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A numerical method to predict the rate-sensitive hardening behaviour of sheet materials using uniaxial and biaxial flow curves

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Abstract.

The use of advanced high strength steel (AHSS) is increasing in the automotive industry due to their remarkable strength-to-weight ratio and formability. In recent years, there has been a keen interest to employ high-energy rate forming processes such as electromagnetic and electrohydraulic forming because they can significantly improve the formability of these materials. However, simulating these forming processes requires reliable hardening functions that can accurately predict their flow behaviour in a wide range of strains and strain rates. One of the limitations of uniaxial tension tests is that the maximum uniform strain is not sufficient to calibrate a hardening function at high strain levels. In this work, a new numerical method is proposed to generate the extended flow curves of DP600 and TRIP780 from uniaxial tension data obtained at strain rates ranging from 0.001s^{-1} to 1000s^{-1} and from balanced biaxial tension data obtained under quasi-static conditions. Then, a 7-parameter strain-rate dependent Voce hardening function, which accounts for stage IV hardening, was fitted to the true stress-strain curves thus generated. Finally, statistical analysis was used to evaluate the goodness of the fit of predicted results.

1. Introduction

Dual phase (DP) and transformation induced plasticity (TRIP) steels are among advanced high-strength steels (AHSS) that have increasingly gained interest in the automotive industry for manufacturing auto-body parts and structural components in order to improve crashworthiness and fuel efficiency by decreasing the overall body weight [1, 2]. Their good ductility, formability and high strength to weight ratio are achieved through their composite microstructure that is usually comprised of soft ferrite and hard martensite islands in DP steels, and small amounts of martensite, retained austenite and bainite in a ductile ferrite matrix in TRIP steels [2, 3]. Due to their superior mechanical properties and deformation behaviour, various forming processes in a wide range of strain rates, such as stamping, drawing, hydroforming, explosive forming (EF) and electro-hydraulic forming (EHF), are employed to produce defect-free automotive components [4–6]. Accordingly, developing a rate-dependent hardening function that can accurately describe the hardening flow behaviour of the material, is the first step to define a complete constitutive plasticity model for finite element analysis of different forming processes [7].

Various phenomenological constitutive hardening models are proposed and utilized to capture the dynamic mechanical response of materials for simulating metal forming processes at different



strain rates and temperatures [8]. Based on the slope of the hardening functions at high strain levels, They can be divided into **unbounded** and **saturated** type models where the hardening rate becomes zero and the flow stress reaches a saturation (constant) value, such as the 3-parameter Voce hardening function [9] in the former while in the latter, the strength of the material increases continuously at higher strains, e.g. the Johnson-Cook [10] and Khan-Huang-Liang (KHL) [11,12] models. However, Sarraf et al. [13,14] evaluated the effect of different rate-dependent hardening equations on the prediction of damage behaviour in DP600 steel sheet specimens and showed that the type of hardening function, whether it is saturated or unbounded, has a significant effect on the prediction of both strain localization at the beginning of instability and final failure geometry in uniaxial tension tests and Marciniak tests. In addition, different hardening models can predict the tensile flow curve of DP600 at a certain strain rate, however, the hardening rate can vary considerably at large deformation in the absence of enough experimental data points to calibrate the hardening function at high strain levels [13]. On the other hand, Hassannejadasl et al. [15] stated that obtaining uniaxial and biaxial flow curves is essential to develop an accurate rate-dependent anisotropy function for finite element simulation of sheet metal forming.

The main objectives of the current research are to propose (a) a new method to predict extended hardening flow curves (beyond uniaxial tension limits) using quasi-static bulge test and (b) an approximation approach to predict rate-dependent biaxial flow curves. Additionally, a new modified strain-rate dependent Voce hardening model is proposed and calibrated for both uniaxial and biaxial flow curves of DP600 and TRIP780, and statistical analysis is used to evaluate the goodness of the fit and accuracy of the predictions.

