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Vibration and Mode Shape Analysis of Functionally Graded Nanocomposite Plates Reinforced by Aggregated Carbon Nanotube

Vahid Tahouneh
Young Researchers and Elite Club, Islamshahr Branch, Islamic Azad University, Islamshahr, Iran.

E-mail: vahid.th1982@gmail.com/vahid.tahouneh@ut.ac.ir

Abstract. In the present work, by considering the agglomeration effect of single-walled carbon nanotubes, free vibration characteristics of functionally graded (FG) nanocomposite plates is presented. The volume fractions of randomly oriented agglomerated single-walled carbon nanotubes (SWCNTs) are assumed to be graded in the thickness direction. To determine the effect of CNT agglomeration on the elastic properties of CNT-reinforced composites, a two-parameter micromechanical model of agglomeration is employed. The effects of geometrical and material parameters on the frequency parameters of the FG nanocomposite plates are investigated. For an overall comprehension on 3-D vibration of rectangular plates, some mode shape contour plots are reported in this research work.

1. Introduction

Nowadays, the use of carbon nanotubes in polymer/carbon nanotube composites has attracted wide attention [1]. A high aspect ratio, low weight of CNTs and their extraordinary mechanical properties (strength and flexibility) provide the ultimate reinforcement for the next generation of extremely lightweight but highly elastic and very strong advanced composite materials. On the other hand, by using of the polymer/CNT composites in advanced multilayered composite materials (sandwich structures) we can achieve structures with low weight, high strength and high stiffness in many structures of civil, mechanical and space engineering. Yas and Tahouneh [2] investigated the free vibration analysis of thick FG annular plates on elastic foundations via differential quadrature method based on the three-dimensional elasticity theory. Tahouneh et al. [3] studied free vibration characteristics of continuous grading fiber reinforced (CGFR) annular plates resting on elastic foundations using DQM. Tahouneh [4] studied the free vibration analysis of thick nanocomposite plates. The author used equivalent continuum model to estimate the material properties of the structure. Tahouneh and Naei [5] used 2D differential quadrature to investigate free vibration and vibrational displacements of elastically supported laminated curved panels with power-law distribution FG layers and finite length. The same authors [6] have considered a 2-D six-parameter power-law distribution for three-dimensional dynamic analysis of thick multi-directional functionally graded rectangular plates with two edges simply supported while the other two edges assumed to be free, clamped and simply supported. Cheng and Batra [7] have studied the buckling and steady state vibrations of a simply supported functionally graded polygonal plate based on Reddy's plate theory. Zenkour [8] presented a two dimensional solution to study the bending, buckling and free vibration of simply supported FG ceramic–metal sandwich plates. Cheng and Batra [9] used first and third order shear deformation theories to relate deflections of a simply supported functionally graded polygonal plate. Reddy [10] presented static and dynamic analysis of the FGM plates based on third order shear deformation theory and by using the theoretical formulation and finite element models. Matsunaga [11] studied vibration analysis of FG plates using 2D higher-order deformation theory. In this work, three-dimensional elasticity theory is used to investigate the free vibration analysis of simply supported functionally graded nanocomposite plates reinforced by aggregated SWCNTs. The applied nanocomposite is assumed a mixture of CNTs (that are randomly oriented and locally aggregated into some clusters) that are embedded in a polymer. The material properties of the nanocomposites plates are assumed to vary along the thickness of plate and estimated through the Mori-Tanaka method. The effects of CNT volume fraction, CNT distribution, cluster distribution and geometric dimensions of plate are investigated on the natural frequency of the FG-CNTRC plates.

