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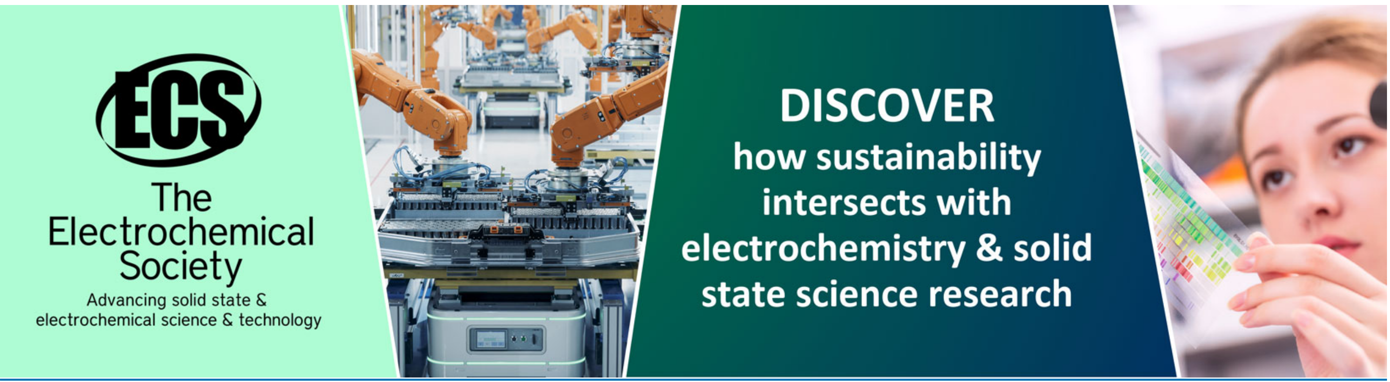

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On polymer loop in a gel under external fields: analytical approach using white noise analysis

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Abstract. This paper presents derivation of the probability distribution for the area enclosed by a polymer loop in a gel and under different external fields using white noise analysis. In this context, the polymer loop is represented by Brownian paths and its immersion in a gel constraints it to occupy a constant area[1]. The external fields considered are electric field, and crossed electric-magnetic fields.

1. Introduction

The Hida-Streit's infinite dimensional analysis, otherwise known as white noise analysis has been used successfully as a framework for stochastic and infinite-dimensional systems. This has been applied to several disciplines, most notably in quantum physics [2,3] and statistical mechanics[4,5]. Its applicationin solving polymer conformation problems was first introduced by C. Bernido and M. V. Bernido where they studied statistical mechanical properties of polymers with length dependent potentials[4,5], polymer chirality [5]and winding probability of entangled polymers[6].

As an extension to the works of C. Bernido and M. V. Bernido, we present in this paper the use white noise analysis in getting the probability distribution for the area enclosed by a polymer loop in a gel under different external fields. In this context, the polymer conformation is viewed as Brownian paths and the steric and topological effects of others polymers in a gel constraints the loop to enclose a fixed area[1,7]. An example of a polymer in a gel under an external field is DNA electrophoresis where the DNA is subjected by an external electric field. In this paper, we generalize the probability distribution to include other possible external fields.

This paper is organized as follows: we give a brief review of the path integral of the probability distribution for the area enclosed by a polymer loop in a gel as presented in [7]. Then we generalize the formulation to include external fields where we consider time-independent electric fieldand crossed electric-magnetic fields. We then translate the probability distribution in the language of white noise analysis and evaluate the distribution using the *T*- and *S-transforms* [5]. We note that the same polymer system with and without electric field was also studied by Khandekar and Wiegel [7,8] using straight forward evaluation of the path integral describing the system.

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2. Path integral for the distribution of the area enclosed by a polymer loop under external fields

An unconstrained polymer may be viewed as a polymer in a dilute solution where the interaction between the solvent and the polymer is negligible. The Wiener integral representation of the probability density of this polymer with ends fixed at $\boldsymbol{r_0}$ and $\boldsymbol{r_1}$ is given by [7]

$$\boldsymbol{P}(\boldsymbol{r_1}, \boldsymbol{N}|\boldsymbol{r_0}, \boldsymbol{0}) = \int_{r_0}^{r_1} \exp\left[-\frac{1}{l^2}\int_0^N \left(\frac{dr}{dv}\right) dv\right] d[r(v)], \quad (1)$$

where for non-interacting polymer,l is the distance between two monomers, and v is the number of monomers with increasing value from 1 to N.