2. Experimental data

Uniaxial tension tests were carried out on DP600 and TRIP780 steel sheet specimens with nominal thickness of 1.5mm at 0, 45, 90° from the rolling direction (corresponding to RD, DD, and TD, respectively). For quasi-static (0.001 and 0.1s⁻¹) tests, ASTM (E8M-04) specimens were used in an Instron 1331 servo-hydraulic testing machine with a ±12.5 mm biaxial extensometer that was used to measure the axial and width strains. In order to perform the tests at intermediate and high strain rates (1, 10, 100 and 1000s⁻¹), a miniature dog-bone shaped specimen has been designed and utilized instead, on a special Hydraulic Intermediate Strain Rate (HISR) apparatus (developed at the University of Waterloo [16]) and tensile split Hopkinson bar (TSHB) machine. Digital Image Correlation (DIC) was employed to measure experimental strain distributions in images recorded with a high speed imaging camera along with necessary data processing software. Here, for the purpose of brevity, the true stress-effective true plastic strain along the rolling direction (RD) of the DP600 and TRIP780 sheet specimens are shown in figure 1. Rahmaan et al. [2] provided a more detailed description of the testing technique, specimens, tools and measurement procedures.

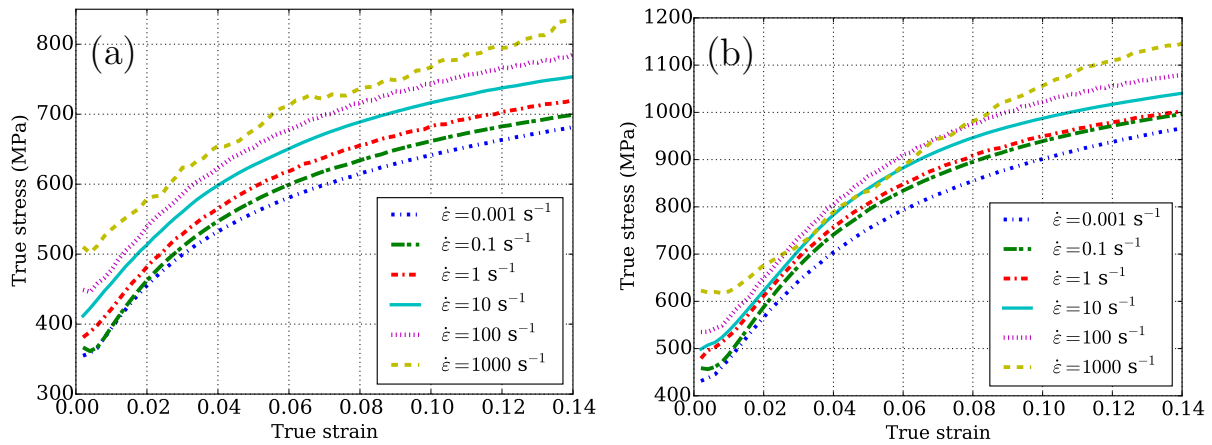


Figure 1. Flow curves of (a) DP600 and (b) TRIP780 in RD at different strain rates ranging from 0.001s^{-1} to 1000s^{-1} .

To obtain higher levels of strain compared to those attained in uniaxial tension, hydrostatic bulge tests were carried out on DP600 and TRIP780 sheet samples using a 240-ton double-action hydraulic press in quasi-static conditions. Circular blanks with a diameter of 230mm was used to allow sufficient space out of 135mm bulge zone in order to perfectly clamp the specimen and prevent the sheet to be drawn into the die. The sheet specimens then were painted in white with subsequent stochastic black speckles so that the DIC technique could be used to measure strain distribution as the speckle pattern changed during the test process. Vasilescu [17] presented a detailed description of the testing procedures, specimens, tools and dies, measurement procedures and obtained results for these experiments. Figure 2 shows the quasi-static flow curves of DP600 and TRIP780 obtained via hydraulic bulge tests.