2. Material Properties in FG-CNTRC Reinforced Composite

2.1. Completely Randomly Oriented CNTs

Consider a CNTRC is made from a mixture of SWCNT (that randomly oriented and locally aggregated into some clusters) and matrix which is assumed to be isotropic. Many studies have been published each with a different focus on mechanical properties of polymer nanotube composites. However, the common theme seems to have been enhancement of Young's modulus. In this section, the effective mechanical properties of the CNT reinforced composite that straight CNTs are oriented randomly or local aggregated in to some clusters are obtained based on the Eshelby-Mori-Tanaka approach. The resulting effective properties for these CNT reinforced composite are isotropic, despite the CNTs are transversely isotropic.When CNTs are completely randomly oriented in the matrix, the composite is then isotropic, and its bulk modulus K and shear modulus G are derived as [12]:

$$K = K_m + \frac{f_r(\delta_r - 3K_m\alpha_r)}{3(f_m + f_r\alpha_r)}, G = G_m + \frac{f_r(\eta_r - 2G_m\beta_r)}{2(f_m + f_r\beta_r)} \tag{1}$$

where subscripts "m" and "r" are referred to matrix and CNT respectively, "f" is volume fraction, K_m and G_m are the bulk and shear moduli of the matrix, respectively. Parameters, δ_r, α_r, η_r and β_r have kind of relation with Hill's elastic moduli for the reinforcing phase (CNTs), [12]. The effective Young's modulus E and Poisson's ratio of the composite is given by:

$$E = \frac{9KG}{3K+G}, \upsilon = \frac{3K-2G}{2(3K+G)} \tag{2}$$

2.2. Effect of CNT Aggregation on the Properties of the Composite

The CNTs were arranged within the matrix in such manner so as to introduce clustering. It has been observed that, due to large aspect ratio (usually >1000), low bending rigidity of CNTs and Van der Waals forces, CNTs have a tendency to bundle or cluster together making it quite difficult to produce fully-dispersed CNT reinforced composites. The effect of nanotube aggregation on the elastic properties of randomly oriented CNTRC is presented in this section. Shi et al. [12] derived a two parameter micromechanics model to determine the effect of nanotube agglomeration on the elastic properties of randomly oriented CNTRC (Fig. 1). It is assumed that a number of CNTs are uniformly distributed throughout the matrix and that other CNTs appear in cluster form because of aggregation, as shown in Fig. 1. The total volume of the CNTs in the representative volume element (RVE), denote by V_r, can be divided into the following two parts:

$$V_r = V_r^{cluster} + V_r^m \tag{3}$$

The two parameters used to describe the aggregation are defined as:

$$\mu = \frac{Vcluster}{V}, \eta = \frac{V_r^{cluster}}{V_r} \quad 0 \le \eta, \mu \le 1 \tag{4}$$

where V is volume of RVE, $V_{cluster}$ is volume of clusters in the RVE. μ is volume fraction of clusters with respect to the total volume V of the RVE, η is volume ratio of the CNTs inside the clusters over the total CNT inside the RVE.
When, $\mu = 1$ this means uniformly distribution of nanotubes throughout the entire composite without aggregation, and with the decreasing in μ ,the agglomeration degree of CNTs is more severe. When $\eta = 1$, all the nanotubes are located in the clusters. The case $\mu = \eta$ means that the volume fraction of CNTs inside the clusters is as same as that of CNTs outside the clusters so all CNTs are located randomly oriented. Thus, we consider the CNT-reinforced composite as a system consisting of clusters of sphere shape embedded in a matrix. We may first estimate respectively the effective elastic stiffness of the clusters and the matrix, and then calculate the overall property of the whole composite system. The effective bulk modulus K and shear modulus G of the cluster can be calculated by Prylutskyy et al. [13], respectively. The effective Young's modulus E and Poisson's ratio υ of the composite can be calculated in the terms of K and G by Eq. (2).

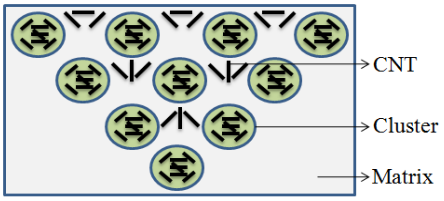

Figure 1. RVE with functionally graded Eshelby cluster model of aggregation of CNTs.