For a polymer in a gel or a polymer solution, the steric and topological effects of others polymers constraints the polymer to enclose a constant area A [1,7] making the probability distribution for the area enclosed as

$$\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{N}) = \boldsymbol{C}\int_{r_0}^{r_1} \delta\left[A - \int_0^N \frac{1}{2}\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right) dv\right] \exp\left[-\frac{1}{l^2}\int_0^N \left(\frac{dr}{dv}\right)^2 dv\right] d[r(v)], \quad (2)$$

where C is the normalization constant and

$$A = \int_0^N \frac{1}{2}\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right) dv \quad (3)$$

is the algebraic area which is enclosed by the polymer loop configuration $\boldsymbol{r}(v)$. The sign convention is that A is counted positive if the enclosed area is located to the left of the curve when traced by increasing number of monomers, v, otherwise it is negative [7].

When this polymeric system is under an external field with potential V, we can write the probability distribution in general form as

$$\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{N}) = \boldsymbol{C}\int_{r_0}^{r_1} \delta\left[A - \int_0^N \frac{1}{2}\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right) dv\right] \exp\left[-\frac{1}{l^2}\int_0^N \left(\frac{dr}{dv}\right)^2 dv - \int_0^N V dv\right] d[r(v)]. \quad (4)$$

The effect of the potential is added on the '*kinetic*' part of the path integral.

Rewriting the Dirac delta as Fourier integral, we have the probability density in equation (4) as

$$\boldsymbol{P}(\boldsymbol{A}, \boldsymbol{N}) = \frac{C}{2\pi}\int_{-\infty}^{+\infty} \widetilde{P}(g, N) \exp(igA)\, dg \quad (5)$$

where

$$\widetilde{P}(g, N) = \int_{r_0}^{r_1} \exp\left(-\int_0^N \tilde{L} dv\right) d[r(v)] \quad (6)$$

and

$$\tilde{L} = \frac{1}{l^2}\left[\left(\frac{dx}{dv}\right)^2 + \left(\frac{dy}{dv}\right)^2\right] + \frac{1}{2} ig\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right) + V. \quad (7)$$

We then evaluate the probability distribution, equation (5), using white noise analysis. In this method, the paths are parametrized in terms of Brownian motion. Essential steps of the evaluation are presented in the next section.

3. Probability distribution in the framework of White Noise Analysis

In writing the probability distribution in equation (5) to the language of white noise analysis, we first parametrize the paths as

$$x(L) = x_0 + lB_x(l) \text{ and} \quad (8)$$

$$y(L) = y_0 + lB_y(l), \quad (9)$$

where $B_x = \int_0^N \omega_x dv$ and $B_y = \int_0^N \omega_y dv$ are Brownian motions in the x and y coordinates respectively parametrized by $v$that runs from 0 to N, $\omega(v)$ is the white noise variable, l is the distance between monomers and L is the length of the polymer loop.

In white noise analysis, the integral over the paths $d[\boldsymbol{r}(v)]$ becomes an integral over the Gaussian white noise $d\mu(\omega)$ with relation given by [5]

$$\boldsymbol{d[r(v)] \to N_{xy}\mathbf{exp}\left[\frac{1}{2}\int_0^N\left(\omega_x{}^2 + \omega_y{}^2\right)dv\right]d\mu(\omega_x)d\mu(\omega_y)}, \tag{10}$$

where $N_{xy}$is an appropriate normalization constant. We further note that in the process of path parametrization done in equations (8) and (9), only the initial point is fixed. Hence we use the Donsker delta functions

$$\boldsymbol{\delta(x(L) - x_1) = \delta(x_0 + lB_x(l) - x_1)}\text{and} \tag{11}$$

$$\boldsymbol{\delta(y(L) - y) = \delta\left(y_0 + lB_y(l) - y\right)} \tag{12}$$

to fix the endpoints. For a polymer loop, the final and initial points are the same simplifying equations (11) and (12) as$\delta(B_x(l))$ and $\delta(B_x(l))$ respectively. These Donsker delta functions are then written into their Fourier integral representations.

