A deterministic aggregate production planning model considering quality of products

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A deterministic aggregate production planning model considering quality of products

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Abstract. Aggregate Production Planning (APP) is a medium-term planning which is concerned with the lowest-cost method of production planning to meet customers’ requirements and to satisfy fluctuating demand over a planning time horizon. APP problem has been studied widely since it was introduced and formulated in 1950s. However, in several conducted studies in the APP area, most of the researchers have concentrated on some common objectives such as minimization of cost, fluctuation in the number of workers, and inventory level. Specifically, maintaining quality at the desirable level as an objective while minimizing cost has not been considered in previous studies. In this study, an attempt has been made to develop a multi-objective mixed integer linear programming model that serves those companies aiming to incur the minimum level of operational cost while maintaining quality at an acceptable level. In order to obtain the solution to the multi-objective model, the Fuzzy Goal Programming approach and max-min operator of Bellman-Zadeh were applied to the model. At the final step, IBM ILOG CPLEX Optimization Studio software was used to obtain the experimental results based on the data collected from an automotive parts manufacturing company. The results show that incorporating quality in the model imposes some costs, however a trade-off should be done between the cost resulting from producing products with higher quality and the cost that the firm may incur due to customer dissatisfaction and sale losses.

1. Introduction
Nowadays, due to huge improvement of operation management methods and emerging new methodologies, companies are faced with intensive competitive circumstances. Under these conditions, companies try to apply appropriate techniques and methods to have a better utilization of their available resources. Facing with expanding competition, companies try to move from disconnected decision making processes towards a more aggregated planning and control of their activities in order to reduce total cost, improve performance, and increase service level [1]. Among all those techniques, the role of aggregate production planning is getting more and more highlighted.

"Aggregate Production Planning" (APP) is a process by which a company determines ideal levels of capacity, production, subcontracting, inventory, stock-out, and even pricing over a specified time horizon [2]. It can also be defined as a medium-term capacity planning often from 3 to 18 months ahead which is concerned with the lowest-cost method of production planning to meet customer’s requirements and to satisfy fluctuating demand over the planning horizon [3].
APP problem has been studied widely, since it was introduced and formulated in 1950. The interest in the problem stems from the ability that such a model provides for effective control of production and inventory costs, which are a substantial portion of the overall cost of the manufacturers [4]. In several conducted studies in the APP area, most of the researchers have concentrated on some common objectives such as minimization of cost, fluctuation in the number of workers, and inventory level. However, due to intensive competitive circumstances, just focusing on some specific objectives can no longer fulfill the requirements of demanding customers. Specifically, to the best of the researchers’ knowledge, maintaining quality at the optimal level as an objective while minimizing cost has not been considered in previous studies. In this study, an attempt has been made to respond to this issue by developing a multi-objective mixed integer linear programming model in which both objectives of minimizing cost and maximizing quality have been considered simultaneously. As the aim of this study is to develop a more practical model, it tries to incorporate both quantitative and qualitative objectives in developing the APP model. In order to test the applicability of the model, it has been applied to an APP problem of an automotive parts manufacturing company in Iran. This paper consists of seven parts. In the next section, a review on some previous studies is presented. The model formulation and the procedure for solving it will be presented in sections 3 and 4 respectively. In Section 5, the model is applied to the company under study. Section 6 presents and discusses the experimental results and finally, the conclusion will be presented in section 7.

2. Literature review

Following the work done by Foote and Ravindran [5], various models with different levels of sophistication have been introduced and many researchers tried to present a classification of them. In 1992 Nam and Ogendar [6] conducted a survey of APP techniques and categorized them into these classes: trial and error method, graphical technique, mathematical technique, linear decision rule, search decision rule, management coefficients method, parametric production planning, production switching heuristic, linear programming, goal programming, mixed integer programming, transportation method and simulation model. Three years later, Pan and Kleiner [7], categorized available models into seven classes including informal approach, mathematical model, linear programming model, linear decision rule [8], [9], heuristic technique, the management coefficients model and search procedure using computer simulation. However, another classification of recently used techniques was given in 2004 by Reay-Chen and Tien-Fu [10]. They categorized conventional models of aggregate planning into six divisions: linear programming model, decision rule method, transportation model, management coefficient approach, search decision rule and simulation method. In the case of dealing with uncertain environments, some approaches including stochastic programming, fuzzy programming, stochastic dynamic programming, and robust optimization have been followed as the main approaches to incorporate uncertainty into the production planning process [11].