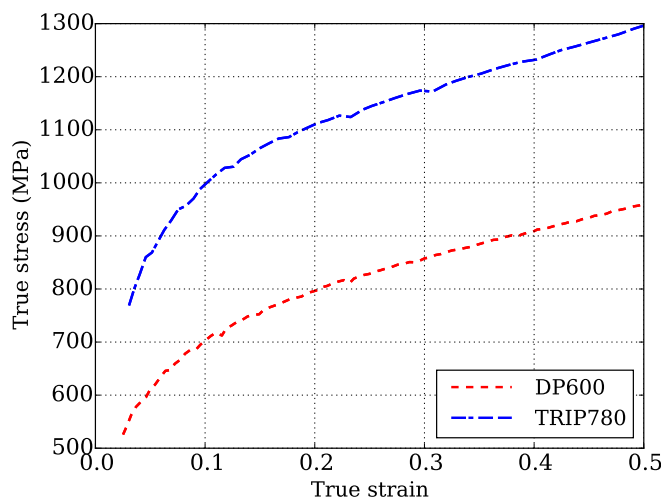


Figure 2. Quasi-static flow curves of DP600 and TRIP780 in biaxial tension (BT).

3. Extended uniaxial and biaxial flow curves

As argued by Sarraf et al.[13], the hardening function cannot be precisely calibrated to predict the hardening rate of the material at high strain levels when no experimental data point is available for that part of the flow curve. On the other hand, it is shown that there is a very good agreement between the hardening flow curves obtained by hydrostatic bulge tests and a combination of flat rolling and uniaxial tension tests [14]. Moreover, the biaxial flow curve

obtained by hydrostatic bulge test can be fitted with the 4-parameter Voce hardening function which consists of a linear term as a function of the equivalent plastic strain ($\theta \varepsilon_p = C_4 \varepsilon_p$) which describes the stage IV hardening behaviour in addition to the saturated 3-parameter Voce model, as shown in equation (1). The performance of the 4-parameter Voce model in this study is somewhat similar to that of the generalized Voce law proposed by Tome et al. [18] and Larour [19] for FCC materials that was extensively used for aluminum alloys in the literature. Therefore, first, a 4-parameter Voce hardening function (Voce-4p) was fitted to the flow curve achieved by hydrostatic bulge test to find the slope of the hardening curve at high strain levels. Then, a linear curve was added to the last few experimental points of each tensile hardening curve for DP600 and TRIP780 at each strain rate and direction such that the slope between those points ($\Delta\sigma/\Delta\varepsilon_p$) found to be the closest to θ , as can be seen in figure 3. In the next step, 4-parameter Voce hardening models were fitted to each individual true stress-true strain curve, hereafter called extended uniaxial flow curves, to make the non-linear to linear transition smoother.

$$\sigma(\varepsilon_p) = C_1 - (C_1 - C_2)(1 - \exp(-C_3 \varepsilon_p)) + C_4 \varepsilon_p \quad (1)$$

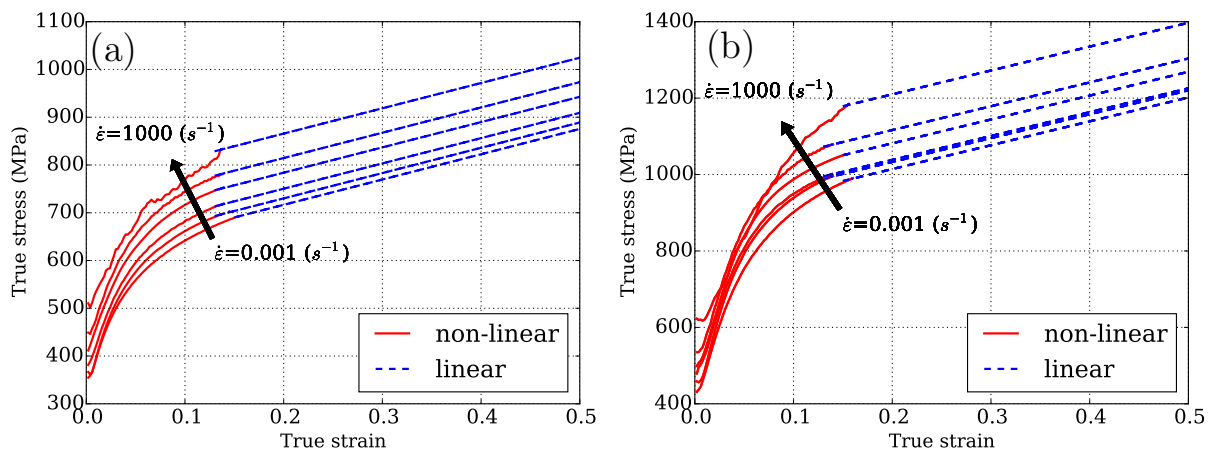


Figure 3. Extending the hardening flow curve of (a) DP600 and (b) TRIP780 in RD by adding linear part of their corresponding biaxial flow curve (blue dashed lines).

