (4)$$

where the angle brackets specify an ensemble average, defined here by $\langle a(\mathbf{r}, t) \rangle = L^{-3} \int d^3\mathbf{x} a(\mathbf{x}, \mathbf{r}, t)$. We find

$$R_{ij}(\mathbf{r}, t) = \sum_{\mathbf{k}} B_i^*(\mathbf{k}) B_j(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}, \quad (5)$$

where the reality conditions ensure that this quantity is real.

We choose to sample this correlation function at a moving probe with position given by $\mathbf{r} = -\mathbf{v}t$; this corresponds to satellite measurements of the solar wind, where the probe is stationary and the solar wind is streaming past the probe at velocity $\mathbf{v}$. Thus, we may determine the reduced magnetic field correlation function, $R^r_{ij}(t) = R_{ij}(\mathbf{r}, t)\big|_{\mathbf{r}=-\mathbf{v}t}$, obtaining the form

$$R^r_{ij}(t) = \sum_{\mathbf{k}} B_i^*(\mathbf{k}) B_j(\mathbf{k}) e^{-i(\mathbf{k}\cdot\mathbf{v} + \omega)t}. \quad (6)$$

The reduced frequency spectrum, defined by $S^r_{ij}(\omega') = (1/2\pi) \int dt' R^r_{ij}(t') e^{i\omega' t'}$, is then given by

$$S^r_{ij}(\omega') = \sum_{\mathbf{k}} B_i^*(\mathbf{k}) B_j(\mathbf{k}) \delta[\omega' - (\mathbf{k}\cdot\mathbf{v} + \omega)]. \quad (7)$$

This demonstrates that the frequency ω' of the fluctuations sampled by the moving probe is the Doppler-shifted frequency $\omega' = \mathbf{k}\cdot\mathbf{v} + \omega$. Note that adopting the Taylor hypothesis (Taylor 1938), as often done in studies of solar wind turbulence, corresponds to dropping ω in Equation (7).

The reduced fluctuating magnetic helicity density is defined by

$$H_m^{\prime r}(k_1) = 2\mathrm{Im}[S_{23}^r(k_1)]/k_1, \tag{8}$$

where the effective wavenumber is calculated from the measured frequency using $k_1 = \omega'/v$, assuming the Taylor hypothesis is satisfied (Matthaeus et al. 1982; Matthaeus & Goldstein 1982), and we have chosen an orthonormal basis with direction 1 along the direction of sampling $\hat{\mathbf{v}} = \mathbf{v}/|\mathbf{v}|$ and directions 2 and 3 in the plane perpendicular to $\hat{\mathbf{v}}$. The normalized reduced fluctuating magnetic helicity density is given by $\sigma_m^r(k_1) = k_1 H_m^{\prime r}(k_1)/|\mathbf{B}(k_1)|^2$, where $|\mathbf{B}(k_1)|^2$ is the trace power.

The relation between the reduced fluctuating magnetic helicity density $H_m^{\prime r}(k_1)$ and the fluctuating magnetic helicity density $H_m'(\mathbf{k})$ can be seen by writing the spectrum in terms of the Doppler-shifted frequency ω' instead of k_1, $H_m^{\prime r}(\omega') \equiv 2\mathrm{Im}[S_{23}(\omega')]/(\omega'/v)$. Using Equation (7) and $2\mathrm{Im}[a^*b] = i(ab^* - a^*b)$, the reduced fluctuating magnetic helicity density can be written as

$$H_m^{\prime r}(\omega') = \sum_{\mathbf{k}} \left(\frac{i[B_2(\mathbf{k})B_3^*(\mathbf{k}) - B_2^*(\mathbf{k})B_3(\mathbf{k})]}{\omega'/v} \right) \times \delta[\omega' - (\mathbf{k}\cdot\mathbf{v} + \omega)]. \tag{9}$$

Equation (9), the experimentally accessible quantity, is in terms of the magnetic field measurements in a frame defined by the solar wind velocity $\mathbf{v}$. To write this in terms of the theoretically calculable $H_m'(\mathbf{k})$ (Equation (2)), we express the magnetic field components B_2 and B_3 in the x, y, z coordinate system. To do so, define the probe velocity in spherical coordinates about the direction of the mean magnetic field: $\mathbf{v} = v\sin\theta\cos\phi\hat{\mathbf{x}} + v\sin\theta\sin\phi\hat{\mathbf{y}} + v\cos\theta\hat{\mathbf{z}}$. The orthonormal basis specified with respect to $\hat{\mathbf{v}}$ can be written as

$$\begin{aligned} \hat{\mathbf{e}}_1 &= \hat{\mathbf{v}} = \sin\theta\cos\phi\hat{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}} \\ \hat{\mathbf{e}}_2 &= \hat{\mathbf{z}}\times\hat{\mathbf{v}}/|\hat{\mathbf{z}}\times\hat{\mathbf{v}}| = -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}} \\ \hat{\mathbf{e}}_3 &= \hat{\mathbf{e}}_1\times\hat{\mathbf{e}}_2 = -\cos\theta\cos\phi\hat{\mathbf{x}} - \cos\theta\sin\phi\hat{\mathbf{y}} + \sin\theta\hat{\mathbf{z}}. \end{aligned} \tag{10}$$