In cases where we are sure about the linearity of objective functions, linear programming techniques can be used [12]. Kanyalkar and Adil [13] developed an APP model using linear programming for a multi-plant, multi-selling-location problem. The solution serves both detailed and aggregated problems at the same time and in a single step. This scheme claims to eliminate the disadvantages of monotheistic plans (huge computational demand and imprecise forecast) as well as hierarchical plans (having infeasibility and sub-optimality). The model formulation consists of some factors such as storage space limitation, availability of raw materials and production capacity of each plant as well as inventory buffer level. Solution to the sample problem was obtained using GLPK optimizer software.

In newer study, Sillekensa et al. [14] applied a Mixed Integer Linear Programming (MILP) model in order to respond to certain challenges in the production planning process of the automotive industry. The challenges came from the possible flexibility in workforces and corresponding working times in APP. Other researchers applied MILP for other purposes. Christoua et al. [15] presented a MILP model and a new approach of hierarchical decomposition of production planning to generate an
aggregate production plan that serves the multi-commodity production and distribution scheduling problem. To improve the hierarchical approach of production planning, Stefansson et al. [16] suggested another new approach based on a "hierarchically structured moving horizon" that can address the problem of considering fixed time horizon and the hypothesis of availability of all data at the time of implementation. Jayaraman [17] proposed a linear programming model called Remanufacturing Aggregate Production Planning (RAPP) to consider the return flow of recoverable materials for manufacturers.

Some researchers incorporated the optimization of operational functions while developing APP. For instance, Vassiliadis et al. [18] studied the interaction between production and maintenance plans and established a MILP model to integrate APP with continuous Markov chain maintenance for a multi-purpose batch plant. However, Suryadiy and Papageorgiou [19] expanded their work by incorporating preventive maintenance and some aspects of design into the model. Both studies provided the solution to the problem using the branch and bound method.

In many cases where a decision maker is faced with a multi-objective mathematical model, "Goal Programming" techniques have been applied in order to obtain the optimal solution that satisfies all objectives simultaneously [20], [21], [22]. Rifai [23] introduced goal programming as an extension of linear programming, which is commonly applied to deal with multi-objective problems. Based on the idea of Leung et al. [20], this approach can manipulate various scenarios with a high degree of flexibility by means of setting either targets or weights. This study also follows a branch of the goal programming approach in order to obtain the solution to the developed multi-objective APP model. This study provides a way of balancing operational resources with fluctuation of customer demand in a deterministic environment while maintaining the cost and quality at the optimal levels. In addition, the developed APP model can provide a better interaction between procurement and production departments in a company.

3. Mathematical Model

This section presents a multi-objective MILP model, was constructed based on a multi-product, multi-period APP problem of an automotive parts manufacturing company in Iran. The company under study produces three different parts in three separate production lines.

3.1. Operational conditions and assumptions of the model

The operational conditions related to the case study and the assumptions of the model are as follows:

- All input data are assumed to be deterministic during the considered time horizon.
- Production lines are balanced and the required number of workers for each production line has been determined and hired at the beginning of the first period. Therefore, the cost of hiring is not included in the variable costs.
- Regular time production, overtime production, and warehouse space cannot exceed their maximum levels determined by the decision maker.
- Demand can be either satisfied or backordered, but backorders must be fulfilled in the next time period.
- No backorder level at the end of the last period is allowed.
- No subcontracting for all types of products is allowed during the time horizon.
- As the company hires permanent workers, firing hired workers because of fluctuation in demand is not allowable. However, based on the required skill level of workers, they will be trained to obtain the required higher level of qualification in different time periods.
- The number of workers with certain skill level in a specified time period is equal to or greater than the number of workers with the same skill level in the previous time period.
- The cost of Work In Process (WIP) inventory is negligible.
- A time horizon of T, consisting of six monthly planning periods has been considered.
Two separate warehouses are available; the first one is for storing finished products of types one and two, and the second one is used to store the products of type three.

