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Mathematical algorithm for calculating an optimal axial preload of rolling bearings with the respect to their life

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Abstract. The article deals with a description of a mathematical algorithm, programmed in the MATLAB environment of the company MathWorks. The algorithm serves a calculation of an optimal axial preload of rolling bearings. So that, the life of a bearing pair, arranged into “X” or “O” arrangement, is maximized according to a given load. Hence angular contact ball bearings and tapered rolling bearings are the subject matter of the article. Moreover, methods, that were used in the optimizing code, such as Hertz’s contact of two bodies and Stribeck’s load-distribution on rolling elements, are described in the article too. The finding is the dependency between total axial preload and life of a bearing pair.

1. Introduction

When a rolling bearing is subjected to a radial loading, rolling elements are not equally loaded. Radial load-distribution on elements of ball bearings was investigated by Stribeck. He stated value of Stribeck’s number to 5.0. It expresses, that the most loaded ball element is loaded 5 times the average loaded ball in a ball bearing with zero internal diametral clearance. Palmgren made the investigation on roller bearings and suggested to use Stribeck’s number with the same value even for roller bearings [1]. Later, Lundberg and Palmgren stated equations for fatigue life of radially loaded rolling bearings. Those relations were simplified relative to the geometry of bearings and incorporated in ISO standard [2, 3]. Harris and Kotzalas attested fatigue life of rolling bearings can be increased by an appropriate internal clearance [1]. Axial preload of angular contact ball bearings and tapered bearings relates to the clearance [4]. The article describes optimizing algorithm created in Matlab that serves the purpose of optimal axial preload calculation with maximal bearing fatigue life. Consequently, life model of Lundberg and Palmgren is compared to ISO281:2007.

2. Stribeck’s radial load-distribution

Stribeck examined rolling element loading variation of radial loaded ball bearings to find out frictional forces [5]. When a rolling bearing with positive internal diametral clearance G_r is loaded radially, the load zone will be less than 180° . Figure 1 explains the internal radial clearance G_r and other geometrical bearing parameters. If a bearing with zero internal clearance is oriented according to the Figure 2 and the radial load F_r acts on the inner ring than the force is distributed on the rolling element labelled 1 and on n carrying pairs of rolling elements. The number of carrying pairs n is given by:

$$n = \frac{(z-1)}{4} \quad (1)$$



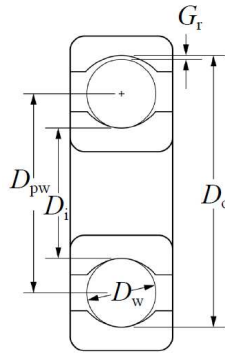


Figure 1. Ball bearing with internal diametral clearance G_r [3].

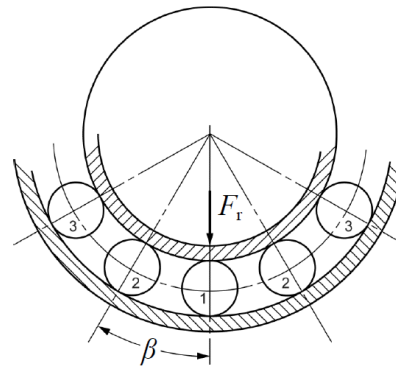


Figure 2. Ball bearing with zero internal diametral clearance subjected to the radial load F_r [1].

where z is the number of rolling elements and n is rounded down to a whole number. From static equilibrium of a radially loaded ball bearing, the following equation for a load-distribution on rolling elements results [1].

$$F_r = Q_1 + 2Q_2 \cos(\beta) + 2Q_3 \cos(2\beta) + \dots + 2Q_{n+1} \cos(n\beta) \quad (2)$$

$Q_1, Q_2, Q_3, \dots, Q_{n+1}$ are normal loads on individual rolling elements. The angle β between 2 nearest rolling elements is shown in the Figure 2 and is given by the equation:

$$\beta = \frac{2\pi}{z} \quad (3)$$

Between load Q_1 and loads Q_{n+1} that act on the other rolling elements, the following relation exists [1]:

$$Q_j = Q_1 \{\cos[(j-1)\beta]\}^\gamma \quad (4)$$

where $j = 2, 3, \dots, n+1$, and the value of exponent $\gamma = 10/9$ for a line contact and for a point contact $\gamma = 3/2$. Hence, the equation (2) can be adjusted to the form:

$$F_r = Q_1 \{1 + 2[\cos(\beta)]^{\gamma+1} + 2[\cos(2\beta)]^{\gamma+1} + \dots + 2[\cos(n\beta)]^{\gamma+1}\} = Q_1 G = Q_{\max} G \quad (5)$$