Subsequently, a new modified version of the Voce hardening model which is comprised of a multiplicative combination of a 4-parameter Voce and a 3-parameter rate-sensitive term, as indicated in equation (2), was fitted to the extended uniaxial flow curves for each material in each direction to capture the dynamic response of each material even at high strain values. It is worth noting that from a mathematical point of view, C_5 can be considered as a scale factor. The experimental flow stresses and those predicted using the modified Voce model along the RD for DP600 and TRIP780 are presented in figure 4. By comparing experimental and predicted values, it can be seen that there is a considerably good agreement between them. Similar quality of fit was observed for other testing directions, i.e. DD and TD. It is worth noting that although the same slope was considered for the linear curve that was added to the end of uniaxial hardening curves, the final hardening slopes predicted by the Voce (7p) follows the same patterns as the original flow curves due to the nature of the multiplicative combination of strain and strain-rate dependent functions.

$$\sigma(\varepsilon_p, \dot{\varepsilon}_p) = [C_1 - (C_1 - C_2)(1 - \exp(-C_3 \varepsilon_p)) + C_4 \varepsilon_p] \left(\frac{\dot{\varepsilon}_p}{C_5} + C_6 \right)^{C_7} \quad (2)$$

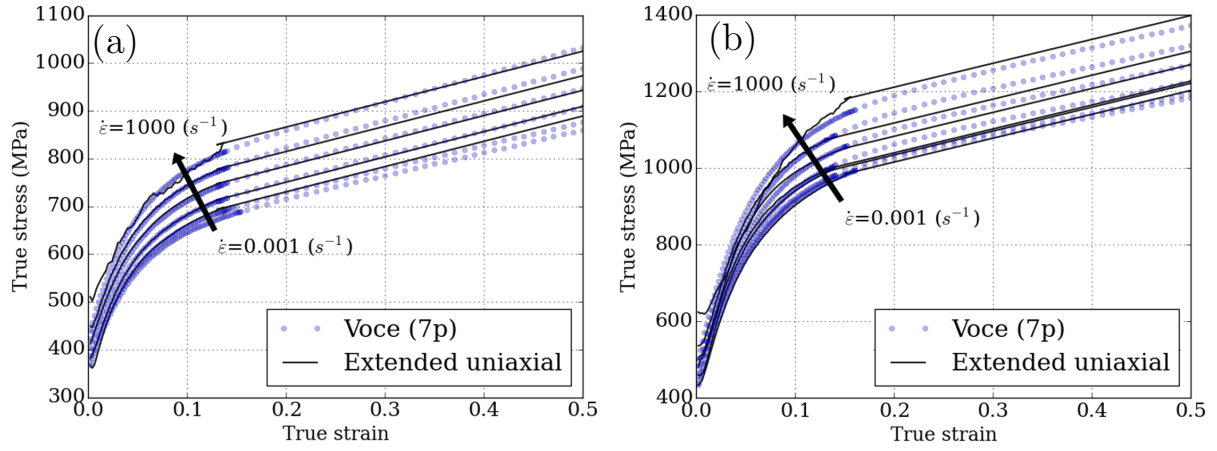


Figure 4. Fitting the strain-rate dependent 7-parameter Voce hardening function to flow curve of (a) DP600 and (b) TRIP780.

In order to estimate the rate-dependent biaxial flow curves, it is assumed that the jump between different biaxial flow stress at various strain rates follows the same trend as a combined effect of strain-rate sensitivity on uniaxial hardening curves in all directions, i.e. the biaxial flow curves just shift up and down by changing the deformation rate [15]. In this regard, a single point is calculated as the average flow stress based on all three test orientations (RD, DD and TD) for each strain rate (as shown in equation (3)) and then, a logarithmic curve is fitted to all average points obtained for different deformation rates. Subsequently, the average flow stress between $\varepsilon_p=0.2-0.4$ is determined as the initial biaxial flow stress point at 0.001s^{-1} which can be considered as the commencing point for deriving rate-dependent (shifted) biaxial hardening curves, as shown in figure 5 with a blue circle. Therefore, the strain-rate dependency of the biaxial flow stress can be obtained by shifting the logarithmic curve on UT points (black solid line in figure 5) to the biaxial starting point (blue dashed line in figure 5). A 7-parameter strain dependent modified Voce hardening function is eventually fitted to the derived biaxial flow stresses from $\dot{\varepsilon}_p=0.001\text{s}^{-1}$ to 1000s^{-1} , as can be seen in figure 6.