Finally, we exploit the fact that the solutions of the Vlasov–Maxwell dispersion relation depend only on the perpendicular and parallel components of the wavevector $k_\perp$ and $k_\parallel$ with respect to the mean magnetic field, and not on the angle about the field; thus, the eigenfunction for a wavevector $\mathbf{k} = k_\perp\hat{\mathbf{x}} + k_\parallel\hat{\mathbf{z}}$ can be rotated by an angle α about the mean magnetic field to yield the solution for any wavevector $\mathbf{k}' = k_\perp\cos\alpha\hat{\mathbf{x}} + k_\perp\sin\alpha\hat{\mathbf{y}} + k_\parallel\hat{\mathbf{z}}$. Using the above, the reduced fluctuating magnetic helicity density $H_m^r(\omega')$ in Equation (9) becomes

$$H_m^{\prime r}(\omega') = \sum_{\mathbf{k}} H_m'(\mathbf{k}) \frac{k_\perp\sin\theta\cos\alpha + k_\parallel\cos\theta}{k_\perp\sin\theta\cos\alpha + k_\parallel\cos\theta + \omega/v} \times \delta[\omega' - (\mathbf{k}'\cdot\mathbf{v} + \omega)], \tag{11}$$

where we have specified the azimuthal angle of the probe velocity $\phi = 0$ without loss of generality. It is clear from this equation that all possible wavevectors $\mathbf{k}'$ that give the same Doppler-shifted frequency ω' will contribute to the sum for the reduced fluctuating magnetic helicity density at the frequency ω'.

4. DISCUSSION

Predicting the values of $H_m^{\prime r}(\omega')$ for solar wind turbulence based on Equation (11) requires understanding three issues: the scaling of the magnetic fluctuation spectrum with wavenumber, the imbalance of Alfvén wave energy fluxes in opposite directions along the mean magnetic field, and the variation of the angle θ between the solar wind velocity $\mathbf{v}$ and the mean magnetic field.

The one-dimensional magnetic energy spectrum in the solar wind typically scales as $k_1^{-5/3}$ in the inertial range and k_1^p in the dissipation range, where $-2 \leqslant p \leqslant -4$ (Smith et al. 2006) and the effective wavenumber is $k_1 = \omega'/v$. It is clear from Equation (11) that, when the plasma frame frequency ω is negligible, the Doppler-shifted observed frequency always results in an effective wavenumber $k_1 \leqslant k$, with equality occurring only when the velocity $\mathbf{v}$ is aligned with the wavevector $\mathbf{k}$. We assume that, for homogeneous turbulence at the dissipation range scales, turbulent energy at fixed $k_\perp$ and $k_\parallel$ is uniformly spread over wavevectors with all possible angles α about the mean magnetic field. Because the fluctuation amplitude deceases for larger effective wavenumbers, the contribution to $H_m^{\prime r}(\omega')$ is maximum at angle $\alpha = 0$; for angles α yielding a Doppler shift to lower effective wavenumbers $k_1 < (k_\perp^2 + k_\parallel^2)^{1/2}$, the higher amplitude fluctuations at those lower wavenumbers will contribute more strongly to $H_m^{\prime r}(\omega')$. An accurate calculation of the magnetic helicity signature based on Equation (11) must take into account the scaling of the magnetic energy spectrum.

To compare to $\sigma_m^r(k_1)$ derived from observations (for example, see Figure 1 of Leamon et al. 1998b), we construct the normalized quantity

$$\hat{\sigma}_m^r(k_1) = \frac{\sum_{\mathbf{k}} H_m'(\mathbf{k}) \frac{\mathbf{k}'\cdot\mathbf{v}}{\mathbf{k}'\cdot\mathbf{v}+\omega} \delta[\omega' - (\mathbf{k}'\cdot\mathbf{v} + \omega)]}{\sum_{\mathbf{k}} [|\mathbf{B}(\mathbf{k})|^2/k]\delta[\omega' - (\mathbf{k}'\cdot\mathbf{v} + \omega)]}. \tag{12}$$

