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A phenomenological model for the viscoelastic behaviour of natural rubber

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Abstract. The present work is concerned with the study of the viscoelastic behaviour of natural rubber in relaxation tests. A phenomenological model is presented to predict the exact viscoelastic behaviour of natural rubber in a finite strain regime to enhance the experimental fit accuracy. A rate dependent multiplicative deformation gradient tensor is splitted into elastic and viscous parts during deformation. Viscoelastic behaviour of natural rubber is introduced through Jeffrey rheological model with Horgan and Saccomandi type energy function. A thermodynamically consistent rate-dependent relaxation time scheme is introduced which results reduced material parameters, complexity level and the computational time. The developed constitutive model is validated with stress relaxation test data that depicts the rate-dependent viscoelastic behaviour of natural rubber.

1. Introduction

Natural rubber is an elastomer which consists of polymers with minor impurities of other organic compounds. Polymeric materials have wide industrial applications and broad range of desirable properties. According to the existing research [1], these materials have three-dimensional network configuration consisting of randomly oriented long molecular chains which are cross-linked, and tangled among themselves with neighbors. The rate dependent viscoelastic behaviour of these materials is investigated by Mullins and Tobin [2] in their experimental work. This observation provides the basis to model the constitutive relation of natural rubber for its nonlinear rate dependent behaviour. Mathematical modeling of rubber-like materials in finite viscoelasticity may be formulated using two different approaches commonly [3]. First, the hereditary integral approach proposed by Simo [4] which utilizes memory functions to investigate the history integral behaviour on the current stress state. The second approach is based on the multiplicative deformation gradient tensor. This framework was initially introduced by Green and Tobolsky [5] and further analyzed by several researchers using either micromechanically motivated models proposed by Bergstrom and Boyce [6] or continuum based approach proposed by Reese and Govindjee [7].

Numerous research have been performed on the viscoelastic behaviour of polymeric materials. The generalized nonlinear viscoelastic model in a compact differential form is proposed by Chung et al. [8]. Subsequently, a multiplicative deformation gradient tensor is used by Amin et al. [9] to obtain the constitutive model of finite strain viscoelasticity. Further Clausius-Duhem inequality is introduced to developed the thermodynamically consistent relationship. The viscoelastic properties are time dependent and become sensitive due to the variation in temperature also.



Knauss et al. [10] coined the term sensitivity associated with small volume change and a constitutive relationship is obtained for nonlinear viscoelastic materials. A phenomenological one-dimensional constitutive relationship is proposed by Abu-Abdeen [11] that characterizes the mechanical behaviour of filled natural rubber. The rate dependent behaviour of filled rubber under cyclic loading condition is analytically presented by Laiarinandrasana et al. [12]. Huber and Tsakmakis [13] proved that the strain field in every linear viscoelastic solid may be decomposed into two different strain fields. The decomposition is associated with two well-known spring dashpot model like Poynting Thomson and Zener model. An experimental work on high damping rubber is performed by Amin et al. [14] and found that there is a fast rate of decaying of stress during relaxation process in compression regime. The viscoelastic property of a filled rubber is analysed experimentally at large deformation under uniaxial cyclic loading condition by Diani et al. [15]. Lejeunes et al. [16] introduced the numerical integration technique for finite viscoelastic model to analyse the material behaviour. The reliability of this technique is investigated with three different rheological models. Holzapfel [17] used Maxwell model to capture the strain induced anisotropy and experimental validation had been performed to observe time-dependent phenomenon of viscoelastic materials. Bergstrom and Boyce [18] proposed a constitutive model to analyse the behaviour of elastomeric material at large strain rate which is the modification of previously reported model [19] by same authors that may capture the experimentally observed material behaviour under various loading conditions.