Because the most heavily loaded rolling element is the element 1 the loading acting on it can be denoted by $Q_1 = Q_{\max}$. The variable G is equal to the expression between the curly braces. Stribeck discovered that for radially loaded ball bearings with zero internal clearance G_r , the division of number of balls z and value of G is close to the constant $z/G = 4.37$ [1]. However, this number changes according to internal diametral clearance, deformation of raceways and loading. Therefore, he suggested to round the Stribeck's number to the value $S_t = 5.0$. Afterwards Palmgren stated constant $z/G = 4.08$ for roller bearings, but he suggested to use the value 5.0 for either ball or roller bearings [1]. The relation for the calculation of maximally loaded element by Stribeck's number can be expressed from the equation (5).

$$Q_{\max} = \frac{F_r}{G} = \frac{S_t F_r}{z} \quad (6)$$

Later Harris and Kotzalas applied an iterative method for the calculation of the maximal loading $Q_{\max}$ by load-distribution integral $J_r(\varepsilon)$. The integral works with the value of the load-zone parameter that is referred to as a load distribution factor ε . Whereby $\varepsilon \leq 0.5$ means that the zone of contact is not more than 180° . In the case, $\varepsilon > 0.5$ the contact zone is more than 180° . And the relation between the load-distribution integral and Stribeck's number is as follows:

$$S_t = 1/J_r(\varepsilon) \quad (7)$$

The dependency of Stribeck's number and the internal diametral clearance is described in the reference [1]. The load distribution factor ε is given by:

$$\varepsilon = \frac{1}{2} - \frac{G_r}{4\delta_T} \quad (8)$$

where G_r is internal clearance and δ_T is the total raceway displacement. Their relative relation is shown in the Figure 3 and is defined as:

$$\delta_T = \delta_{\max} + G_r/2 \quad (9)$$

Generally, the relation between the loading and the deflection in the contact of a rolling element and its raceway is expressed by the equation:

$$Q = K\delta^\gamma \quad (10)$$

Q denotes loading, K is the stiffness of the system and δ is the deflection in a contact. The total deflection between both raceways under the loading separated by rolling element is given by the sum of both deflections of the inner and the outer raceway [5].

$$\delta_n = \delta_i + \delta_o \quad (11)$$

The total stiffness of this system is given by: [5]

$$K_n = \left[\left(\frac{1}{K_i} \right)^\gamma + \left(\frac{1}{K_o} \right)^\gamma \right]^{-\gamma} \quad (12)$$

And the consequential normal load is:

$$Q_n = K_n \delta_n^\gamma \quad (13)$$

Hence, the maximal load $Q_{\max}$ can be found by combining equations (9) and (13).

$$Q_{\max} = K_n (\delta_T - G_r/2)^\gamma \quad (14)$$

Then the relation for δ_T is acquired by substituting equations (7) and (14) in the equation (6). Parameter δ_T is an important constituent of mentioned iteration method for calculating values of load distribution factor ε .

$$\delta_T = \frac{G_r}{2} + \left(\frac{F_r}{z K_n J_r(\varepsilon)} \right)^{\frac{1}{\gamma}} \quad (15)$$

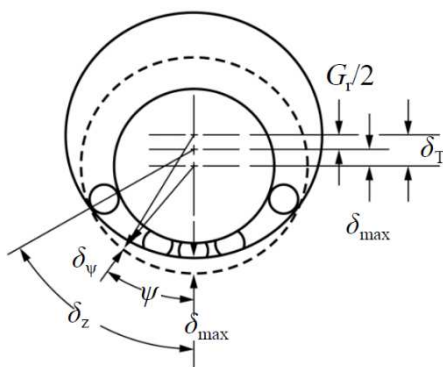


Figure 3. Radially loaded rolling bearing with internal clearance G_r and elastic deformation $\delta_{\max}$ [1].

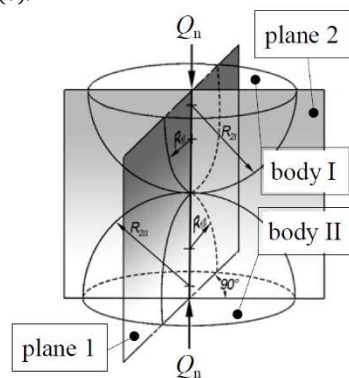


Figure 4. Contact of two bodies pressed by normal load Q_n to each other and curvature radii lying in main planes [7].