In evaluating Equation (12), we assume a model anisotropic one-dimensional energy spectrum[4] $E(k) = B^2(k)/k$ that fills *only* the MHD Alfvén and kinetic Alfvén wave regimes ($k_\perp > k_\parallel$ and $k_\parallel\rho_i < 1$) and scales as $k^{-5/3}$ for $k\rho_i \ll 1$ and $k^{-7/3}$ for $k\rho_i \gg 1$, consistent with theories for critically balanced turbulence (Goldreich & Sridhar 1995; Howes et al. 2008a; Schekochihin et al. 2009) and solar wind observations (Smith et al. 2006). In Figure 2, we plot $\hat{\sigma}_m^r(k_1)$ versus effective wavenumber $k_1 = \omega'/v$ for plasma parameters $\beta_i = 1$, $T_i/T_e = 1$, $v_{\mathrm{th}_i}/c = 10^{-4}$, $\theta = 60°$, and $v/v_A = 10$. The contributions to $\hat{\sigma}_m^r(k_1)$ for all angles α of each wavevector are collected in 120 logarithmically spaced bins in the Doppler-shifted frequency. The results are rather insensitive to the scaling of the one-dimensional magnetic energy spectrum over the range from k^{-1} to k^{-4}. The solid line in Figure 2 corresponds to the model spectrum assumed above, while the dashed line corresponds to a k^{-1} energy spectrum. Figure 2 demonstrates that turbulence consisting of Alfvén and kinetic Alfvén waves produces a positive (right-handed) magnetic helicity signature in the dissipation range at $k_1\rho_i \gtrsim 1$.

The analysis presented in Figure 2 considers only waves with $k_\parallel > 0$, so all of the waves in the summation in Equation (11) are traveling in the same direction. If there were an equal Alfvén wave energy flux in the opposite direction—a case of balanced energy fluxes, or zero cross helicity—the net $\hat{\sigma}_m^r(k_1)$ would be zero due to the odd symmetry of $H_m'(\mathbf{k})$ in $k_\parallel$. It is often observed, at scales corresponding to the inertial range, that the energy flux in the anti-sunward direction dominates,

[4] On 150×150 logarithmic gridpoints over $k_\perp\rho_i, k_\parallel\rho_i \in [10^{-3}, 10^2]$, the model weights B^2 as a function of $k = (k_\perp^2 + k_\parallel^2)^{1/2}$ using $B^2(k) = B_0^2\{[(k\rho_i)^{-1/3} + (k\rho_i)^{4/3}]/[1 + (k\rho_i)^2]\}^2$.

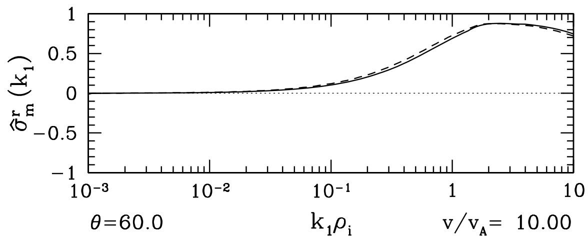

Figure 2. Normalized reduced fluctuating magnetic helicity $\hat{\sigma}^r_m(k_1)$ vs. effective wavenumber k_1 due to a turbulent spectrum of kinetic Alfvén waves with $\theta = 60°$. The solid line corresponds to the model one-dimensional energy spectrum, while the dashed line corresponds to a k^{-1} spectrum.

leading to a large normalized cross helicity (Leamon et al. 1998a). If this imbalance of energy fluxes persists to the smaller scales associated with the dissipation range, a non-zero value of $\hat{\sigma}^r_m(k_1)$ is expected. However, theories of imbalanced MHD turbulence (Chandran 2008, and references therein) predict that the turbulence is "pinned" to equal values of the oppositely directed energy fluxes at the dissipation scale. This implies that, at sufficiently high wavenumber k_1, the value of $\hat{\sigma}^r_m(k_1)$ should asymptote to zero. Thus, $\hat{\sigma}^r_m(k_1)$ in Figure 2 would likely drop to zero more rapidly than shown, leaving a smaller positive net $\hat{\sigma}^r_m(k_1)$ around $k_1\rho_i \sim 1$, consistent with observations (Goldstein et al. 1994; Leamon et al. 1998b; Hamilton et al. 2008). We defer a detailed calculation of the effects of imbalance to a future paper.

The angle θ between $\mathbf{B}_0$ and $\mathbf{v}$ is likely to vary during a measurement; this angle does not typically sample its full range $0 \leqslant \theta \leqslant \pi$, but has some distribution about the Parker spiral value. Calculations of $\hat{\sigma}^r_m(k_1)$ over $0 \leqslant \theta \leqslant \pi$ yield results that are qualitatively similar to Figure 2, so this averaging will not significantly change our results.