Numerous authors have proposed various form of viscoelastic model for polymeric materials, but most of them lack of simplicity in their formulation and also fit in accuracy with experimental data. The research area of viscoelasticity is broad and still open for researchers across different fields for the accurate prediction of this time-dependent mechanical behaviour of viscoelastic material. Following the modeling techniques of previous literatures for polymeric materials, a new simplified mathematical model is developed to study the viscoelastic behaviour of natural rubber in relaxation. The main goal is to predict the stress-time and stress-stretch curves using model equations that combine enough mathematical simplicity and may allow its use in describing the complex nonlinear behaviour of natural rubber in different engineering problems. The material parameters appear in the evolution equation may be easily identified using the least squares regression method. The main contribution of this work is to present a simple but reliable alternative parallel approach to predict the viscoelastic behaviour of the natural rubber.

2. Experimental results

2.1. Aim of the experiment

The aim of the experiment is to understand the practical behaviour of the natural rubber in multistep relaxation test. The practical information of the viscoelastic behaviour of natural rubber is used to understand the physical mechanism of the chain network clearly. Then accordingly, a mathematical model is developed to study the rate dependent behaviour of the natural rubber during deformation.

2.2. Material

A natural rubber material was used for the stress relaxation test. A few dumbell specimens of Natural rubber were made according to ASTM specification D6746-15 and the experiment is conducted at room temperature. The standard gage length of the test specimen was 26 mm as shown in Figure 1. Tinius Olsen H5KS universal testing machine with 250 N load cell capacity was used to test the viscoelastic property of the natural rubber.

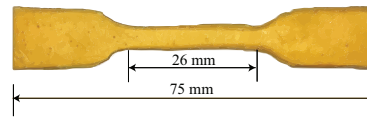


Figure 1: The standard test specimen of the natural rubber

2.3. Relaxation test procedure

Multistep cyclic relaxation tests were designed to investigate the time dependent behaviour of natural rubber with 50 s of relaxation time at each step. The stretch steps were taken as 150%, 200%, and 250%. The specimen was first stretched up to 150% at strain rate 0.05 s^{-1} and hold there for 50 s. Then it was again stretched up to 200%, and was hold there for predefined period and again stretched up to the final value of 250%. This completes half cycle of loading. The whole process for unloading cycle repeated immediately at the predefined strain rate and relaxation time. The same experiment was performed for same stretch value at different strain rates like 0.07 s^{-1} and 0.10 s^{-1} also. The experimental data is reported in the Figure 2.

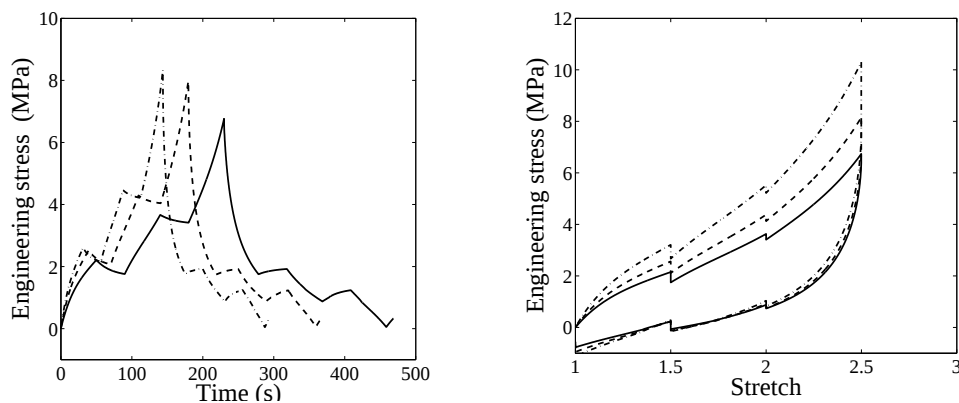


Figure 2: Multi step stress relaxation plot (a) Stress Time and (b) Stress Stretch at strain rate 0.05 s^{-1} (solid curve), 0.07 s^{-1} (dash curve) and 0.10 s^{-1} (dash-dot curve) at stretch value = 1.5, 2.0, 2.5 with relaxation time 50 s.