Inventory level at the beginning of the first period is assumed to be zero.

Each production line is assigned to just one type of product.

All raw material components can be purchased from all suppliers.

Reject rates and costs of raw materials, purchased from different suppliers are different.

The company does not pay for rejected raw materials and the associated cost is incurred by the corresponding supplier.

Salary of workers is not included in the cost of producing one unit of product and is considered separately. Workers with higher skill levels are paid more.

3.2. Parameters definition

The related parameters have been defined as follows:

- $T$: Planning time horizon consisting of six time periods
- $t$: Time period
- $m$: Type of product to be produced in a given time horizon $T$ ($m \in M$)
- $k$: Supplier index ($k \in K$)
- $i$: Component (items) of raw materials ($i \in I$)
- $n_{im}$: Coefficient of using component $i$ in product type $m$
- $h$: Type of production time (regular ($h=1$) or overtime ($h=2$)) ($h \in H$)
- $sk$: Level of workers' skill (ordinary ($sk=1$), good ($sk=2$) and excellent ($sk=3$))
- $S_{st}$: Salary of workers with skill level of $sk$
- $C_{ik}$: Cost of component $I$ purchased from supplier $k$ (cost of placing orders and receiving materials)
- $\theta_i$: Reject rate of component $i$ purchased from supplier $k$
- $D_{mt}$: Forecasted demand of product type $m$ in period $t$
- $\tau_m$: Ideal cycle time for producing one unit of product type $m$
- $C_{ph}$: Cost of producing one unit of product type $m$ in production time $h$
- $C_{it}$: Inventory carrying cost per unit of product $m$ in period $t$
- $C_m^B$: Backorder cost per unit of product $m$
- $C_{ht}$: Cost to train one worker in period $t$
- $L_{wt}$: Number of workers to be assigned for producing product type $m$ in time period $t$
- $Max_{w1m}^m$: Maximum warehouse space for storage of product types 1 and 2
- $Max_{w2m}^m$: Maximum warehouse space for storage of product type 3
- $Max_{w1}^m$: Maximum allowable regular time ($h=1$) or overtime ($h=2$) production in period $t$
- $W^m$: Required warehouse space per unit of product $m$

All input data related to each defined parameter will be made available upon request.

3.3. Decision variables

The outputs of the model are:

- $P_{mth}$: Unit of product type $m$ to be produced in production time $h$ (regular or overtime) in time period $t$
- $L^t_m$: Number of workers to be trained in period $t$ for product type $m$
- $B_{mt}$: Backorder level at the end of period $t$ for product type $m$
3.4. Objective functions

For the process of production planning in the company under study, qualitative objective function has been considered in the model as well. The objective functions of the model are as follows:

**Quantitative objective functions:**
- Minimization of total cost

\[
\min Z_1 = \sum_{m=1}^{M} \sum_{t=1}^{T} (C_{pt}^m B_{mt} + C_{ht}^m I_{mt}) + \sum_{i=1}^{I} \sum_{m=1}^{M} C_{it}^m L^b_{it} + \sum_{k=1}^{K} \sum_{l=1}^{L} C_{lk}^m L^s_{lk} + \sum_{s=1}^{S} C_{qs}^m Q_{skt} \tag{1}
\]

The first element is cost of producing products in regular and overtime production hours, the second element demonstrates backorder cost and inventory cost, the third element is related to cost of training, and the last two elements are costs that the company should incur for salary of workers and purchasing raw materials respectively.