These are the initial preliminary steps in writing a Wiener integral representation of the polymer loop in the language of white noise analysis. The bulk of the evaluation of the probability distribution is done via the *T-transform*, the mathematical tool used to evaluate the integration over the Gaussian white noise measure $d\mu(\omega)$.

In the following subsections, we consider different systems in applying white noise analysis.

3.1. Polymer in a gel in the absence of an external field

When there is no external field, the distribution for the area enclosed by the polymer loop is given by

$$\boldsymbol{P_0(A,N) = \frac{C}{2\pi}\int_{-\infty}^{+\infty}\widetilde{P}_0(g,N)\mathbf{exp}(igA)\,dg} \tag{13}$$

where

$$\boldsymbol{\widetilde{P}_0(g,N) = \int_{r_0}^{r_1}\mathbf{exp}\left(-\int_0^N \tilde{L}dv\right)d[r(v)]} \tag{14}$$

and

$$\boldsymbol{\tilde{L} = \frac{1}{l^2}\left[\left(\frac{dx}{dv}\right)^2 + \left(\frac{dy}{dv}\right)^2\right] + \frac{1}{2}ig\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right)}. \tag{15}$$

Writing this distribution in the language of white noise analysis yields

$$\boldsymbol{P(A,N) = \frac{C_1}{8\pi^3}\int_{-\infty,g}^{+\infty}\int_{-\infty,p_x,p_y}^{+\infty}\int_{\mathrm{r}_0}^{\mathrm{r}_1}\mathbf{exp}\left[-\frac{1}{2}\int_0^N\left(\omega_x{}^2+\omega_y{}^2\right)dv - \frac{igl^2}{2}\int_0^N\left(B_xdB_y - B_ydB_x\right)dv\right]}$$

$$\boldsymbol{\mathrm{x}\,\mathbf{exp}\left\{\int_0^N\left[\frac{-igl}{2}\left(x_0\omega_y - y_0\omega_x\right) + \left(ip_xl\omega_x + ip_yl\omega_y\right)\right]dv\right\}\mathbf{exp}(igA)\,d\mu(\omega)dp_xdp_{x,}dg}, \tag{16}$$

where C_1 is the collective normalization constant, the variablesp_x and p_y came from the Fourier integral representations of the Donsker delta functions in equations (11) and (12), and$\int_0^N \frac{1}{2}\left(B_xdB_y - B_ydB_x\right)dv$ corresponds to the Levy's stochastic areawhich is equal to A[9].

Notice that the probability distribution in equation (16) is in two dimensional Brownian motion. This can be realized in the probability space of one-dimensional white noise as presented in [9]resulting to

$$P_0(A,N) = \frac{C_1}{8\pi^3} \int_{-\infty,g}^{+\infty} \int_{-\infty,p_x,p_y}^{+\infty} \int_{r_0}^{r_1} \exp\left[-\frac{1}{2} < \omega,\omega > -igl^2 < \omega, F_s\omega >\right]$$

$$\text{x } \exp\left[\left(i < \omega, \left(p_y l + \frac{1}{2} g l x_0\right) X_{[-N,0]} + \left(p_x l + \frac{1}{2} g l y_0\right) X_{[0,N]}. >\right)\right] d\mu(\omega) dp_x dp_x \exp(igA) dg. \quad (17)$$

The term $< \omega, F_S\omega >$ which can be expanded as $\int_{R^2} \omega(v) F_S(v,v')\omega(v)'\, dv dv'$ corresponds to the Levy's stochastic area [9].