2.4. Experimental observations

Following observations are concluded from the experiment

- During loading the amount of stress relaxation is observed to be more as compare to unloading cycle. From this observation it may be concluded that at loading the chain alignment allows the flow of chain inside the network due to decrease in entropy level. This results decrease in stress level at loading relaxation. While at unloading there is increase in entropy level and it will hinder the flow of chain inside the network. This results increase in stress level at unloading relaxation.
- During deformation intermolecular slippage occurs in polymeric network, results reversible breaking or swapping of different chain bonds. The require timespan to complete this phenomenon is called the relaxation time.
- Since there is a range of chain lengths in polymeric network that would be continuous spectrum of relaxation time. The rearrangement of kinks and convolution entirely depends on the rate of deformation. Hence strain rate is a dominant factor for stress relaxation phenomenon.

3. Physics of the chain network deformation

Following the performed experimental observations in the preceding section, the physics of rubber elasticity related to viscoelastic behaviour of natural rubber is understood through the mechanism of chain network deformation from microscopic point of view. Figure 3 represents the schematic diagram of chain assimilation during deformation of the natural rubber. In the network of chains within the natural rubber, the free chain also participates with dangling chain and entangled chain during deformation process. The rearrangement of kinks and convolution of polymeric chain is restricted with relaxation time which depends on the rate of deformation process.

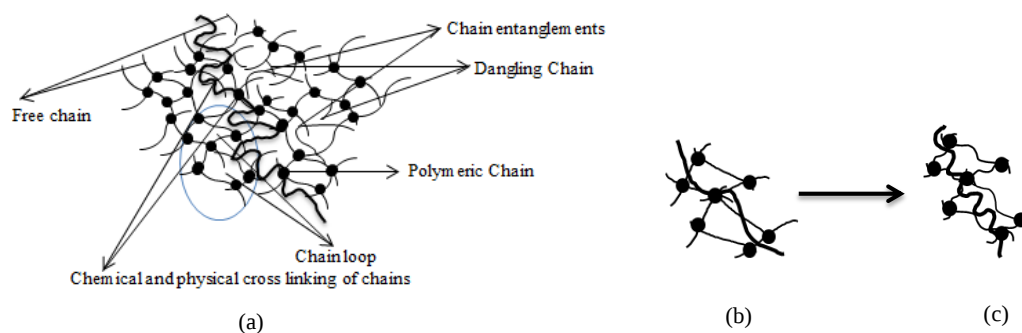


Figure 3: Schematic diagram showing chain assimilation with deformation (a) Reference network (b) Deformed chain network (c) Relaxed deformed network

From microscopic point of view, intermolecular slippage occurs during deformation in the polymeric network due to reversible breaking and swapping of intermolecular bonds. The polymeric chains get aligned due to applied load which results decrease in entropy level. It allows the flow of different chains associated in polymeric network. The state up to the maximum stretch level is reached without breaking of any chain and the reversibility condition prevails; the material will regain its original configuration upon unloading. Beyond that state, polymeric chain starts to break and this results increase in entropy level. It provides hindrance to the flow of the chain inside the polymeric network results the different values of viscous coefficient. The orientation of different chains in deformed and relaxed configuration of polymeric network may also be seen in the Figure 3. However, the deformation is resisted by the free chain in polymeric network as they gain their peak stretch value during deformation process and the free chains may not attain the same value of stretch level at all.

According to the affine network analogy, the entangled points of molecular chains are assumed to be connecting points and the number of chains per unit volume and the average number of kinks in a single chains are unchanged during deformation. But in real deformation process, the number of entangled points may increase or decrease according to the local deformation of the polymeric chain by Botto et. al. [20]. Increase in the entangled points reduces the number of kinks in polymeric chains results increase in the relative stiffening of the materials. On the other hand, decrease in the entangled points results the opposite effect.