$$\sigma_{ave | \dot{\varepsilon}_p = cte.} = \frac{1}{n} \left[\sum_{RD, DD, TD} \sum_{\varepsilon_p=0.08}^{0.14} \sigma(\varepsilon_p) \right] \quad (3)$$

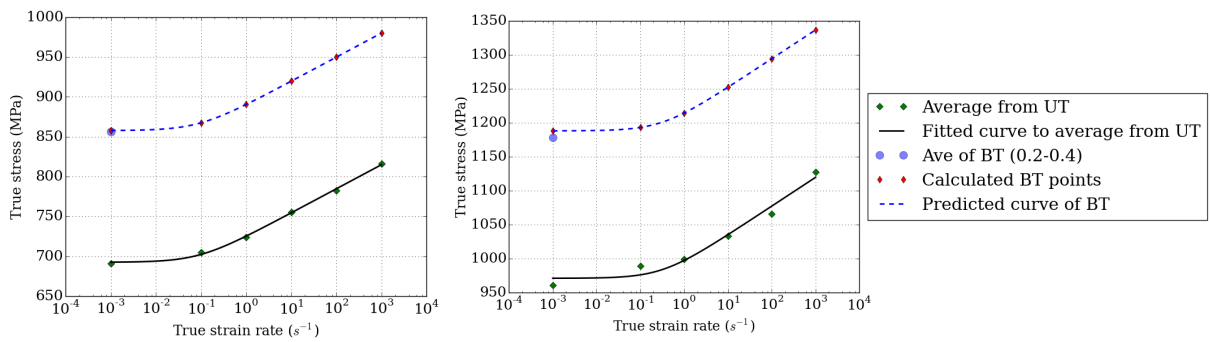


Figure 5. Prediction of the strain-rate dependency of the biaxial flow curves of DP600 (left panel) and TRIP780 (right panel).

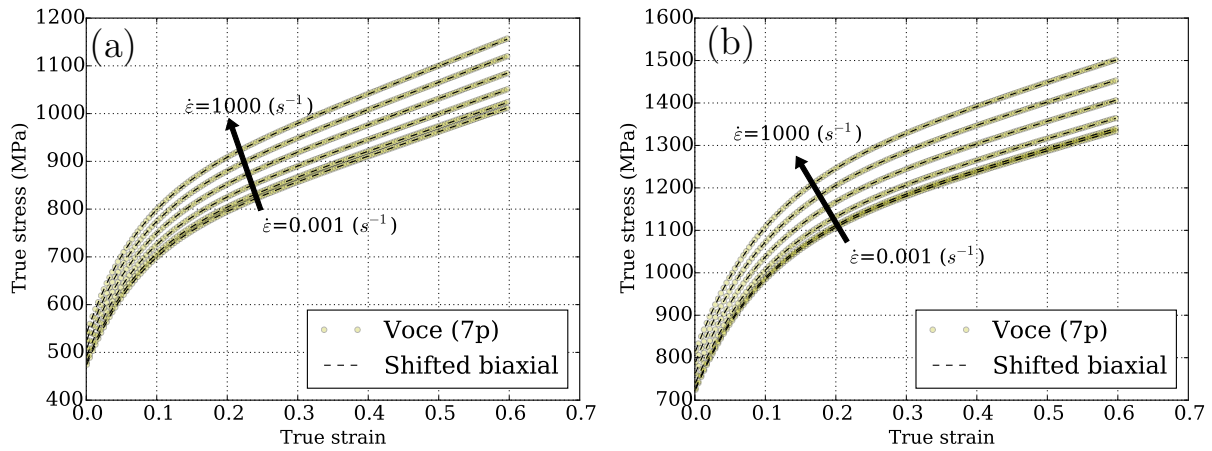


Figure 6. Prediction of biaxial flow curves of (a) DP600 and (b) TRIP780 at 0.1, 1, 10, 100, 1000s⁻¹ (black dashed lines) and the fitted 7-parameter Voce hardening function (yellow circles).

