PT-symmetric transport in non-PT-symmetric bi-layer optical arrays

To cite this article: J Ramirez-Hernandez et al 2016 J. Opt. 18 09LT01

View the article online for updates and enhancements.

Related content

- Relativistic Zitterbewegung in non-Hermitian photonic waveguide systems
  Guanglei Wang, Hongya Xu, Liang Huang et al.

- Dissimilar directional couplers showing $\mathcal{PT}$-symmetric-like behavior
  Wiktor Walasiak, Chicheng Ma and Natalia M Litchinitser

- $\mathcal{PT}$-symmetry breaking in the steady state of microscopic gain-loss systems
  Kosmas V Kepesidis, Thomas J Milburn, Julian Huber et al.

Recent citations

- Notable advances in photonics: the JOPT Highlights of 2016
  Jarlath McKenna
Letter

\( \mathcal{PT} \)-symmetric transport in non-\( \mathcal{PT} \)-symmetric bi-layer optical arrays

J Ramirez-Hernandez\(^1\), F M Izrailev\(^{1,4}\), N M Makarov\(^2\) and D N Christodoulides\(^3\)

\(^1\)Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apdo. Post. J-48, Puebla, Pue., 72570, Mexico

\(^2\)Instituto de Ciencias, Benemérita Universidad Autónoma de Puebla, Priv. 17 Norte No. 3417, Col. San Miguel Hueyotlipan, Puebla, Pue., 72050, Mexico

\(^3\)College of Optics and Photonics-CREOL, University of Central Florida, 32816, USA

E-mail: felix.izrailev@gmail.com and makarov.n@gmail.com

Received 28 February 2016, revised 30 April 2016
Accepted for publication 5 May 2016
Published 27 July 2016

Abstract

We study transport properties of an array created by alternating \((a, b)\) layers with balanced loss/gain characterized by the key parameter \(\gamma\). It is shown that for non-equal widths of \((a, b)\) layers, i.e., when the corresponding Hamiltonian is non-\(\mathcal{PT}\)-symmetric, the system exhibits the scattering properties similar to those of truly \(\mathcal{PT}\)-symmetric models provided that without loss/gain the structure presents the matched quarter stack. The inclusion of the loss/gain terms leads to an emergence of a finite number of spectral bands characterized by real values of the Bloch index. Each spectral band consists of a central region where the transmission coefficient \(T_N\) and two side regions with \(\bar{T}_N\). At the borders between these regions the unidirectional reflectivity occurs. Also, the set of Fabry–Perot resonances with \(T_N = 1\) are found in spite of the presence of loss/gain.

Keywords: PT-symmetry, unidirectional reflectivity, quarter stacks

(Some figures may appear in colour only in the online journal)

Introduction

The possibility of deliberately intermixing loss and gain in optical structures as a means to attain physical properties that are otherwise out of reach in standard arrangements has recently been explored in a number of studies [1–8]. In general, such non-conservative configurations can display surprising behavior resulting from the possible presence of exceptional points [9, 10] and mode bi-orthogonality. A particular class of non-Hermitian systems that has recently attracted considerable attention is that respecting parity-time (PT) symmetry [1–5]. Such a symmetry emerges if a Hamiltonian is symmetric under the combined action of space reflection \(\mathcal{P}\) and time reversal \(\mathcal{T}\) operators. In this case the spectrum happens to be real in spite of the fact that the potential involved is complex [9, 10]. Once however a loss/gain parameter exceeds a critical value, this symmetry can spontaneously break and consequently the eigenvalue spectrum ceases to be real and hence enters the complex domain [9].

The availability of optical attenuation and amplification makes optics an ideal ground where \(\mathcal{PT}\)-symmetric effects can be experimentally observed and investigated [3, 4, 11]. In the paraxial regime, a necessary (but not sufficient) condition to impose this symmetry in photonics is that the complex
refractive index distribution obeys \( n(x) = n^\mu(-x) \) [1, 2]. This latter condition directly implies that the real part of the refractive index potential must be an even function of position while its imaginary component must be antisymmetric. On the other hand, in the case of scattering problems, \( \mathcal{PT} \)-symmetric effects can be investigated based on scattering matrix formalisms or transfer matrices—avenues that have remained so far relatively unexplored.