We note that equation (17) can be written as

$$P(A,N) = \frac{C_1}{8\pi^3} \int_{-\infty,g}^{+\infty} \int_{-\infty,p_y}^{+\infty} \int_{-\infty,p_x}^{+\infty} \quad T(\Phi) dp_x dp_x \exp(igA) dg\,, \quad (18)$$

where $T(\Phi)$ is the T-transform of the functional $\Phi(\omega)$:

$$T(\Phi) = \int \exp\,(i < \omega, \xi >) \Phi(\omega)\, d\mu(\omega) \quad (19)$$

with

$$\Phi(\omega) = \exp\left[-\frac{1}{2} < \omega,\omega > -igl^2 < \omega, F_s\omega >\right] \quad (20)$$

and

$$\xi = \left(p_y l + \frac{1}{2} g l x_0\right) X_{[-N,0]} + \left(p_x l + \frac{1}{2} g l y_0\right) X_{[0,N]}. \quad (21)$$

Calculations of the T-transform of Φ and evaluation of the integrals in equation (18) yields

$$P_0(A,N) = \left[2Nl^2 \cosh^2\left(\frac{2\pi A}{Nl^2}\right)\right]^{-1}. \quad (22)$$

This agrees with the result obtained by Wiegel and Khandekar [7] where they used straightforward integration over $d[r(v)]$.

3.2. Polymer in a gel subjected to a uniform electric field

In this system, we consider the uniform electric field to be along x and y axes. The contribution of the electric field on the probability distribution of the area enclosed by the polymer loop is added on the '*kinetic*' part making the expression for the probability distribution as

$$P_E(A,N) = C \int_{r_0}^{r_1} \delta\left[A - \int_0^N \frac{1}{2}\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right) dv\right] \exp\left[-\frac{1}{l^2}\int_0^N \left(\frac{dr}{dv}\right)^2 dv\ + q\int_0^N \mathbf{E}\cdot\mathbf{r} dv\right] d[r(v)], \quad (21)$$

where $\mathbf{E} = \mathrm{E}_1\hat{\imath} + \mathrm{E}_2\hat{\jmath}$, $\mathbf{r} = x\hat{\imath} + y\hat{\jmath}$ and q is $\frac{q'}{k_B T}$ with q' as the charge of each monomer and T as the absolute temperature.

Writing the Dirac delta function in its Fourier integral form, the probability distribution becomes

$$P_E(A,N) = \frac{C}{2\pi}\int_{-\infty}^{+\infty} \widetilde{P}_E(g,N) \exp(igA)\, dg, \quad (22)$$

where

$$\widetilde{P}_E(g,N) = \Gamma_E \int_{r_0}^{r_1} \exp\left\{-\int_0^N \frac{1}{l^2}\left[\left(\frac{dx'}{dv}\right)^2 + \left(\frac{dy'}{dv}\right)^2\right] dv + \int_0^N \frac{1}{2}\lambda\left(x'\frac{dy'}{dv} - y'\frac{dx'}{dv}\right) dv\right\} d[r(v)] \quad (23)$$

and

$$\Gamma_E = \exp\left[-\frac{2}{l^2}\alpha(x'_N - x'_0) - \frac{2}{l^2}\beta(y'_N - y'_0) - \frac{N}{l^2}(\alpha^2 + \beta^2) - \frac{qN^2}{2}(E_1\alpha + E_2\beta)\right]. \quad (24)$$

Using the preliminary steps, presented in equations (8-12), we have the probability distribution in white noise analysis as

$$\boldsymbol{P_E(A,N)=\frac{C_1}{8\pi^3}\int\limits_{-\infty,g}^{+\infty}\int\limits_{-\infty,p_x,p_y}^{+\infty}\int\limits_{r_0}^{r_1}\exp\left[ip'_y(y'_0-y'_1)+ip'_x(x'_0-x'_1)+igA\right]}$$

$$\text{x}\exp\left[-\frac{2}{l^2}\alpha(x'_N-x'_0)-\frac{2}{l^2}\beta(y'_N-y'_0)-\frac{N}{l^2}(\alpha^2+\beta^2)-\frac{qN^2}{2}(E_1\alpha+E_2\beta)\right]$$
$$\text{x}\exp\left\{\int_0^N\left[-\frac{1}{2}\left({\omega_{x'}}^2+{\omega_{y'}}^2\right)+\frac{\lambda l}{2}\left(x'_0\omega_{y'}-y'_0\omega_{x'}\right)+ip_{x'}l\omega_{x'}+ip_{y'}l\omega_{y'}\right]dv\right\}$$
$$\textbf{x}\ \boldsymbol{\exp\left\{\int_0^N\left[\frac{\lambda l^2}{2}\left(B_{x'}dB_{y'}-B_{y'}dB_{x'}\right)\right]dv\right\}d\mu(\omega_{x'})d\mu(\omega_{y'})dp_{x'}dp_{y'}dg}, \tag{25}$$

where C_1 is the collective normalization constant, the variables p_x and p_y came from the Fourier integral representations of the Donsker delta functions in equations (11) and (12), and $\int_0^N\frac{1}{2}\left(B_{x'}dB_{y'}-B_{y'}dB_{x'}\right)dv$ corresponds to the Levy's stochastic area[9].