Table 1 shows the coefficients of the 7-parameter modified Voce hardening function that is fitted to rate-dependent uniaxial and biaxial flow curves. These coefficients are determined through a combination of non-linear regression (NLR) as the fitting procedure and Markov chain Monte Carlo (MCMC) method-Metropolis Hastings algorithm (MH) as the parameter optimization method [13].

Table 1. Coefficients of the Voce (7p) hardening function for DP600 and TRIP780 in RD, DD, TD and BT.

Material	Test	C_1 (MPa)	C_2 (MPa)	C_3	C_4 (MPa)	C_5	C_6	C_7
DP600	RD	373.030	658.387	24.1975	499.704	1.164	0.054	0.0189
	DD	374.523	669.215	22.5536	483.507	0.949	0.120	0.0174
	TD	397.825	684.390	23.9744	488.497	1.283	0.036	0.0154
	BT	474.225	714.008	13.6935	506.700	0.105	0.647	0.0141
TRIP780	RD	419.214	955.077	19.4872	491.720	0.391	0.362	0.0168
	DD	409.225	936.615	17.0147	456.183	0.673	2.491	0.0203
	TD	475.554	997.994	19.9872	504.450	0.519	0.017	0.0135
	BT	720.738	1055.222	10.4103	463.373	0.269	1.034	0.0144

The accuracy of the proposed hardening function in predicting flow curves of DP600 and TRIP780 steel sheets for each testing procedure and direction is determined using adjusted R -squared value and root mean square error (RMSE) [13] and is shown in figure 7. It can be seen that the modified 7-parameter Voce hardening function can successfully predict strain and strain-rate dependent flow behaviour of DP600 and TRIP780 for both extended uniaxial and biaxial flow curves since the calculated $\bar{R}^2$ values are greater than 0.97 in all cases. However, comparing RMSE values reveals that the proposed hardening model showed better performance for DP600 sheet specimens since its corresponding RMSE is slightly more than that of TRIP780.

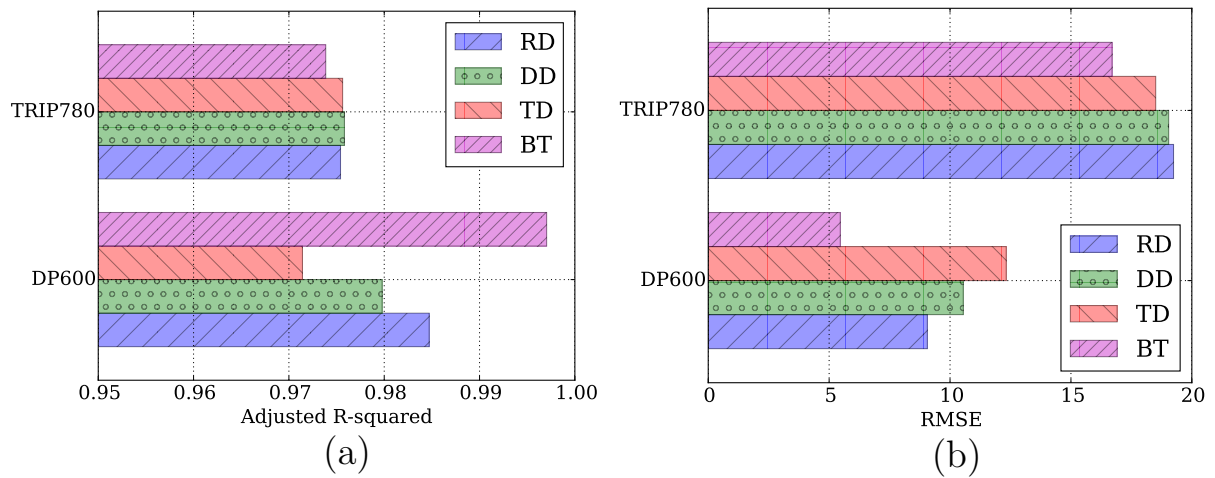


Figure 7. Comparison of accuracies of the proposed Voce(7p) hardening function through (a) cumulative adjusted R -squared value and (b) RMSE (MPa) for DP600 and TRIP780.