3. Surface and subsurface stress

In the contact of rolling elements and a raceway of a loaded bearing, the elastic deformation is present. Miniature contact surfaces occur and distribute loading to individual rolling elements. Because of the size of the surfaces, implicated contact stress is relatively high. Hertzian theory of elastic deformation provides relations for calculation of contact stress and deformations. It is based up on these assumptions:

- The material is homogeneous and isotropic.
- Contact surface is continuous and non-conforming and the area of contact is much smaller than the characteristic radius of the body. This surface is submitted to normal load only, not to shear.
- The elastic limit of the material is not exceeded.

Even though rolling bearings do not meet the assumptions entirely, the theory applied to this field gives adequate accuracy. And many publications affirm that calculated values of deformation and contact area dimensions correspond to experimentally achieved measurements.

In Hertzian theory, when two bodies with curved surfaces are pressed to each other, characteristic radius ρ is defined as the reciprocal of the curvature radius r [4]:

$$\rho = 1/r \quad (16)$$

Each of both bodies is characterized by curvature in main planes, as in the Figure 4, which are orthogonal to each other and in which are located maximal and minimal curvatures. If the curvature centre is situated inside the body ρ is positive. In opposite situation ρ is negative.

In the case of rolling bearings with point contact, the contact area commonly has an elliptical shape. Contact ellipse is defined by the length of major and minor axes:

$$2a = 2\mu \sqrt[3]{\frac{1}{E}(1-m^2)\left(\frac{3Q_n}{\Sigma\rho}\right)} \quad (17)$$

$$2b = 2\nu \sqrt[3]{\frac{1}{E}(1-m^2)\left(\frac{3Q_n}{\Sigma\rho}\right)} \quad (18)$$

where $2a$ is the length of the major axis, $2b$ is the length of the minor axis, E is Young's modulus, m is Poisson's ratio, Q_n is normal load, μ and ν are Hertz's coefficients, that characterized stress distribution in the contact area. In the case of line contact, the width of the contact area is given with sufficient accuracy by:

$$2b = 2\sqrt{\frac{1}{E}(1-m^2)\left(\frac{8Q_n}{\pi L_{we}\Sigma\rho}\right)} \quad (19)$$

Whereby the sum of curvature is calculated just from one main plane. The maximal contact stress p_0 for point contact is located in the centre of the contact area and is defined as:

$$p_0 = \frac{1,5}{\pi\mu\nu} \sqrt[3]{\left[\frac{E}{(1-m^2)}\frac{\Sigma\rho}{3}\right]^2 Q_n} \quad (20)$$

For line contact, the maximal contact stress p_0 can be calculated by the following equation, which is applicable for roller and tapered bearings [8].

$$p_0 = \sqrt{\left(\frac{E}{(1-m^2)}\right)\left(\frac{Q_n\Sigma\rho}{\pi 2L_{we}}\right)} \quad (21)$$

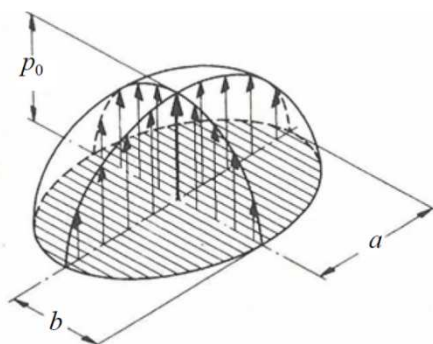


Figure 5. Distribution of surface stress in elliptical contact area – point contact [7].

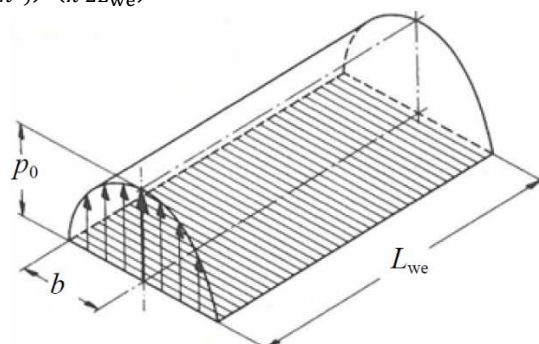


Figure 6. Distribution of surface stress in rectangular contact area – line contact [7].