Thus far, the ramifications of parity-time symmetry on optical transport have been systematically considered in several theoretical and experimental investigations. Processes emerging from \( \mathcal{PT} \)-symmetry, like double refraction, band merging, unidirectional invisibility, abrupt phase transitions and nonreciprocal wave propagation have been predicted and observed [1, 3, 11–14]. In addition this same symmetry was recently utilized to enforce single-mode behavior in laser micro-cavities as well as to judiciously control wave dynamics around exceptional points and photonic crystals [15]. Finally, similar concepts are nowadays used in other fields, like for example opto-mechanics, acoustics, nonlinear optics, plasmonics, metamaterials, and imaging to mention a few [16]. In view of these developments, the following question naturally arises. To which extent could a system depart from an exact parity-time symmetry and still exhibit \( \mathcal{PT} \)-symmetric characteristics? Recently, a possibility of merging, unidirectional invisibility, abrupt phase transitions and nonreciprocal wave propagation have been predicted and observed [1, 3, 11–14]. In addition this same symmetry was recently utilized to enforce single-mode behavior in laser micro-cavities as well as to judiciously control wave dynamics around exceptional points and photonic crystals [15]. Finally, similar concepts are nowadays used in other fields, like for example opto-mechanics, acoustics, nonlinear optics, plasmonics, metamaterials, and imaging to mention a few [16]. In view of these developments, the following question naturally arises. To which extent could a system depart from an exact parity-time symmetry and still exhibit \( \mathcal{PT} \)-symmetric characteristics? Recently, a possibility of synthesizing complex potentials that support entirely real spectra despite the fact that they violate the necessary condition for \( \mathcal{PT} \)-symmetry has been discussed in [17] within the context of optical supersymmetry (see, also, [18]).

In this paper we show that it is indeed possible to observe \( \mathcal{PT} \)-scattering behavior even in systems that strictly speaking lack this symmetry. This is explicitly demonstrated in perfectly matched multilayer non-Hermitian arrangements having unequal layer widths. In this case, the incorporation of loss/gain domains leads to a finite number of bands—all associated with a real Bloch index that is reminiscent of actual \( \mathcal{PT} \)-symmetric lattices [12]. Special attention has been paid to an emergence of unidirectional reflectivity. Our results indicate that a \( \mathcal{PT} \)-symmetric response can be quite robust and therefore can be observed in more involved structures even in the absence of a strict \( \mathcal{PT} \)-symmetry.

The model

We consider the propagation of an electromagnetic wave of frequency \( \omega \) through a regular array of \( N \) identical unit \((a, b)\) cells embedded in a homogeneous \( \varepsilon \) -medium, see figure 1. Each cell consists of two dielectric \( a \) and \( b \) layers with the thicknesses \( d_a \) and \( d_b \), respectively, so that \( d = d_a + d_b \) is the unit-cell size. The \( a \) and \( b \) layers are made of the materials absorbing and amplifying the electromagnetic waves, respectively. The loss and gain in the layers are incorporated via complex permittivities, while the magnetic permeabilities \( \mu_{a,b} \) are assumed to be real and positive constants. The refractive indices \( n_{a,b} \), impedances \( Z_{a,b} \) and wave phase shifts \( \varphi_{a,b} \) of \( a \) and \( b \) layers are presented as follows,

\[
\begin{align*}
    n_a &= n_a^{(0)}(1 + i\gamma), \quad Z_a = Z(1 + i\gamma)^{-1}, \quad \varphi_a = \varphi(1 + i\gamma)/2, \\
    n_b &= n_b^{(0)}(1 - i\gamma), \quad Z_b = Z(1 - i\gamma)^{-1}, \quad \varphi_b = \varphi(1 - i\gamma)/2,
\end{align*}
\]

where

\[
Z = \mu_a/n_a^{(0)} = \mu_b/n_b^{(0)}, \quad \varphi = 2\omega n_0^{(0)}d_a/c = 2\omega n_0^{(0)}d_b/c.
\]