We again note that the probability distribution in equation (25) is in two dimensional Brownian motion. This can be realized in the probability space of one-dimensional white noise as inequation (17) resulting to

$$\boldsymbol{P_E(A,N)=\frac{C_1}{8\pi^3}\int\limits_{-\infty,g}^{+\infty}\int\limits_{-\infty,p_x,p_y}^{+\infty}\int\limits_{r_0}^{r_1}\exp\left[-\frac{1}{2}<\omega,\omega>+\lambda l^2<\omega,F_s\omega>\right]}$$

$$\text{x}\exp\left[-\frac{2}{l^2}\alpha(x'_N-x'_0)-\frac{2}{l^2}\beta(y'_N-y'_0)-\frac{N}{l^2}(\alpha^2+\beta^2)-\frac{qN^2}{2}(\mathrm{E}_1\alpha+\mathrm{E}_2\beta)\right]$$
$$\text{x}\exp\left\{(i<\omega,\left(p_{y'}l+\frac{1}{2}i\lambda lx'_0\right)X_{[-N,0]}+\left(p_{x'}l+\frac{1}{2}i\lambda ly'_0\right)X_{[0,N]}>)\right\}$$
$$\textbf{x}\ \boldsymbol{\exp\{+ip'_x(x'_0-x'_1)+ig\mathrm{A}\}d\mu(\omega)dp_{x'}dp_{y'}dg}. \tag{26}$$

Furthermore, equation (26) can be rewritten, as in equation (17), as

$$\boldsymbol{P_E(A,N)=\frac{C_1}{8\pi^3}\int_{-\infty,g}^{+\infty}\int_{-\infty,p_x,p_y}^{+\infty}T(\Phi)dp_{x'}dp_{y'}\exp(igA)dg} \tag{27}$$

with$T(\Phi)$ given in equation (19) but with functional

$$\boldsymbol{\Phi(\omega)=N_0\exp\left[-\frac{1}{2}<\omega,\omega>\ +\lambda l^2<\omega,F_s\omega>\right]} \tag{28}$$

and

$$\boldsymbol{\xi=\left(p_{y'}l+\frac{1}{2}i\lambda lx'_0\right)X_{[-N,0]}+\left(p_{x'}l+\frac{1}{2}i\lambda ly'_0\right)X_{[0,N]}}. \tag{29}$$

Performing the *T-transform* of Φ and integrating over$dp_{x'}$ anddp_yin equation (27) yields

$$\boldsymbol{P(A,N)=\frac{C_1\lambda}{32\pi^4\sin\left(\frac{\lambda l^2N}{4}\right)}\int\limits_{-\infty,g}^{+\infty}\exp\left[-\frac{\lambda}{2}\alpha\left(x'_0y'_1-y'_0x'_1\right)+-\frac{N}{l^2}(\alpha^2+\beta^2)-\frac{qN^2}{2}(E_1\alpha+E_2\beta)+igA\right]}$$

$$\textbf{x}\ \boldsymbol{\exp\left\{-\frac{\lambda}{4\tan\left(\frac{\lambda l^2N}{4}\right)}\left[(x'_0-x'_1)^2+\ (y'_0-y'_1)^2\right]-\frac{2\alpha}{l^2}(x'_N-x'_0)-\frac{2\beta}{l^2}(y'_N-y'_0)\right\}dg}. \tag{30}$$