It ensures that there must be inhomogeneity associated during deformation which affect the viscoelastic properties of polymeric materials. The deformed chain network and the relaxed deformed network shown in Figures 3(b) and 3(c) may identify such effect. In order to consider the inhomogeneity during deformation we may consider the Jeffrey rheological model to analyse the viscoelastic behaviour of polymeric materials under deformation. Figure 3(b) shows the

deformed state below the maximum possible deformation and this allows to relax the whole network. The elastic response of the material does not depend on broken chain which relaxes together with the free chain. The reptile motion of free chains occurs through space provided by broken chains and results the rate dependent deformation process. The schematic representation shown in the Figure 3 assures that chain density increases in relaxed deformed network as compared to the reference network in loading path. Therefore, the time require to regain its initial configuration in reference path is more compared to the relaxed path. Since there is a range of chain length in the polymeric network there would be prolonged series of relaxation time. This relaxation time depends on the rate of deformation because of readjustment of kinks and convolution is restricted with respect to chain stiffening property.

4. Material modeling

4.1. Deformation mapping

In Figure 4 the reference configuration of the material is Ω_0 at time $t = 0$ and due to finite rate of deformation the material undergoes the current configuration Ω_t at time $t = t$. If the material is allowed to relax in the current configuration then it will never regain its original configuration due to the irreversibility associated within the material. Therefore, a hypothetical intermediate configuration must exist which lies between initial and current configuration termed as the relaxed configuration Ω_r . This relaxed state may be gained by direct unloading of the material. Figure 4 also shows the compound mapping of reference configuration to current configuration

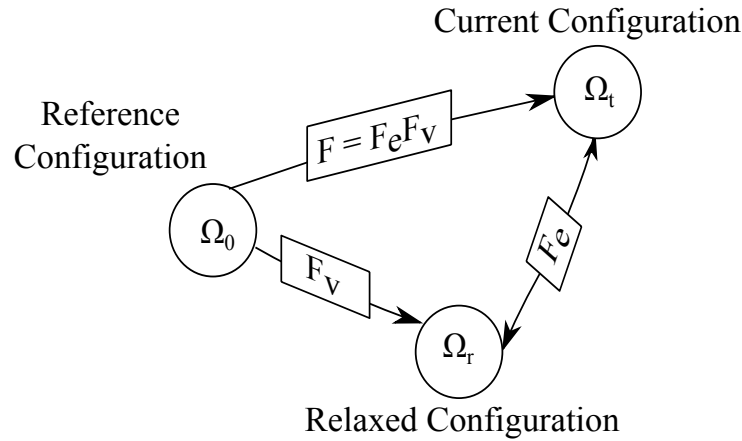


Figure 4: Schematic diagram of multiplicative deformation gradient tensor

through the intermediate relaxed configuration by splitting the physical deformation gradient $\mathbf{F}$ in elastic and viscous part as $\mathbf{F}_e$ and $\mathbf{F}_v$, respectively. The deformation gradient $\mathbf{F}$ is invertible and its components $\mathbf{F}_e$ and $\mathbf{F}_v$ are also invertible and related as $\mathbf{F} = \mathbf{F}_e \mathbf{F}_v$. The incompressibility constraint in the deformation suggests $\det \mathbf{F} = 1$, $\text{tr} \mathbf{L} = \text{tr} \mathbf{D} = 0$ and the decomposed deformation gradient tensor consists additional incompressibility constraints as

$$\det \mathbf{F}_e = \det \mathbf{F}_v = 1, \quad (1)$$

Where $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$ is the rate of deformation gradient tensor and its symmetric part represents the stretch rate tensor $\mathbf{D}$. The intermediate configuration Ω_r may suffer due to rigid body rotation $\mathbf{R}_e$. Therefore, through the polar decomposition theorem the viscous stretch tensor $\mathbf{D}_v^R$ may be written as

$$\mathbf{D}_v^R = \mathbf{R}_e \mathbf{D}_v \mathbf{R}_e^T. \quad (2)$$

4.2. Viscoelastic model

We may recognize that the polymeric material comprise from different types of chain network which are interlinked through each other results viscoelastic effects during its deformation. The schematic representation of Figure 3 may be mathematically modeled in many possible ways. The Jeffrey rheological model is introduced to ensure the the instantaneous stress response due to the proximity to attain high stretch level in faster way with high accuracy. Figure 5 represents the Jeffrey rheological model that comprise two nonlinear dashpot. These dashpots have the ability to consider inhomogeneity associated in material deformation. The inhomogeneity arises due to slipping and breaking of different chains in the polymeric network during its deformation. Since the model consists two different elements in which a nonlinear dashpot element is in series with Kelvin-voigt element. Therefore, the stresses in both the elements are equal while the strains are additive.