Here the parameter \( \gamma > 0 \) measures the strength of balanced loss and gain inside \( a \) and \( b \) layers, respectively, and \( \varphi = \varphi_a + \varphi_b \) is the purely real phase shift of the wave passing each of the unit cells. In the case of no loss/gain (\( \gamma = 0 \)) the structure is known as the matched quarter stack for which the \( a \) and \( b \) layers are perfectly matched and have equal optic paths, \( n_a^{(0)}d_a = n_b^{(0)}d_b \). As one can see, for \( \gamma = 0 \) and \( d_a = d_b \) the corresponding Hamiltonian has no \( \mathcal{PT} \)-symmetry. A particular \( \mathcal{PT} \)-symmetric realization is achieved only when \( d_a = d_b \) with \( \varepsilon_a^{(0)} = \varepsilon_b^{(0)} \), and \( \mu_a = \mu_b \).

Within any layer an electromagnetic wave propagates along the \( x \)-direction (figure 1) perpendicular to the stratification according to the 1D Helmholtz equation. Its general solution inside every unit cell can be presented as a superposition of traveling forward and backward plane waves. By combining these solutions with the boundary conditions for the wave at the interfaces between \( a \) and \( b \) layers, one can obtain the unit-cell transfer matrix \( \hat{M} \) relating the wave amplitudes of two adjacent unit cells (see, e.g., [19]). For our model the elements of the \( M \)-matrix read

\[
\begin{align*}
    M_{11} &= \frac{\exp(i\varphi) + \gamma^2\exp(-\gamma\varphi)}{1 + \gamma^2}, \\
    M_{12} &= \frac{i\gamma}{1 + \gamma^2}[\exp(-i\varphi) - \exp(\gamma\varphi)], \\
    M_{21} &= \frac{i\gamma}{1 + \gamma^2}[\exp(i\varphi) - \exp(-\gamma\varphi)], \\
    M_{22} &= \frac{\exp(-i\varphi) + \gamma^2\exp(\gamma\varphi)}{1 + \gamma^2}.
\end{align*}
\]

Note that \( \det \hat{M} = 1 \) in accordance with the general condition inherent for transfer matrices. On the other hand, the transfer matrix \( \hat{M}(\gamma) \) has specific symmetry,

\[
M_{22}(\gamma) = M_{11}^*(\gamma), \quad M_{21}(\gamma) = M_{12}^*(\gamma),
\]

where ‘*’ stands for the complex conjugation. This symmetry can be compared with that emerging for various \( \mathcal{PT} \)-symmetric Hamiltonians [7, 20–22]. Clearly, it differs from the standard one, \( M_{22} = M_{11}^*, \quad M_{21} = M_{12}^* \), that originates from time-reversal symmetry.
Two curves $\gamma = \pm 0.25$ with different values of $N$. Spectral bands and gaps are shown by shaded areas.

**Frequency band structure**

The dispersion relation for the Bloch index $\mu_B$ determined through the eigenvalues $\exp(\pm i\mu_B)$ of the transfer $\tilde{M}$-matrix, is defined by the matrix trace, $2 \cos \mu_B = M_{11} + M_{22}$. Thus, one obtains

$$\cos \mu_B = \frac{\cos \varphi + \gamma^2 \cosh(\gamma \varphi)}{1 + \gamma^2}. \quad (3)$$

This relation determines the dependence of the Bloch wave number $\kappa = \mu_B/d$ on the frequency $\omega$ (or, on the phase shift $\varphi \propto \omega$) for the wave inside the scattering region. One of the important conclusions that follows from equation (3), is a finite number $N_{\text{band}}$ of spectral bands defined by the condition $|\cos \mu_B| < 1$. This number $N_{\text{band}}$ depends on the parameter $\gamma$ only, and for $\gamma < 1$ can be estimated as

$$N_{\text{band}} \approx (1/\pi) \ln(1/\gamma). \quad (4)$$

Within these bands the Bloch index $\mu_B$ is real in spite of the presence of loss and gain. This property has been widely discussed in view of the nature of $PT$-symmetric systems. As is known, the real-valued solutions $\mu_B$ to the dispersion equation $2 \cos \mu_B = M_{11} + M_{22}$ can exist only when the trace $M_{11} + M_{22}$ of the transfer $\tilde{M}$-matrix is real-valued, i.e. if

$$\text{Im}M_{12} = -\text{Im}M_{21}. \quad (5)$$

In our model this determinative condition is provided not due to the $PT$-symmetry but merely due to the balance between loss and gain.