Rewriting this back to *x* and *y* variables using the relations [8]

$$\boldsymbol{x = x' + \alpha v} \text{ and} \tag{31}$$

$$\boldsymbol{y = y' + \beta v} \tag{32}$$

with$\alpha = \frac{qE_2}{\lambda}$ and $\beta = -\frac{qE_1}{\lambda}$results to

$$\boldsymbol{P(A,N) = \frac{C_1\lambda}{32\pi^4 sin\left(\frac{\lambda l^2 N}{4}\right)} \int_{-\infty,g}^{+\infty}} \textbf{exp}\left[\boldsymbol{\frac{qN}{2}\left(E_1x_0 + E_2y_0\right) + \frac{N}{l^2}(\alpha^2+\beta^2)}\right]$$

$$\textbf{x exp}\left\{\boldsymbol{-\frac{\lambda N^2}{4\,tan\left(\frac{\lambda l^2 N}{4}\right)}(\alpha^2+\beta^2) + iga}\right\}\boldsymbol{dg}, \tag{33}$$

where $\lambda = -ig$.

From equation (33), we can consider two limiting cases. For weak electric field, the probability density is given by the expression

$$\boldsymbol{P_{weakE}(A,N) \cong P_0(A,N)\left[1 + \frac{4\pi aE^2}{N^2 l^4} - 3NlP_0(A,N)\right]}, \tag{34}$$

where$a = \frac{\pi(\mathrm{N}l^2)^4 \mathrm{Q}^2}{23040 \mathrm{k_B^2 T^2}}$ and $P_0(A,N)$ represents the distribution for the area enclosed by the polymer loop in the absence of electric field given in equation (22).

For strong electric field, the probability density is

$$\boldsymbol{P_{strongE}(A,N) \cong \frac{24k_BT\left(\frac{5}{\pi}\right)^{\frac{1}{2}}}{(\mathrm{N}l^2)^{\frac{3}{2}}QE}} \textbf{exp}\left[\boldsymbol{-\frac{bA^2}{E^2}}\right], \tag{35}$$

where $b = \frac{2880\mathrm{k_B^2T^2}}{\mathrm{Q}^2N^2l^4}$.

Equations (34) and (35)agree with the works of Khandekar and Wiegel in reference [8].

3.3. Polymer loop in a gel subjected to crossed electric-magnetic fields

In this system, the polymer loop which encloses a constant area is subjected to crossed electric-magnetic fields. For this case, the probability distribution can be written as

$$\boldsymbol{P_{EB}(A,N) = C\int_{r_0}^{r_1} \delta\left[A - \int_0^N \frac{1}{2}\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right)dv\right]} \textbf{exp}\left[\boldsymbol{-\frac{1}{l^2}\int_0^N \left(\frac{dr}{dv}\right)^2 dv}\right]$$

$$\textbf{x exp}\left[\boldsymbol{+q\int_0^N (\mathbf{E}\cdot\mathbf{r} + \mathbf{B}\cdot\dot{\mathbf{r}})dv}\right]\boldsymbol{d[r(v)]}, \tag{36}$$

where$q$is$\frac{q'}{k_BT}$withq' as the charge of each monomer, $\mathbf{E} = \mathrm{E}_1\hat{\imath} + \mathrm{E}_2\hat{\jmath}$ and $\mathbf{B} = \frac{\mathrm{q}}{2}(H\,\mathrm{x}\dot{\mathbf{r}})$ with H as the uniform magnetic field along the z axis.

Following the same procedure prescribed in the previous subsections, equation (36) can be written as

$$\boldsymbol{P_{EB}(A,N) = \frac{C}{2\pi}\int_{-\infty}^{+\infty} \widetilde{P}_{EB}(g,N)}\textbf{exp}\boldsymbol{(igA)\,dg} \tag{37}$$

where

$$\boldsymbol{\widetilde{P}_{EB}(g,N) = \int_{r_0}^{r_1}} \textbf{exp}\left(\boldsymbol{-\int_0^N \tilde{L}dv}\right)\boldsymbol{d[r(v)]} \tag{38}$$

and

$$\tilde{L}_{EB} = \frac{1}{l^2}\left[\left(\frac{dx}{dv}\right)^2 + \left(\frac{dy}{dv}\right)^2\right] + \frac{1}{2}\lambda\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right)$$

$$-q(E_1x - E_2y) + \frac{qH}{2}\lambda\left(x\frac{dy}{dv} - y\frac{dx}{dv}\right); \lambda = -ig. \quad (39)$$

This expression can be simplified by using the relations

$$x = x' + \alpha v \text{ and} \quad (40)$$

$$y = y' + \beta v, \quad (41)$$

where$\alpha = \frac{qE_2}{\lambda}$ and $\beta = -\frac{qE_1}{\lambda}$.