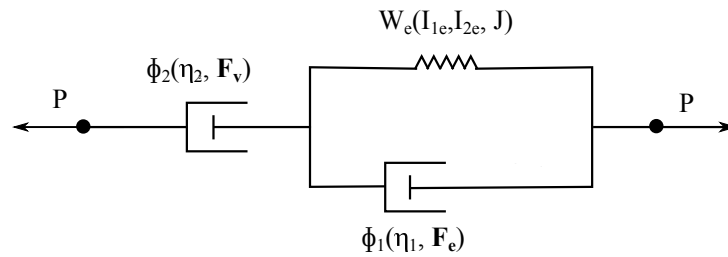


Figure 5: Jeffrey rheological model

In Jeffrey rheological model we may consider that the chain entanglement and the dangling chain both are the separate network from free chain. The free chains undergo relatively small stretch compare to other polymeric chain under loading condition. It ensures that different polymeric chains are interlinked through chemical or physical cross linking in chain network. The first and second invariant of the left Cauchy-Green deformation tensors $\mathbf{B}$ are denoted by I_1 and I_2 . The material parameter J measures the degree of limiting chain extensibility during its deformation. The material parameter J is an alternative limiting chain extensibility parameter which may improve the implication of maximum principal stretch limit due to the singularity at first invariant value in the simplest Gent model [5]. Horgan et. al. [21] modified the previous singularity $\max[\lambda_1, \lambda_2, \lambda_3] < \lambda_*$ with an alternative concept as $\max[f(\lambda_1), f(\lambda_2), f(\lambda_3)] < J$. Where λ_* is a constant depends on the deformation being considered and J is an alternative limiting chain extensibility parameter.

4.3. Constitutive relation

The total energy function for Jeffrey rheological model may be represented as

$$W_e = W_e(I_{1e}, I_{2e}, J). \quad (3)$$

This energy function also consists second invariant I_2 which plays a significant role in nonhomogeneous deformation and strain stiffening property at high stretch level [22]. The defined energy function has resemblance with the energy function given by Horgan and Saccomandi [23]. Therefore, in order to derive constitutive relationship for stress field Horgan and Saccomandi type energy function is considered. The viscous stretch rate tensor is decomposed in order to account inhomogeneity effect associated during its deformation properly. The time derivative of energy function from equation (3) may be obtained as

$$\dot{W}_e = 2\mathbf{B}_e \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial \mathbf{B}_e} : \mathbf{D} - 2\mathbf{B}_e \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial \mathbf{B}_v} : \mathbf{D}_v. \quad (4)$$

We begin our formulation using Clausius-Duhem inequality for thermodynamically consistent viscoelastic model. The inhomogeneity is considered due to slipping and breaking of different chain in the polymeric network. In order to account this effect the viscous stretch rate tensor $\mathbf{D}_v$ may be decomposed in $\mathbf{D}_{v1}$ and $\mathbf{D}_{v2}$. Further a rate dependent relaxation time scheme may be incorporated which results reduce in required material parameters and also the complexity level associated in calibration of relaxation time. The thermodynamically consistent constitutive relation for viscoelastic material may be derived through Clausius-Duhem inequality. Introducing the Clausius-Duhem inequality, the mechanical dissipation associated in the deformation process may be expressed as

$$\Gamma - \dot{W}_e \geq 0, \quad (5)$$

where Γ represents stress power density and W_e is energy absorption capacity of material during its deformation. With the assumption that material properties are invariant due to change in temperature [22] during deformation, we may rewrite the following inequality as

$$\mathbf{S} - 2\mathbf{B}_e \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial \mathbf{B}_e} : \mathbf{D} - 2\mathbf{B}_e \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial \mathbf{B}_v} : (\mathbf{D}_{v1} + \mathbf{D}_{v2}) \geq 0. \quad (6)$$