2.3. CNT Volume Fraction Distribution

Consider a rectangular FG-CNTRC plate with thickness of *h*, and the edges are parallel to axes *x* and *y*, as shown in Fig. 2. The volume fractions of CNTs, f_r, are varied along the thickness of plate as the following:

$$f_r = (f_r^u - f_r^d)(1 + z/h)^p + f_r^d, -h/2 < z < h/2 \tag{5}$$

$$f_m = 1 - f_r \tag{6}$$

where p $(0 \le p < \infty)$ is the volume fraction exponent and f_r^u and f_r^d are the values of CNT volume fraction at upper (*z=h/2*) and downer surfaces (*z=-h/2*), respectively. Fig. 3 shows the variation of CNT volume fraction along the thickness of plate for different values of the volume fraction exponent, *p*. Therefore the effective Young's modulus *E* and Poisson's ratio υ are obtained from Eq. (2).

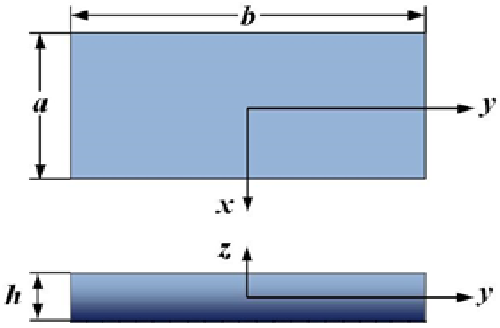

Figure 2. Schematic of the CNTRC plate.

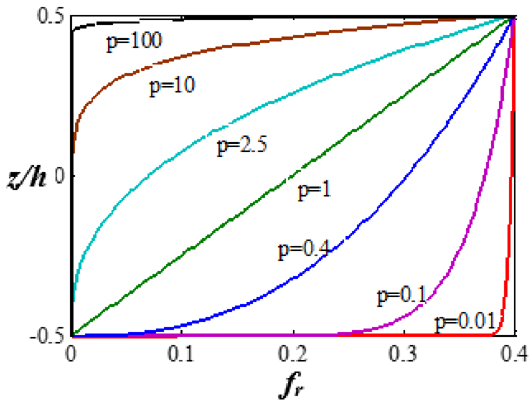

Figure 3. Variation of properties along the thickness of cylinders for different values of p according to Eq. (5)

2.4. Governing Equations

The mechanical constitutive relations that relate the stresses to the strains are as follows:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{7}$$

where λ and μ are the Lame constants, ε_{ij} is the infinitesimal strain tensor and δ_{ij} is the Kronecker delta. The infinitesimal strain tensor is related to the displacements as follows:

$$\begin{aligned} &\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}, \\ &\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{aligned} \tag{8}$$

where u, v and w, are displacement components along the x, y and z axes, respectively. Upon substitution (8) into (7) and then into the governing equations, the equations of motion are obtained in terms of displacement components. The related boundary conditions at z=-h/2 and h/2 are as follows:

$$\sigma_{zx} = \sigma_{zy} = \sigma_{zz} = 0 \tag{9}$$

Using the method of separation of variables, it is possible to seek solutions that are harmonic in time. The displacements can be written as follows:

$$\begin{aligned} &u = \sum_m^\infty \sum_n^\infty U(z) \cos(\beta_m x) \sin(p_n y) e^{i\omega t}, \\ &v = \sum_m^\infty \sum_n^\infty V(z) \sin(\beta_m x) \cos(p_n y) e^{i\omega t}, \\ &w = \sum_m^\infty \sum_n^\infty W(z) \sin(\beta_m x) \sin(p_n y) e^{i\omega t}, \\ &\beta_m = m\pi/a, p_n = n\pi/b, (m,n = 1,2,...) \end{aligned} \tag{10}$$

ω is the natural frequency and i(= $\sqrt{-1}$) is the imaginary number. Substituting for displacement components from (10) into the equations of motion which obtained in terms of displacement components, the coupled partial differential equations are reduced to a set of coupled ordinary differential equations (ODE). The geometrical and natural boundary can also be simplified, however, for brevity purpose they are not shown here. The differential quadrature method can be applied to discretize the equations of motion and the boundary conditions. One can find out how to use differential quadrature for discretizing the equations in [2-4] .The discretized form of the equations of motion and the related boundary conditions can be represented in the matrix form, and through solving an eigenvalue system of equations the natural frequencies and mode shapes of the plate can be calculated.