Outside the bands, where $|\cos \mu_B| > 1$, the index $\mu_B$ is purely imaginary, thus creating spectral gaps. Here the waves are known as the evanescent Bloch states, attenuated on the scale of the order of $|\mu_B|^{-1}$. Therefore, for a sufficiently long structure, $N|\mu_B| > 1$, the transmission is exponentially small. It should be noted that in the absence of loss/gain ($\gamma = 0$), there are no spectral gaps since all spectral bands are touching.

When $\gamma$ exceeds the critical value $\gamma_{cs} = 1$, there are no spectral bands since the Bloch index $\mu_B(\omega)$ becomes imaginary for any frequency $\omega$. Two examples of the dependence $\mu_B(\omega)$ are shown in figure 2 for different values of $\gamma$. It is quite instructive that within any spectral band the maximal value of the Bloch index $\mu_B^{\max}$ is less than $\pi$ for any nonzero $\gamma$. The smaller the parameter $\gamma$, the closer to $\pi$ can be the Bloch index $\mu_B$. On the other hand, for any given $\gamma$ from the interval $0 < \gamma < 1$ the higher the band the smaller the value of $\mu_B^{\max}$.

**Transmittance**

With the standard transfer matrix approach we derived an analytical expression for the transmittance $T_N$ of our model consisting of $N$ unit cells and connected to perfect $c$-leads by the impedance $Z_T$,

$$T_N = \left[1 - \frac{\gamma^2 \sin^2(N\mu_B)}{4(1 + \gamma^2)^2 \sin^2 \mu_B} \mathcal{F}(\gamma, \varphi) \mathcal{F}(-\gamma, \varphi)\right]^{-1}, \quad (6)$$

where $\mathcal{F}(\gamma, \varphi) = \gamma^2 \sinh(\gamma \varphi) - \gamma \sin \varphi + 2(\cos \varphi - e^{-\gamma \varphi})$. \quad (7)

Here $\mathcal{F}(\gamma, \varphi) < 0$ for any value of $\phi$, unlike $\mathcal{F}(\gamma, \varphi)$ that can be either positive or negative as a function of $\varphi$. Two examples of dependence $T_N(\varphi)$ are given in figure 3.

The analysis shows that a distinctive property of the transmittance is that every spectral band consists of the central region with $T_N \geq 1$ and those with $T_N \leq 1$, see figures 3 and 4. At the borders between these regions (circles in figure 3) the transmission is perfect, $T_N = 1$, and this happens due to vanishing the function $\mathcal{F}(\gamma, \varphi)$. The corresponding values $\phi_0(\gamma)$ for which $\mathcal{F}(\gamma, \varphi_0) = 0$ can be compared with exceptional points emerging in the $PT$-symmetric models (see, e.g., [23] and references therein). However, in our case the Bloch index $\mu_B(\gamma, \varphi_0)$ does not vanish, in contrast to truly exceptional points for which both conditions, $T_N = 1$ and $\mu_B = 0$, are fulfilled. These conditions are known as that providing invisibility of a model in the scattering process. The only situation for the invisibility to occur in our setup is when two conditions, $\mathcal{F}(\gamma, \varphi) = 0$ and $\mu_B(\gamma, \varphi) = 0$, meet. As one can see in figure 4, the sequence of exceptional points $\varphi = \varphi_0(\gamma)$ emerges for the corresponding sequence of $\gamma = \gamma_0(\varphi)$ for which two curves $\varphi = \varphi_0(\gamma)$ and $\mu_B(\gamma, \varphi) = 0$ touch each other. Note that at every fixed value of $\gamma = \gamma_0$ the
corresponding exceptional point $\varphi_{sp}$ is located on the top of the highest band. Apart from the specific points $\varphi_s(\gamma)$, the perfect transmission, $T_N = 1$, occurs due to Fabry–Perot resonances located within spectral bands. They are defined by the condition $\sin(N\mu_B)/\sin \mu_B = 0$ specifying the resonant values $\mu_B^{\text{res}} = m\pi/N$ ($m = 1, 2, 3, \ldots, N - 1$) of the Bloch index $\mu_B$. Since the spectrum $\mu_B(\omega)$ is restricted by $\mu_B^{(\text{max})}$, the total number of such resonances in a given spectral band can vary from zero to $2(N - 1)$, depending on the band number and loss/gain parameter $\gamma$. Specifically, for a fixed $\gamma$ the higher spectral band contains the smaller number of Fabry–Perot resonances. This result is principally different from what happens in an array of bi-layers with real and positive optical parameters, where $N - 1$ number of Fabry–Perot resonances emerge in any spectral band.