From the first part of inequality (6) the Cauchy stress tensor $\mathbf{S}$ for incompressible, isotropic hyperelastic material may be obtained as

$$\mathbf{S} = -p\mathbf{I} + 2\mathbf{B}_e \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial I_{1e}} + 2\mathbf{B}_e(I_{1e} - \mathbf{B}_e) \frac{\partial W_e(I_{1e}, I_{2e}, J)}{\partial I_{2e}}. \quad (7)$$

The second invariant and an alternative limiting chain extensibility parameter are additionally included in the Horgan and Saccomandi type energy function [23] which makes it more versatile in finite deformation. The expression of the energy function is given as

$$W_e(I_{1e}, I_{2e}, J) = -\frac{\mu}{2} \frac{(J-1)^2}{J} \ln \left(\frac{J^3 - J^2 I_{1e} + J I_{2e} - 1}{(J-1)^3} \right). \quad (8)$$

Introducing this energy function in equation (7), the Cauchy stress tensor in Jeffrey rheological model may be obtained as

$$\mathbf{S} = -p\mathbf{I} + \mu \frac{(1 - \frac{1}{J})^2}{\left(1 - \frac{\lambda_{1e}^2}{J}\right) \left(1 - \frac{\lambda_{2e}^2}{J}\right) \left(1 - \frac{\lambda_{3e}^2}{J}\right)} \left(\mathbf{B}_e + \frac{\mathbf{B}_e^{-1}}{J} \right), \quad (9)$$

where p is the arbitrary hydrostatic pressure associated due to incompressibility constraint and μ is the material constant during deformation.

In order to derive the evolution equation for viscoelastic material, there must be existence of a dissipation pseudo potential function postulated by Boukamel et. al. [24]. This dissipative potential function Φ describes the thermodynamics of irreversibility associated due to growth of entropy rate that may be written as

$$\Phi = \frac{\eta}{2} (\mathbf{D}_v^R : \mathbf{D}_v^R), \quad (10)$$

where η is a viscosity coefficient, hence derivative of potential function is given as

$$\frac{\partial \Phi}{\partial \mathbf{D}_{v1}} = \eta_1 \mathbf{D}_{v1} \quad \text{and} \quad \frac{\partial \Phi}{\partial \mathbf{D}_{v2}} = \eta_2 \mathbf{D}_{v2}. \quad (11)$$

Introducing equation (11) into the second part of inequality (6) leads to balance the rate of dissipative forces as

$$\mathbf{D}_{\mathbf{v}1} = \frac{2}{\eta_1} \left(\mathbf{B}_e \frac{\partial W_e}{\partial \mathbf{B}_v} - q \mathbf{I} \right) \quad (12)$$

similarly;

$$\mathbf{D}_{\mathbf{v}2} = \frac{2}{\eta_2} \left(\mathbf{B}_e \frac{\partial W_e}{\partial \mathbf{B}_v} - q \mathbf{I} \right), \quad (13)$$

where q is the arbitrary hydrostatic pressure associated due to dissipative stretch rate tensor. The generalized evolution equation of viscoelastic material from Jeffrey rheological model may be obtain as

$$\dot{\mathbf{B}}_e = 2\mathbf{B}_e \mathbf{D} - 4\mathbf{B}_e \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) \left[B_e \frac{\partial W_e}{\partial B_e} - (\mathbf{S} - u \mathbf{I}) \right]. \quad (14)$$

The above evolution equation consists of two different viscous coefficients η_1 and η_2 respectively. These coefficients ensure the degree of inhomogeneity associated during deformation as there is slippage and breakage of chain in polymeric network. Where u is indeterminate hydrostatic pressure arising from the incompressibility constraint associated with Cauchy stress.