Rahmaan et al. [20] performed several shear tests on DP600 and AA5182 sheet specimens in a broad range of strain rates from quasi-static conditions (0.01 s^{-1}) to elevated strain rates (600 s^{-1}) in order to investigate the strain rate sensitivity of these materials at higher strain levels. Figure 8 compares the effective true stress-effective true plastic strain curves obtained from the shear test results (so they can be compared to uniaxial tension test data) and those predicted by the proposed 7-parameter Voce hardening function. Overall, a very good agreement between shear experimental hardening curves and the numerically obtained results through the combination of tensile+bulge tests and Voce (7p) can be observed. Therefore, the proposed method and hardening model can be considered as effective approach to predict the extended rate-sensitive flow curves of materials.

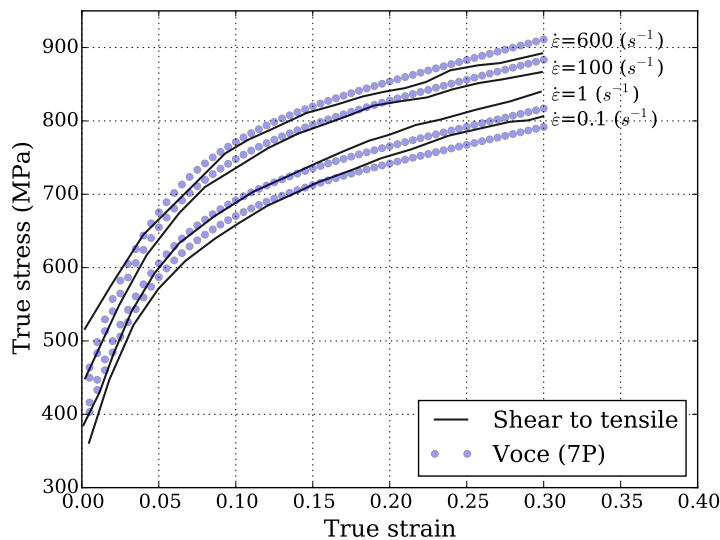


Figure 8. Comparison of the extended flow curves obtained through shear experiments [20] and those predicted by the Voce (7P).

4. Summary and conclusions

Two methods were proposed (a) to generate extended uniaxial flow curves to higher strain levels using quasi-static biaxial flow stress and (b) to calculate rate-dependent biaxial flow curves using uniaxial tension tests for DP600 and TRIP780 sheet metal alloys in three testing orientations

(RD, DD, TD) in a wide range of strain rates, from quasi-static conditions to high strain rates. In addition, a modified 7-parameter Voce function that takes the stage IV hardening into account was introduced and its accuracy in predicting hardening flow curves was evaluated.

In the first approach, the linear part of the biaxial flow curve is appended to the last few data points on the flow curve of each material at each strain rate and material orientation obtained through uniaxial tension tests. In order to smooth the transition from non-linear to linear curve, a 4-parameter Voce model was fitted to each hardening flow stress and subsequently, a modified rate dependent Voce function was fitted to generated flow curves of DP600 and TRIP780 in RD, DD and DD. Furthermore, the average of uniaxial flow stresses for each material at a constant strain rate in a certain range of strains was used to determine the approximate upward or downward shift that quasi-static biaxial flow stress may encounter to generate rate-dependent biaxial flow curves. The modified Voce model with a multiplicative combination of strain and strain-rate dependent terms was assessed through statistical analysis of $\bar{R}^2$ and RMSE and shown to be able to successfully predict the rate-dependent flow curves of DP600 and TRIP780 steel sheets subjected to uniaxial tension and hydrostatic bulge tests.

The proposed approaches and hardening model can be used in finite element simulations of different metal forming processes where the deformation exceeds the uniaxial strain levels with various strain rates. Moreover, combined approaches can effectively contribute to future advances in developing strain and strain-rate dependent anisotropic plasticity functions.

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