- Maximization of products' quality

\[
\min Z_2 = \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{t=1}^{T} \theta_{skt}^i + \sum_{m=1}^{M} \sum_{l=1}^{L} L^w_{lmt} \tag{2}
\]

This objective function consists of two elements. The first element tries to maximize the quality of raw materials by minimizing the quantity of raw materials with higher reject rates to be purchased from different suppliers and the second element focuses on applying higher skilled workers by minimizing the number of lower skilled workers. These two elements have a tremendous effect on the quality of finished products. One can add other elements that are effective on the quality of finished products, depending on the situation of the company under study.

**Qualitative objective function:**
- Customer service level should be "rather high".

The third objective function is a linguistic term defined by the decision maker. For this qualitative objective function, the linguistic term "customer service level should be rather high" is represented by a membership function [25] which has been defined based on the expectation of the decision maker of the company under study as shown in Figure 1.

![Figure 1. Membership functions for different "Backorder Level Percentages" (BLP).](image-url)
As depicted in Figure 1, the linguistic set has been defined as: \( A = \{ VL \text{ (Very Low)}, L \text{ (Low)}, RL \text{ (Rather Low)}, M \text{ (Medium)}, RH \text{ (Rather High)}, H \text{ (High)}, VH \text{ (Very High)} \} \).

So, the membership function corresponding to the defined qualitative objective function can be formulated as:

\[
\mu_{BLP} = \begin{cases} 
1 & \text{BLP}_t \leq 25 \\
\frac{30 - \text{BLP}_t}{5} & 25 < \text{BLP}_t < 30 \\
0 & 30 \leq \text{BLP}_t 
\end{cases} 
\] (3)

in which, \( \text{BLP}_t \) is defined as backorder level percentage at the end of period \( t \) and is formulated as:

\[
\text{BLP}_t = \frac{\text{Backorder level at the end of period } t}{\text{Demand in period } t} \times 100
\] (4)

Therefore, for the third objective function, the following expression can be considered:

\[
Z_3 = \text{Max} \sum_{t=1}^{T} \frac{30 - \text{BLP}_t}{5}
\] (5)

3.5. Constraints

Based on the operating conditions and assumptions of the model described earlier, the following constraints have been considered:

\[
\sum_{m=1}^{2} P_{mht} = D_{mt} - I_{m(t-1)} - B_{mt} + I_{mt} + B_{m(t-1)}; \forall t > 1, \forall m
\] (6)

\[
\sum_{m=1}^{2} P_{mht} = D_{mt} - B_{mt} + I_{mt}; \forall m, t = 1
\] (7)

\[
\sum_{sk=1}^{3} (L_{skmt} - L_{skm(t-1)}) = L^h_{skm(t-1)}; \forall t > 1, \forall m
\] (8)

\[
L_{skmt} \geq L_{skm(t-1)}; \forall t > 1, \forall m, \forall sk
\] (9)

\[
\sum_{sk=1}^{3} L_{skmt} = L^w_{skm}; \forall m, \forall t
\] (10)

\[
P_{mht} \tau_m \leq \text{Max}^h; \forall m, \forall t, \forall h
\] (11)

\[
B_{mt} \leq D_{mt}; \forall t, \forall m
\] (12)

\[
B_{mt} = 0; \forall m, t = 6
\] (13)

\[
\sum_{m=1}^{2} W_{m} I_{mt} \leq \text{Max}^w; \forall t
\] (14)
\[ W_m I_m \leq \max_{t, m=3} \quad \forall t, m = 3 \] (15)

\[ \sum_{k=1}^{K} Q_{ikt} = \sum_{m=1}^{M} \sum_{i=1}^{N} (P_{mil} \times n_{im}); \quad \forall t, \forall i \] (16)