5. Experimental validation and model comparison

The generalized stress response relationship for the natural rubber is obtained from equation (7) along with the stretch response function from the evolution equation (14). The author's multistep relaxation tests for uniaxial loading are shown in Figure 2. The validation of proposed mathematical model with the experimental test data may be predicted by physical time dependent uniaxial stretch in principal coordinate as $\lambda_1 = \lambda(t)$ and $\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda(t)}}$. The internal state variables λ_e and λ_v denote the elastic and viscous stretch, respectively. They are related as $\lambda(t) = \lambda_e(t)\lambda_v(t)$. These variables satisfy the incompressibility constraint illustrated in equation (1). The engineering stress response for Horgan and Saccomandi type material may be obtained from the equation (7) for uniaxial loading condition as

$$P_{11} = \mu \frac{\left(1 - \frac{1}{J}\right)^2 \left(\frac{\lambda_e^2}{\lambda} - \frac{1}{\lambda \lambda_e}\right)}{\left(1 - \frac{\lambda_e^2}{J}\right) \left(1 - \frac{1}{J \lambda_e}\right)}. \quad (15)$$

Expanding all the elements of equation(14) we may get

$$\dot{B}_{e11} = 2B_{e11} \frac{\dot{\lambda}}{\lambda} - 4 \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) B_{e11} \left[\frac{\frac{\mu}{2} \left(1 - \frac{1}{J}\right)^2}{\left(1 - \frac{\lambda_e^2}{J}\right) \left(1 - \frac{1}{J \lambda_e}\right)^2} B_{e11} - (\mathbf{S}_{11} - u) \right] \quad (16)$$

and

$$\dot{B}_{e22} = -B_{e22} \frac{\dot{\lambda}}{\lambda} - 4 \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) B_{e22} \left[\frac{\frac{\mu}{2} \left(1 - \frac{1}{J}\right)^2}{\left(1 - \frac{\lambda_e^2}{J}\right) \left(1 - \frac{1}{J \lambda_e}\right)^2} B_{e22} - (\mathbf{S}_{22} - u) \right]. \quad (17)$$

Now, applying the incompressibility constraint $B_{e11}B_{e22}B_{e33} = 1$ from the equations (16) and (17) the evolution equation of stretch rate may be obtained as

$$\dot{\lambda}_e = \frac{\dot{\lambda}}{\lambda} \lambda_e - \frac{2\mu}{3} \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) \left[\frac{\left(1 - \frac{1}{J}\right)^2 \left[1 + \frac{1}{J} \left(1 - \frac{2}{\lambda_e}\right)\right]}{\left(1 - \frac{\lambda_e^2}{J}\right) \left(1 - \frac{1}{J \lambda_e}\right)^2} (\lambda_e^3 - 1) \right]. \quad (18)$$

The above equation represents the generalized rate dependent stretch response of the natural rubber. The viscoelastic stretch response is derived to analyse the time dependent behaviour of natural rubber during its deformation under loading condition. Laiarinandrasana et al. [12] used a polynomial type Rivlin energy function for elastic and viscous network separately:

$$\begin{aligned} W_e &= C_{10e}(I_{1e} - 3) + C_{01e}(I_{2e} - 3) + C_{20e}(I_{2e} - 3)^2, \\ W_v &= C_{10v}(I_{1v} - 3) + C_{01v}(I_{2v} - 3), \end{aligned} \quad (19)$$

where C_{10e} , C_{01e} and C_{20e} are positive material parameters for elastic network and C_{10v} , C_{01v} for viscous network.