Stress analysis according to Hertz only covers inspection of surface stress caused by concentrated load normal to the surface. But experimental findings prove that fatigue failure of rolling bearings material is mediated mainly by subsurface stress [7]. Therefore, determination of the subsurface stresses is important. The following basic points result from the subsurface stress analysis of point contact according to reference [7]:

- Maximal normal stress $\sigma_{\max}$ is oriented in the direction of acting load.
- This maximal normal stress $\sigma_{\max}$ is equal to maximal contact stress p_0 .
- Maximal shear stress $\tau_{\max}$ is located in the symmetry plane y-z and the value is approximately $\tau_{\max} \approx 0.3 \sigma_{\max}$.
- Orthogonal shear stress τ_{yz} comprise of two same maximal values, but one of them is positive and the other negative. The value is around $\tau_{yz} \approx \pm 0.25 \sigma_{\max}$ and they are in depth approximately $z_{\tau_{yz}} \approx 0.5b$ under the surface of contact and in the distance from symmetry plane approximately $y \approx 0.9b$.

In accordance with reference [7], fatigue phenomenon directly refers to the amplitude of stress. ISO and AFBMA suggest to use maximal orthogonal shear stress τ_{yz} as the fatigue contact criterion.

4. Lundberg and Palmgren model

The model is the basic theoretical formulation of rolling bearings fatigue life. Lundberg and Palmgren adapted Weibull statistical strength theory. They supposed that the depth in which is located the critical shear stress influences a crack initiation and fatigue life. Orthogonal shear stress τ_{yz} was designated as critical shear stress. They stated the relation for bearing rings subjected to the cyclic load, which determines the probability S that N cycles will not occur failure of the raceway [8].

$$\ln \frac{1}{S} = A \left(\frac{N^e \tau_{yz}^c V}{z_{\tau_{yz}}^h} \right) \quad (22)$$

Where τ_{yz} is maximal orthogonal shear stress, $z_{\tau_{yz}}$ is corresponding depth in which this stress acts. Parameters A , c and h are material characteristics obtained experimentally. Parameter e is Weibull slope. Stressed volume of material V is defined as [3]:

$$V = a l_L z_{\tau_{yz}} \quad (23)$$

$$V = L_{we} l_L z_{\tau_{yz}} \quad (24)$$

If considering point contact, a is length of the semi-major axis of contact ellipse. In the case of line contact L_{we} is effective length of roller element. And l_L is the length of the raceway.

$$l_L = \pi(D_{pw} \pm D_{we} \cos \alpha) \quad (25)$$

The equation is universal for calculation of raceway length l_L of either bearing rings (inner and outer). D_{pw} denotes pitch diameter of rolling elements and D_{we} is diameter of rolling element.

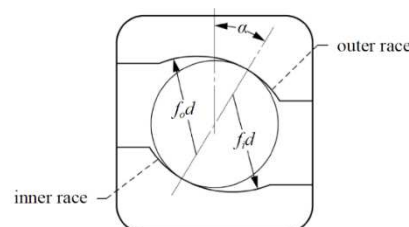


Figure 7. Contact angle α of angular contact ball bearing [3].

The number of revolutions N over which will not occur defect on the raceway, in contact of rolling element and raceway, with 90% probability is expressed from the equation (22).

$$N = L_{10_o} = L_{10_i} = \left(\frac{z_{\tau yz}^h \ln \frac{1}{0.9}}{A \tau_{yz}^c V} \right)^{\frac{1}{e}} \quad (26)$$

Whereas a bearing is a system compounded of several components, and each of them has a different life, the total basic rating life is defined as:

$$\frac{1}{L_{10}^e} = \frac{1}{L_{10_i}^e} + \frac{1}{L_{10_o}^e} \quad (27)$$

5. ISO 281

Following relations was derived by Lundberg and Palmgren too and was incorporated into the standard ISO281:2007. It is a standardized guide for calculation of bearing life. The next equation was obtained by substituting for τ_{yz} , $z_{\tau yz}$, and V in terms of the bearing dimensions and contact load in equation (22) [8].