The probability distribution in terms of these new variables is then

$$P_{EB}(A,N) = \frac{C}{2\pi}\int_{-\infty}^{+\infty}\Gamma_{EB}\int_{r_0}^{r_1}\exp\left\{\int_0^N\left[-\frac{1}{l^2}\left(\frac{dx'}{dv}\right)^2 - \frac{1}{l^2}\left(\frac{dy'}{dv}\right)^2\right]dv\right\}$$

$$\exp\left\{\int_0^N\left[\frac{\lambda}{2}\left(x'\frac{dy'}{dv} - y'\frac{dx'}{dv}\right)\right]dv\right\}d[r(v)]dg, \quad (42)$$

where

$$\Gamma_{EB} = \exp\left[-\frac{2}{l^2}\alpha(x'_N - x'_0) - \frac{2}{l^2}\beta(y'_N - y'_0) + igA\right]$$

$$\text{x}\exp\left[-\frac{N}{l^2}(\alpha^2+\beta^2) - \frac{qN^2}{2}(E_1\alpha + E_2\beta) - \frac{igN}{2}(\alpha y'_N - \beta x'_N) + \frac{qHN}{2}(\alpha y'_N - \beta x'_N)\right]. \quad (43)$$

Equation (42) can be expressed in terms of white noise variables, that is, following the procedure prescribed in the previous sections. So we have

$$P_{EB}(A,N) = \frac{C_1}{8\pi^3}\int_{-\infty,g}^{+\infty}\int_{-\infty,p_xp_y}^{+\infty}\int_{r_0}^{r_1}\Gamma_{EB}\exp\left[\int_0^N\left[({\omega_{x'}}^2 + {\omega_{y'}}^2) + ip_{x'}l\omega_{x'} + ip_{y'}l\omega_{y'}\right]dv\right]$$

$$\text{x}\exp\left\{\int_0^N\left[\frac{(-ig+qH)l}{2}(x'_0\omega_{y'} - y'_0\omega_{x'}) + ip_{x'}l\omega_{x'} + ip_{y'}l\omega_{y'}\right]dv\right\}dp_{x'}dp_{y'}$$

$$\text{x}\ \exp\left\{(-ig+qH)l^2\int_0^N\left[\frac{1}{2}(B_{x'}dB_{y'} - B_{y'}dB_{x'})\right]dv\right\}d\mu(\omega_{x'})d\mu(\omega_{y'})dg, \quad (44)$$

whereC_1 is the collective normalization constant.

In the probability space of one-dimensional white noise as presented in reference [9], the probability distribution may be written as

$$P_E(A,N) = \frac{C_1}{8\pi^3}\int_{-\infty,g}^{+\infty}\int_{-\infty,p_xp_y}^{+\infty}\int_{r_0}^{r_1}\Gamma_{EB}\exp\left[-\frac{1}{2} < \omega,\omega > +(\lambda + qH)l^2 < \omega, F_s\omega >\right]$$

$$\text{x}\ exp\left\{i\left\langle\omega, \left(p_{y'}l + \frac{1}{2}i\lambda l x'_0\right)X_{[-N,0]} + \left(p_{x'}l + \frac{1}{2}i\lambda l y'_0\right)X_{[0,N]}\right\rangle\right\}d\mu(\omega)dp_{x'}dp_{y'}dg. \quad (45)$$