Constraints 6 and 7 describe the total quantity of products to be produced in both regular and overtime production hours. Constraint 8 determines the number of workers that should be trained to obtain the required higher level of skill in different periods. Constraint 9 is constructed based on the operational condition of the company that forbids the firing of workers during the time horizon under study. Therefore, once an operator is trained in a certain period, she/he has the same or higher skill level (due to training) in the following periods. Constraint 10 states that the total number of workers with different skill levels assigned to a specific production line should be equal to the dedicated number of workers of that production line in each period. Constraint 11 puts a limitation on the quantities of products to be produced based on the capacity of the corresponding production line. Constraint 12 is a reasonable constraint, which limits the level of backorder quantities in each period; whereas constraint 13 puts a barrier on having any backorder level at the end of the specified time horizon (the time horizon consists of 6 time periods). Constraints 14 and 15 specify the space limitations of the available warehouses for storing all three types of finished products (inventory) at the end of each period. Constraint 16 determines the total quantity of each component (raw material) to be purchased from different suppliers based on the usage coefficients of that component in different products and the quantity of products to be produced in each period.

4. Providing solution to the model

To solve the resulted multi-objective MILP model, the Fuzzy Goal Programming approach has been applied. This approach involves determining a goal value for each objective function, defining a membership function for each of them and finally transforming them to an equivalent single objective using an aggregation operator. Consider an objective function \( Z_i \) (with minimization objective), the corresponding membership function is presented in Figure 2.

![Figure 2. A typical membership function for a minimization objective.](image)

\( Z_i^{PIS} \) and \( Z_i^{NIS} \) are positive and negative ideal solutions of objective function \( Z_i \), respectively. The main problem in defining a membership function is to determine positive and negative ideal solutions with respect to each objective function \( Z_i \). The approach followed by Waiel and Sang [24] can best answer this issue. Based on this approach, the maximum aspiration level of an objective function \( Z_i \) (PIS) is the solution obtained by solving the model for each \( Z_i \) separately. However, the negative ideal solution of an objective function is obtained by one of the following equations:

\[ Z_i^{NIS} = \max \left\{ Z_i \left( v' \right); \ i \neq j \right\} \text{ in case of having a minimization objective} \] (17)

\[ Z_i^{NIS} = \min \left\{ Z_i \left( v' \right); \ i \neq j \right\} \text{ in case of having a maximization objective} \] (18)
where $v^*_j$ is the positive ideal solution of objective function $Z_j$. Once all membership functions are constructed, the FGP model can be formulated.

5. Applying the model to the company under study

Applying the data gathered from the company under study and following the approach described in section 4, the positive and negative ideal solutions of each objective function were obtained using IBM ILOG CPLEX Optimization Studio (version 12.4) software as shown in Table 1.

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$v^*_1$</th>
<th>$v^*_2$</th>
<th>$v^*_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
<td>5457721944</td>
<td>9859252719</td>
<td>6604138849</td>
</tr>
<tr>
<td>$z_2$</td>
<td>6.05</td>
<td>0.25</td>
<td>3.727</td>
</tr>
<tr>
<td>$z_3$</td>
<td>5.808</td>
<td>4.791</td>
<td>6</td>
</tr>
</tbody>
</table>

Therefore from the table above and equations 17 and 18, it is concluded that:

$Z^{NIS}_1 = 9859252719$, $Z^{NIS}_2 = 6.05$, $Z^{NIS}_3 = 4.791$, $Z^{PIS}_1 = 5457721944$, $Z^{PIS}_2 = 0.25$, $Z^{PIS}_3 = 6$.

The constructed membership functions along with their formulations are as depicted in Figures 3 to 5 and Equations 19 to 21.