$$L_{10} = \left(\frac{C}{P} \right)^p \quad (28)$$

Where L_{10} is basic rating life in million revolutions, C is basic dynamic radial or axial load rating, P is dynamic equivalent load. Exponent $p = 3$ for point contact and $p = 10/3$ for modified line contact. Because the paper deals only with a fraction of rolling bearings, just necessary equations are mentioned. Angular contact ball bearings and tapered bearings are normally subjected to radial and axial load simultaneously. In this situation, the dynamic equivalent radial load P_r has to be determined [6]:

$$P_r = X F_r + Y F_a \quad (29)$$

Where F_r and F_a are acting radial and axial load respectively, X is dynamic radial load factor, Y is dynamic axial load factor, and both of them are stated in tables of the standard. Basic dynamic radial load rating C_r of angular contact ball bearings is defined as [6]:

$$C_r = 1.3 f_c (i \cos \alpha)^{0.7} z^{\frac{2}{3}} D_w^{1.8} \quad \text{for } D_w \leq 25.4 \quad (30)$$

$$C_r = 4.7411 f_c (i \cos \alpha)^{0.7} z^{\frac{2}{3}} D_w^{1.4} \quad \text{for } D_w > 25.4 \quad (31)$$

and tapered bearings [6]:

$$C_r = 1.1 f_c (i L_{we} \cos \alpha)^{\frac{7}{9} \frac{3}{4}} D_{we}^{\frac{29}{27}} \quad (32)$$

Where coefficient f_c is tabular value and is dependent on bearing geometry, the accuracy of its components and material. α is nominal contact angle, i number of rows, z number of rolling elements, D_w nominal diameter of ball, D_{we} and L_{we} are the calculating roller diameter and its effective length.

6. Optimization algorithm

The optimization problem is to find the optimal total axial preload of a pair of angular contact bearings or tapered bearings oriented into X or O arrangement, for the purpose of rolling system life maximization. Lundberg and Palmgren fatigue life model is stated as optimization function. And design variable is axial total preload, on which depends internal diametral clearance. The inputs of algorithm involve required bearing geometrical parameters, material constants and duty cycles.

The algorithm was developed and encoded in the environment Matlab of company MathWorks. In the Figure 8, the shown diagram schematically represents the sequence of individual algorithm steps. Running this code, the program reads the entry data from the external files, calculates additional parameters and creates the total preload range of δ_a to be explored. The range is divided by the increment into m portions. In the diagram, the outer rectangle is then a cycle that is repeated m times, successively for each δ_{ai} ($i = 1, 2, 3, \dots, m$).

In the first step, the distribution of the axial preload δ_{ai} on bearings A and B is calculated using the axial stiffness of the individual bearings. Then, according to the equation (10), the respective loads $F_{\delta_{ai}}$ caused by these preloads are determined. In the step number 2, a range of axial loads according to

the duty cycles and the forces of the preload is created for each bearing, considering both directions of the axial loads.

Then the algorithm enters into another nested cycle which repeats itself l times. Where l represents the number of duty cycles. In the diagram, it is depicted as an inner rectangle. In step 2.1., the actual axial load $F_{aai k}$ and the actual axial deflection $\delta_{aai k}$ for each bearing are determined ($k = 1, 2, 3, \dots, l$). If the axial load of k duty cycle $F_{ak} = 0$, only the forces from the initial preload δ_{ai} act on the bearings in this direction. If $F_{ak} > 0$ or $F_{ak} < 0$ then the values of the actual axial forces $F_{aai k}$ are obtained by interpolation from the load range of step number 2, and their respective deflections $\delta_{aai k}$ are calculated.

In the next step, the actual axial bearing deflection $\delta_{aai k}$ is transformed by the following equation for calculating the internal diametrical clearance G_{rik} of each bearing A and B:

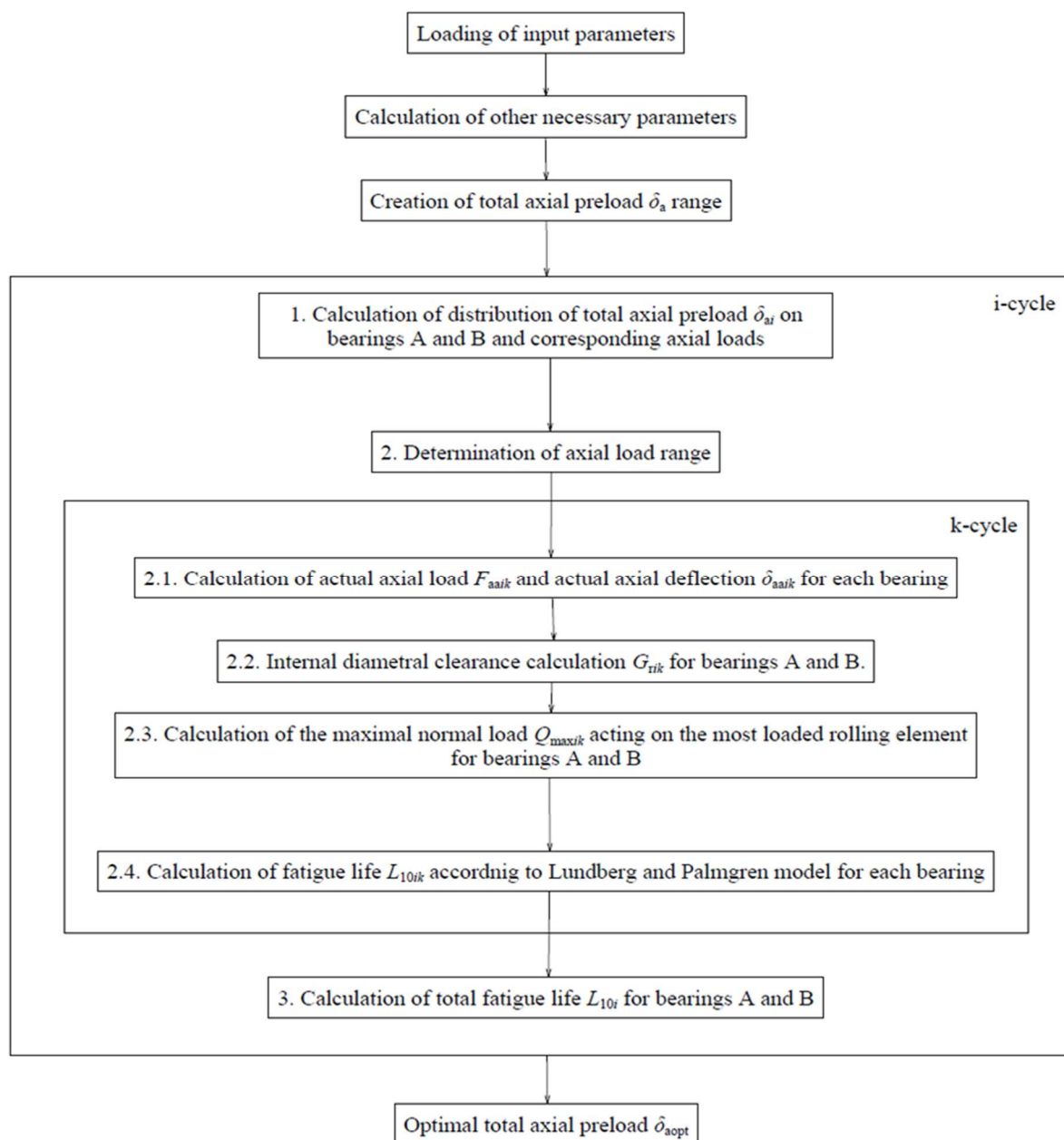


Figure 8. Optimization algorithm diagram.

$$G_{rik} = -\delta_{aalk} \tan \alpha \quad (33)$$

In step 2.3. the calculation of load on the most loaded rolling element is based on Stribeck's theory. To obtain the Stribeck number, it uses the load-distribution integral $J_r(\varepsilon)$. For each calculation of $J_r(\varepsilon)$, it is necessary to use the separate numerical method described in the reference (Zaretsky). Generally, this method starts with an estimate of load distribution factor ε , then the functional value of $J_r(\varepsilon)$ is obtained and is substituted, along with G_r value, in the equation (2). The obtained values are used in the equation (3) to calculate a new ε value. This repeats until the accuracy condition or the maximum number of iterations is satisfied. The output is $J_{rik}(\varepsilon_{ik})$ which is used to calculate the most loaded rolling element according to the relation (20), where the transformation to the normal load is applied:

$$Q_{maxik} = \frac{F_{rk}}{z J_{rik}(\varepsilon_{ik}) \cos \alpha} \quad (34)$$

Step 2.4. consists of calculations from the section of the surface and subsurface stress theory. The maximum contact stress and contact area parameters are calculated according to the equations (16 - 21) for the contact of the element with the outer and inner rolling raceways for each bearing. Then, for each contact, the fatigue life L_{10ik} and L_{10oik} are calculated according to the relation (26). And the total life L_{10ik} of bearings A and B according to equation (27). This nested cycle terminates when $k = l$.