Integration over $d\mu(\omega)$ can then be evaluated using the *T-transform* of Φ and we can write equation (45) as

$$P_E(A,N) = \frac{C_1}{8\pi^3}\int_{-\infty,g}^{+\infty}\int_{-\infty,p_x,p_y}^{+\infty}T(\Phi)dp_{x'}dp_{y'}\exp(igA)dg\ , \quad (46)$$

where$T(\Phi)$is given by equation (19). For this case, the white noise functional Φhas the form

$$\boldsymbol{\Phi(\omega) = N_0 \exp\left[-\frac{1}{2} < \omega, \omega > +(\lambda + qH)l^2 < \omega, F_s\omega >\right]} \tag{47}$$

with

$$\boldsymbol{\xi = \left(p_{y'}l + \frac{1}{2}i\lambda l x'_0\right) X_{[-N,0]} + \left(p_{x'}l + \frac{1}{2}i\lambda l y'_0\right) X_{[0,N]}}. \tag{48}$$

Calculations of the *T-transform* of Φresults to

$$\boldsymbol{T(\Phi) = \left[\cos\frac{(\lambda + qH)l^2N}{4}\right]^{-1} \exp\left\{\frac{-1}{(\lambda + qH)l^2}\tan\left(\frac{(\lambda + qH)l^2N}{4}\right)\left[p_{y'}^2 l^2 - i(\lambda + qH)l^2 {x'_0}^2 p_{y'}\right]\right\}}$$

$$\boldsymbol{\mathrm{x}\exp\left\{\frac{-1}{(\lambda+qH)l^2} tan\left(\frac{(\lambda+qH)l^2N}{4}\right)\left[p_{x'}^2 l^2 - i(\lambda + qH)l^2 {y'_0}^2 p_{x'}\right] - \frac{(\lambda+qH)^2 l^2({x'_0}^2+{y'_0}^2)}{4}\right\}}. \tag{49}$$

The integrations over $dp_{x'}$ and $dp_{y'}$in equation (46) are done using one-dimensional Gaussian integral. Rewriting the probability distribution, equation (52), back to *x* and *y* variables and identifying the normalization constant [8] results to

$$\boldsymbol{P_{EB}(A,N) = K(E,H)\int_{-\infty,g}^{+\infty} \frac{u}{sinhu}\exp\left[\frac{4iuA}{Nl^2} - \frac{N^3l^2q^2E^2}{72}u^2\right]du}, \tag{50}$$

where $u = Nl^2(g + qHi)$ and

$$\boldsymbol{K(E,H) = \frac{8\sin\left(\frac{Nl^2qH}{4}\right)}{N^2l^4qH\pi}\exp\left[\frac{N^3l^2q^2E^2}{48} + qHA + \frac{E^2N}{l^2}\left(-\frac{1}{H^2} + \frac{Nl^2q}{4H\tan\left(\frac{Nl^2qH}{4}\right)}\right)\right]}. \tag{51}$$

Similar in section 3.2, the structure of equation (51) is complicated and so limiting cases are considered [8]. We further note that the integrand in equation (51) only affects electric field which means limiting cases are for electric field only.

In the presence of constant magnetic field and weak electric field, the probability density is given by the expression

$$\boldsymbol{P_{E_wB}(A,N) \cong K(E,H)\{p_0(\xi) - \beta[-\pi^2 p_0(\xi)] + 3p_0^2(\xi)\}}, \tag{52}$$

where$\beta = \frac{\mathrm{Nl^2Q^2E^2}}{720\mathrm{k_B^2T^2}}$, $\xi = \frac{4\mathrm{A}}{\mathrm{Nl^2}}$ and $\mathrm{p_0}(\xi) = \frac{\pi^2}{2\cosh\left(\frac{\pi\xi}{2}\right)}$. And the probability distribution in the presenceof magnetic field and strong electric field is given by

$$\boldsymbol{P_{E_sB}(A,N) \cong K(E,H)\left(\frac{k_BT}{lQE}\sqrt{\frac{720\pi}{N}}\right)\exp\left[-\frac{4(720)\mathrm{k_B^2T^2}}{N^3\mathrm{l^6Q^2}}\left(\frac{A^2}{E^2}\right)\right]}. \tag{53}$$

For both limiting cases, results agree with reference [8] when magnetic field is switched off.

4. Conclusion

In this paper, we have obtained the expression for the probability distribution of the area enclosed by a polymer loop under different external fields: constant electric field and crossed electric-magnetic fields. The method used in the evaluation of the probability distribution is white noise analysis where the polymer loop is represented by various paths of a Brownian motion.

5. References

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