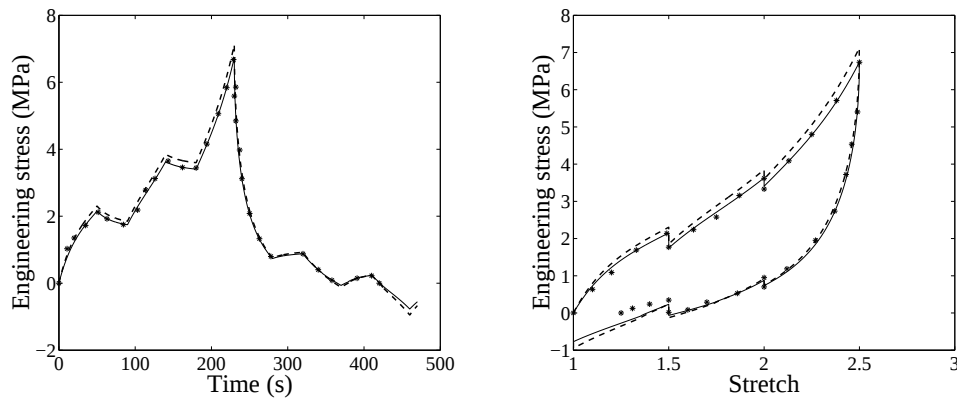


Figure 6: Multistep stress relaxation test data at strain rate 0.05 s^{-1} (star dots) (a) Stress Time and (b) Stress Stretch plot compared with proposed rheological model (solid line) and Laiarinandrasana et. al. model (dotted line)

The proposed mathematical model for the natural rubber is presented by equation (9) together with the evolution equation (14) consists only four material parameters for uniaxial deformation presented in the equations (15) and (18). Experiment was carried out from zero stress level to peak stress for multistep stress relaxation test and the evolution criteria is governed by the relation (14). The state variable λ_e may be obtained through numerical integration of rate dependent evolution equation (19). The plot of multistep relaxation test for strain rate 0.05 s^{-1} is shown in Figure 6 by star dots. The model parameters appear in Laiarinandrasana et al. and proposed mathematical model may be obtained easily using the least squares regression method by fitting the author's multistep relaxation test data is presented in Table 1.

Table 1: Material parameters obtained from the constitutive models

Proposed model	$\eta_1 = 39.23 \text{ MPas}$	$\eta_2 = 43.65 \text{ MPas}$	$\mu = 6.41 \text{ MPa}$
	$J = 21.37$		
Laiarinandrasana et al.	$C_{10e} = 2.81 \text{ MPa}$	$C_{01e} = 0.73 \text{ MPa}$	$C_{20e} = 0.47 \text{ MPa}$
	$C_{10v} = 2.76 \text{ MPa}$	$C_{01v} = 1.03 \text{ MPa}$	$\eta = 59 \text{ MPas}$

The stress-time and stress-stretch plots shows the comparison between the proposed rheological model (solid line) and Laiarinandrasana et. al. model (dotted line) in Figure 6. The comparison of the proposed mathematical model shows a good agreement with the experimental test data and Laiarinandrasana et al. model. An interesting merit of the proposed mathematical model lies in the fact that it may provide the physical parameters values with no complexity.

The experiments have performed at three different strain rates like 0.05 s^{-1} , 0.07 s^{-1} and 0.1 s^{-1} respectively. The comparison of proposed model with experimental data has been done at strain rate 0.05 s^{-1} to identify the time dependent behaviour of viscoelastic materials. The other strain rate may also be used for the model comparison then accordingly value of the material parameters will vary. Moreover, the simple and clear mathematical formulation allows us to understand how it is possible to generalize constitutive model considering a chain stiffening parameter J beyond classical model reported by other researchers which are quite complex.

6. Concluding remarks

A generalized viscoelastic model is presented to predict the viscoelastic behaviour of natural rubber with reduced complexity. The success of the derived evolution equation (14) characterises the viscoelastic effect with reduced material parameters and complexity level. Moreover, the proposed model requires only four material parameters for their evolution with experimental data. The shear modulus μ signifies about rigidity of material depends on the cross linking of chains, chain realignment and chain entanglements. The other parameter η_1 and η_2 are strains rate coefficients measure the degree of flow in the chains to its initial undeformed state and J represents the limiting chain extensibility parameter of material. In contrast, other viscoelastic models require more material parameters for their evolution with experimental data and also demands more computational time. Finally, our simple four parameter evolution equation and the stress response with second invariant I_2 focuses on the physically relevant model development and experimental validation with relaxation test data. The results obtained are in good agreement with experimental data and with Laiarinandrasana et. al. model as well. The material model including damage effect will be presented in our future work.

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