3. Numerical Results and Discussion

In this stage simply supported FG-CNTRC square plates are considered. In these plates the volume fraction of randomly oriented CNT, f_r, varies from zero to 0.4 according to Eq. (5) along the thickness of plate. Fig. 4 shows the first natural frequency parameters, Ω_{11}, that are calculated by the following equation based on the mechanical properties of CNT for various values of p and b/h.

$$\Omega = \omega b/h\sqrt{\rho^{CNT}/E^{CNT}} \tag{11}$$

This figure shows that increasing the ratio of width to thickness plates, b/h, or decreasing of the volume fraction exponent, p, can lead to increase Ω_{11}. Fig. 5 illustrates variation of Ω_{11} versus μ for various values of η in UD-CNTRC square plate with a/h=10 and f_r=0.4. It can be seen that frequency parameter increases as the μ increases or especially η decreases. Also, Fig. 6 shows the first natural frequency of the same plates with η=1 and various values of μ and a/h. This figure shows that frequency parameters are increased by increasing of μ or decreasing of the ratio of length to thickness, a/h. Table 1 shows various modes of frequency parameters, Ω_{11}, Ω_{12}, Ω_{22}, Ω_{13} for various values of p. it can be concluded that aggregation of CNTs deeply decreases frequency parameters in all modes. In the all of the above FG-CNTRC plates, volume fraction of CNT was changed but FG-CNTRC plates can be made by changing of the volume fraction of clusters. Consider CNTRC square plates with a/h=10, f_r =0.2 and η=1. Volume fraction of clusters of CNTs, μ, varies from zero to 0.4 according to Eq. (5) along the thickness of plate. Table 2 shows various modes of frequency parameters, Ω_{11}, Ω_{12}, Ω_{22}, Ω_{13}, for various values of volume fraction exponent of clusters, p. This table discloses that increasing of p decreases frequency parameters in all modes. Finally, considers a UD-CNTRC square plate with a/h=10, f_r =0.2, η=1 and μ=0.5. Fig. 7 shows mode shapes of the plate at mode numbers of (1,1), (2,1), (1,2) and (3,1).

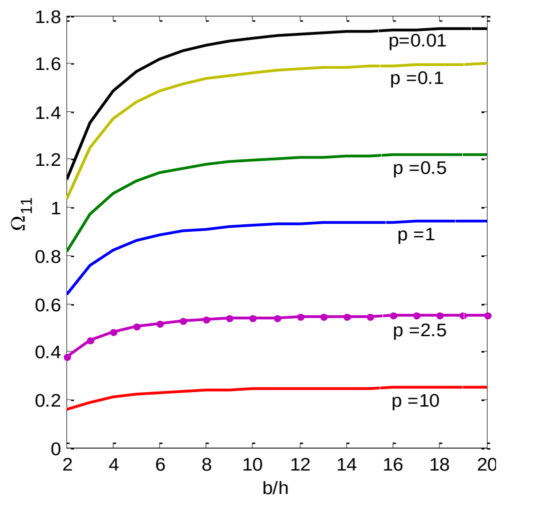

Figure 4. First frequency parameters versus b/h for FG-CNTRC square plates with fully dispersed CNT and f_r =0→0.4.

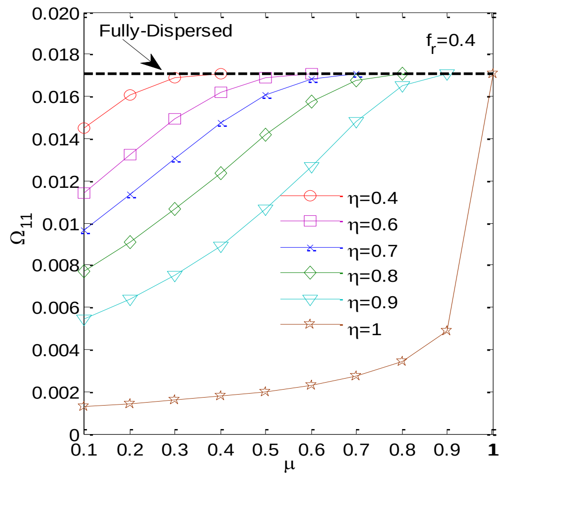

Figure 5. Variation of properties along the thickness of cylinders for different values of p according to Eq. (5)