### Unidirectional reflectivity

The expressions for the left and right reflectances, $R_N^{(L)}$ and $R_N^{(R)}$, read

$$R_N^{(L)} = \frac{\gamma^2 \sin^2(N\mu_B)}{4(1 + \gamma^2)^2 \sin^2 \mu_B} \mathcal{F}^2(-\gamma, \varphi),$$

$$R_N^{(R)} = \frac{\gamma^2 \sin^2(N\mu_B)}{4(1 + \gamma^2)^2 \sin^2 \mu_B} \mathcal{F}^2(-\gamma, \varphi).$$

Note that $T_N(-\gamma) = T_N(\gamma)$ and $R_N^{(L)}(-\gamma) = R_N^{(R)}(\gamma)$. It is remarkable that $T_N(\gamma)$ and $R_N^{(L)}(\gamma)$, $R_N^{(R)}(\gamma)$ satisfy the famous relation,

$$|1 - T_N| = \sqrt{R_N^{(L)} R_N^{(R)}}.$$
is not the $\mathcal{PT}$-symmetric one. Our analysis exhibits that such a situation emerges in the bi-layer optical setup which without loss/gain is known as the quarter stack, provided all layers are perfectly matched (no reflections due to boundary conditions).

In the absence of loss/gain terms, all spectral bands are touched thus creating a perfect transmission for any wave frequency. When the balanced loss and gain are alternatingly included in all layers, the spectral gaps emerge between the bands, and the total number of bands is defined by the loss/gain parameter $\gamma$ only.

Inside the spectral bands the Bloch index $\mu_B$ appears to be real as it happens in the known models with the $\mathcal{PT}$-symmetric Hamiltonians. The analysis shows that each of the spectral bands consists of a central region where the transition coefficient is greater than one, $T_N > 1$, and two side regions with $T_N \leq 1$. At the borders between these regions the transmission is perfect, $T_N = 1$, however, $\mu_B$ does not typically vanish as happens in the known $\mathcal{PT}$-symmetric models. These borders are of specific interest since one of the reflectances vanishes, thus leading to the so-called ‘unidirectional reflectivity’, the effect which is important both from theoretical and experimental points of view. The ratio between left and right reflectances is specified by the loss/gain parameter $\gamma$ and the wave frequency $\omega$, being independent of the number $N$ of cell units. It is important that a strong difference between left and right reflectances occurs in a relatively large wave-propagation region where the Bloch index $\mu_B$ is real.

The analytical expressions display that inside the spectral bands the Fabry–Perot resonances emerge in spite of the presence of loss and gain. These resonances exist both in the regions with $T_N > 1$ and $T_N < 1$. It is also worthwhile that the invisibility (when both relations, $T_N = 1$ and $\mu_B = 0$ hold) is realized only for a quite specific values $\gamma_{ip}$ and for the corresponding wave frequencies located in the highest spectral bands.

In conclusion, our study unexpectedly manifests that the $\mathcal{PT}$-symmetric properties may occur in non-$\mathcal{PT}$-symmetric systems, a fact that may be important in view of experimental realizations of matched quarter stacks, as well as for the theory of the $\mathcal{PT}$-symmetric transport.

Acknowledgments

The authors are thankful to L. Deych, T. Kottos and A. Li-syansky for fruitful discussions. This work was supported by the CONACYT (México) grant No. CB-2011-01-166382 and by the VIEP-BUAP grant IZF-EXC15-G. DNC acknowledges the support by NSF (Grant No. ECCS-1128520), and AFOSR (Grant No. FA9550-14-1-0037).

References


[22] Chang L et al 2014 Nat. Photon. 8 524