In the step number 3, the total fatigue life L_{10ik} of both bearings for each load stage is converted to L_{10i} . It corresponds to the life of these bearings for the entire duty conditions with the given axial preload δ_{ai} . From this step, the algorithm returns to step 1 and the cycle ends when $i = m$. The output of this cycle is a matrix with the dimensions $m - by - 3$, where the first column contains δ_a values, the second bearing life L_{10A} of the bearing A and the last column are the values L_{10B} of the bearing B.

The final step of the whole algorithm is to find the maximum bearing life of bearings A and B. From these two values, the minimum value of life is selected, and the corresponding axial preload is then the optimal preload δ_{aopt} of the whole rolling construction system.

7. Comparison of L-P model and ISO281

A particular rolling construction system was selected where tapered roller bearings of type 33209 and 33207 were mounted in X arrangement. This system had defined certain operating modes. Subsequently, the calculation was performed using the above-mentioned algorithm, which uses the L-

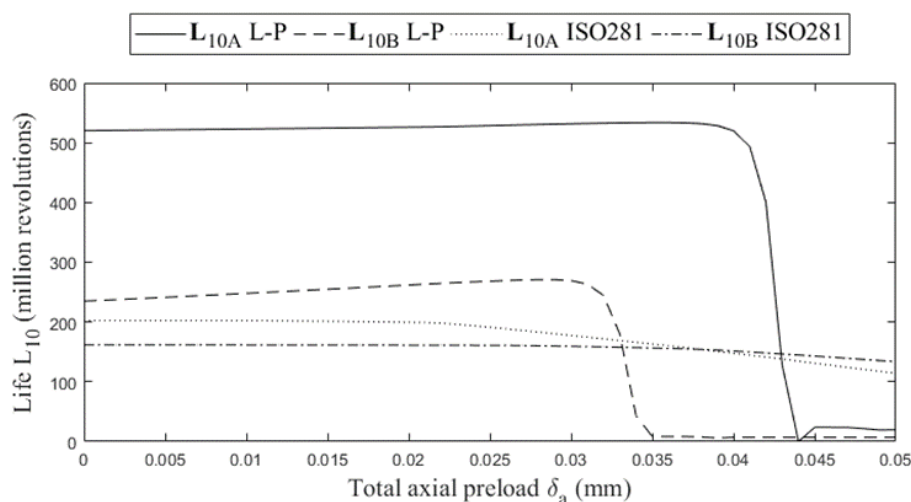


Figure 9. Dependence of life L_{10} and total axial preload δ_a of two bearings mounted in X arrangement, obtained by two different methods, namely Lundberg and Palmgren method and ISO281.

P model to obtain $L_{10A/B}$ bearing life. From the calculated values, the dependence of bearing life $L_{10A/B}$ and axial preload δ_a was established for the bearing A and B. Then, the same rolling system was calculated according to the ISO 281 standard, and the same dependence was obtained from the calculated values. This dependence and comparison of the two procedures are shown in the Figure 9. As can be seen from the line graph, according to the $L_{10A/B}$ curve of ISO:281, an optimal preload cannot be determined, as its trend is approximately horizontal and then begins to decrease. It has no response to axial preload change. However, for curves obtained using the L-P model, this value can be determined. And in this particular case, the optimal total axial preload $\delta_{a_{opt}} = 0.028mm$.

8. Conclusion

The paper summarizes the basic theory that was necessary to create the optimization algorithm programmed in the Matlab environment. This algorithm search for an optimal axial preload of a rolling bearings system, that consists of two angular contact ball bearings or tapered bearings mounted in X or O arrangement, to maximize bearing life. The Lundberg and Palmgren model is used to evaluate rolling bearing life. This model was based on the Stribeck's radial load-distribution on the rolling elements and the theory of surface and subsurface stress. At the end of the paper are described the individual steps of the algorithm. Also, the particular example is demonstrated, where the bearing life was calculated by Lundberg and Palmgren method and the method presented in ISO281:2007. The dependence on bearing life and axial bearing preload was depicted graphically. And from the data in this graph, it was estimated that it is not possible to determine the optimal axial bearing preload using the ISO281 standardized procedure. On the other hand, using the Lundberg and Palmgren model, it is obvious that the axial bearing preload influences bearing fatigue life and can be determined the optimal preload value. Therefore, this model was selected in the optimization algorithm.

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