As the final step for solving the fuzzy goal programming model, the max-min operator of Bellman-Zadeh [26] was applied as an aggregation operator to convert the multi-objective linear model to an equivalent single objective one. Therefore the following model can be constructed as the final model.
\[ \text{Max } \Phi \]
\[ S.t. \]
\[ \Phi \leq \mu_1; \Phi \leq \mu_2; \Phi \leq \mu_3 \]
\[ \mu_1 = \frac{9859252719 - Z_1(x)}{9859252719 - 5457721944}; \mu_2 = \frac{6.05 - Z_2(x)}{6.05 - 0.25}; \mu_3 = \frac{Z_3(x) - 4.791}{6 - 4.791} \]  

Constraints 6 to 16;
\[ x \geq 0 \]
\[ 0 \leq \Phi \leq 1. \]

Using IBM ILOG CPLEX Optimization Studio software (version 12.4), the solutions to the above MILP model were obtained as presented in the following section.

6. Experimental Results

As shown in Table 2, incorporating quality as an objective function in the model imposes an increase of 36510000 units in the total cost. The increase stems from purchasing some raw materials from suppliers with lower reject rates and higher prices and also applying higher skilled workers with higher salaries when considering the objective of quality in the model. Due to space limitation, we just present a general comparison related to the skill levels of workers in Figures 6 and 7. From the figures, it can be found that, in the presence of quality as an objective function, higher qualified workers are required to produce the products.

Although including quality in the APP process imposes additional cost, it has tremendous effect on enhancing customer satisfaction due to the higher reliability of products. Therefore, a firm should perform a trade-off between the additional cost resulted from enhancing the reliability of products and the cost incurred because of customer dissatisfaction and sale losses.

**Figure 6.** Required number of workers with different skills in different time periods without considering the objective of quality.

**Figure 7.** Required number of workers with different skills in different time periods considering the objective of quality.

Table 3 presents the obtained results for other decision variables including inventory, backorder and production levels in different time periods. The results shown in this table obviously indicate that bearing some inventories or and having overtime production hours are more preferred than having some backorders. The reason lies in the fact that the cost of having backorder is far higher than keeping inventory and/or producing during overtime production hours in the case under study. In addition, the objective of optimizing customer service level has its own effect on obtaining such results.
Table 2. Comparison of obtained results with and without considering quality as an objective.

<table>
<thead>
<tr>
<th>Considering the objective of quality</th>
<th>Without considering the objective of quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 ), ( Z_2 ), ( Z_3 )</td>
<td>( Z_1 ), ( Z_2 ), ( Z_3 )</td>
</tr>
<tr>
<td>5532400000</td>
<td>5495890000</td>
</tr>
<tr>
<td>0.28</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 3. Obtained results for decision variables of backorder, inventory and production levels.

<table>
<thead>
<tr>
<th>Backorder unit</th>
<th>Inventory unit</th>
<th>Production unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( h=1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( h=2 )</td>
</tr>
<tr>
<td>( t )</td>
<td>( m_1 )</td>
<td>( m_2 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In addition to the results discussed above, the outputs related to the decision variable "required number of workers to be trained in different time periods" recommend having no training (the values are zero in all time periods) for workers during the specified time horizon. In other words, it is more profitable for the firm to hire workers with the required qualification level instead of training them to acquire higher skills.

7. Conclusion
In this study, an attempt has been made to present a multi-objective MILP model that best serves those planners who aim at optimizing cost and quality simultaneously. The application of qualitative objective functions in the model makes it more valuable and practical for those who deal with linguistic terms in the process of production planning. To solve the constructed model, the fuzzy goal programming approach along with the aggregation operator of Bellman-Zadeh was used. The numerical results were obtained using IBM ILOG CPLEX Optimization Studio (version 12.4) software based on the data gathered from an automotive parts manufacturing company. The results show that incorporating quality as an objective into the model can impose additional cost. However, the firm should perform a trade-off between the additional cost resulted from producing products with higher quality and the cost incurred because of customer dissatisfaction and sale losses.

For the company under study, the results also show that it is more profitable to have some level of inventories and/or overtime production rather than having backorders. Furthermore, the numerical outputs of the model emphasize on hiring workers with the required qualification level instead of training them to acquire demanded skills.

References