Taken together, we have demonstrated that a solar wind dissipation range composed of kinetic Alfvén waves produces a magnetic helicity signature consistent with observations, as presented in Figure 2. The underlying assumption of the cyclotron damping interpretation of magnetic helicity measurements, an interpretation that dominates the solar wind literature (Goldstein et al. 1994; Leamon et al. 1998a, 1998b; Hamilton et al. 2008), is the slab model, $\mathbf{k} = k_\parallel \hat{\mathbf{z}}$ and $k_\perp = 0$, i.e., purely parallel wavevectors. As shown in Figure 1, only in the limit $k_\parallel \gg k_\perp$ does the Alfvén wave root generate a left-handed helicity $\sigma_m \to -1$ as $k_\parallel \rho_i \to \sqrt{\beta_i}$; in the same limit, the fast/whistler root generates a right-handed helicity $\sigma_m \to +1$ in a quantitatively similar manner (see Figure 9 of Gary 1986). Strong ion cyclotron damping of the Alfvén/ion cyclotron waves as $k_\parallel \rho_i \to 1$ (Gary & Borovsky 2004) would leave a remaining spectrum of right-handed fast/whistler waves, as proposed by cyclotron damping interpretation. However, only if the majority of the turbulent fluctuations have $k_\parallel \gtrsim k_\perp$ is the slab limit applicable, and only if significant energy resides in slab-like fluctuations are the conclusions drawn about the importance of cyclotron damping valid. There is, on the other hand, strong theoretical and empirical support for the hypothesis that the majority of the energy in solar wind turbulence has $k_\perp \gg k_\parallel$ (see Howes et al. 2008a, and references therein). In this case, there is a transition to kinetic Alfvén wave fluctuations at the scale of the ion Larmor radius. This Letter demonstrates that a dissipation range comprised of kinetic Alfvén waves produces a reduced fluctuating magnetic helicity signature consistent with observations.

G.G.H. thanks Ben Chandran for useful discussions. G.G.H. was supported by the DOE Center for Multiscale Plasma Dynamics, Fusion Science Center Cooperative Agreement ER54785. E.Q. and G.G.H. were supported in part by the David and Lucille Packard Foundation. E.Q. was also supported in part by NSF-DOE Grant PHY-0812811 and NSF ATM-0752503.

REFERENCES

Boldyrev, S. 2006, Phys. Rev. Lett., 96, 115002
Chandran, B. D. G. 2008, ApJ, 685, 646
Cho, J., & Vishniac, E. T. 2000, ApJ, 539, 273
Gary, S. P. 1986, J. Plasma Phys., 35, 431
Gary, S. P., & Borovsky, J. E. 2004, J. Geophys. Res., 109, 6105
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
Goldstein, M. L., Roberts, D. A., & Fitch, C. A. 1994, J. Geophys. Res., 99, 11519
Hamilton, K., Smith, C. W., Vasquez, B. J., & Leamon, R. J. 2008, J. Geophys. Res., 113, A01106
Horbury, T. S., Forman, M., & Oughton, S. 2008, Phys. Rev. Lett., 101, 175005
Howes, G. G., Cowley, S. C., Dorland, W., Hammett, G. W., Quataert, E., & Schekochihin, A. A. 2006, ApJ, 651, 590
Howes, G. G., Cowley, S. C., Dorland, W., Hammett, G. W., Quataert, E., & Schekochihin, A. A. 2008a, J. Geophys. Res., 113, A05103
Howes, G. G., Dorland, W., Cowley, S. C., Hammett, G. W., Quataert, E., Schekochihin, A. A., & Tatsuno, T. 2008b, Phys. Rev. Lett., 100, 065004
Leamon, R. J., Matthaeus, W. H., Smith, C. W., & Wong, H. K. 1998a, ApJ, 507, L181
Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., & Wong, H. K. 1998b, J. Geophys. Res., 103, 4775
Matthaeus, W. H., & Goldstein, M. L. 1982, J. Geophys. Res., 87, 6011
Matthaeus, W. H., Goldstein, M. L., & Smith, C. 1982, Phys. Rev. Lett., 48, 1256
Podesta, J. J. 2009, ApJ, 698, 986
Schekochihin, A. A., Cowley, S. C., Dorland, W., Hammett, G. W., Howes, G. G., Quataert, E., & Tatsuno, T. 2009, ApJS, 182, 310
Smith, C. W., Hamilton, K., Vasquez, B. J., & Leamon, R. J. 2006, ApJ, 645, L85
Stix, T. H. 1992, Waves in Plasmas (New York: AIP)
Taylor, G. I. 1938, Proc. R. Soc. A, 164, 476
Woltjer, L. 1958a, Proc. Natl Acad. Sci., 44, 489
Woltjer, L. 1958b, Proc. Natl Acad. Sci., 44, 833