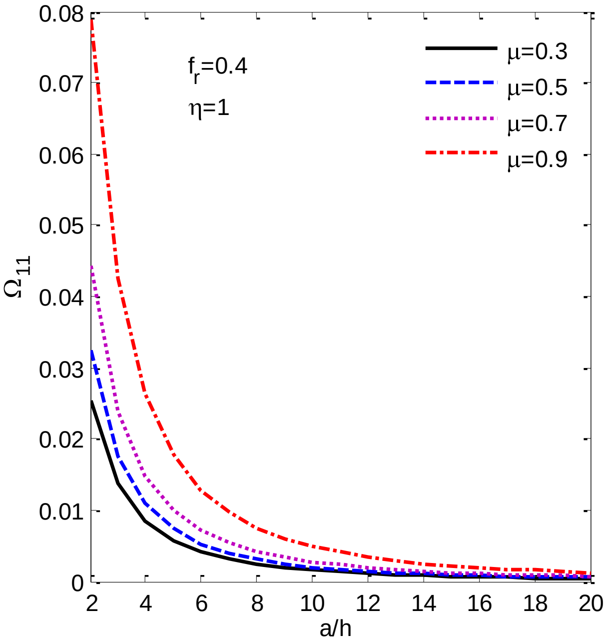

Figure 6. First frequency parameters versus *a/h* for UD-CNTRC square plates with aggregated CNT and f_r =0.4.

4. Conclusion

In this research work, generalized differential quadrature method is employed to obtain a highly accurate semi-analytical solution for free vibration of functionally graded (FG) nanocomposite rectangular plates. The study is carried out based on the three-dimensional, linear and small strain elasticity theory. The Mori-Tanaka approach is implemented to estimate the effective material properties of the nanocomposite plate.

The agglomeration effect of single-walled carbon nanotubes, is considered in this study and it is shown that the natural frequencies of structure are seriously affected by the influence of CNTs agglomeration. Results presented the fact that mechanical properties and therefore free vibrations of plates are seriously affected by CNTs agglomeration.

The effects of various geometrical parameters, different material profiles along the thickness of rectangular plates are investigated. Results show that by increasing the ratio of width to thickness plates, *b/h*, or decreasing of the volume fraction exponent, *p*, the non-dimensional frequency of the plate increases. It can be concluded that aggregation of CNTs deeply decreases frequency parameters in all modes. It is observed that increasing of p decreases frequency parameters in all modes. It can be seen that frequency parameter increases as the μ increases or especially η decreases.

Results reveal that natural frequency of the same plates with η=1 and various values of μ and *a*/*h*. It shows that frequency parameters are increased by increasing of μ or decreasing of the ratio of length to thickness, *a*/*h*.

Table 1. Frequency Parameters of FG-CNTRC Square Plates with a/h =10, μ=0.5, η=1 and f_r =0→0.4.

(m,n)	p=0.01	p=0.1	p=0.4	p=1	p=2.5	p=10	p=100
(1,1)	0.2002	0.2008	0.2016	0.1986	0.1774	0.1520	0.1327
(1,2)	0.4778	0.4790	0.4811	0.4743	0.4225	0.3612	03143
(2,2)	0.7332	0.7352	0.7385	0.7285	0.6559	0.5528	0.4796
(1,3)	0.8935	0.8959	0.9000	0.8882	0.8013	0.6725	0.5826

Table 2. Frequency Parameters of UD-CNTRC Square Plates with aggregated CNT, a/h=10, f_r =0.2, η=1 and μ=0→0.4.

(m,n)	p=0.01	p=0.1	p=0.4	p=1	p=2.5	p=10	p=100
(1,1)	0.1791	0.1714	0.1569	0.1455	0.1376	0.1290	0.1203
(1,2)	0.4271	0.4091	0.3746	0.3468	0.3268	0.3058	0.2863
(2,2)	0.6554	0.6281	0.5753	0.5318	0.4995	0.4668	0.4385
(1,3)	0.7986	0.7655	0.7013	0.6478	0.6073	0.5672	0.5336

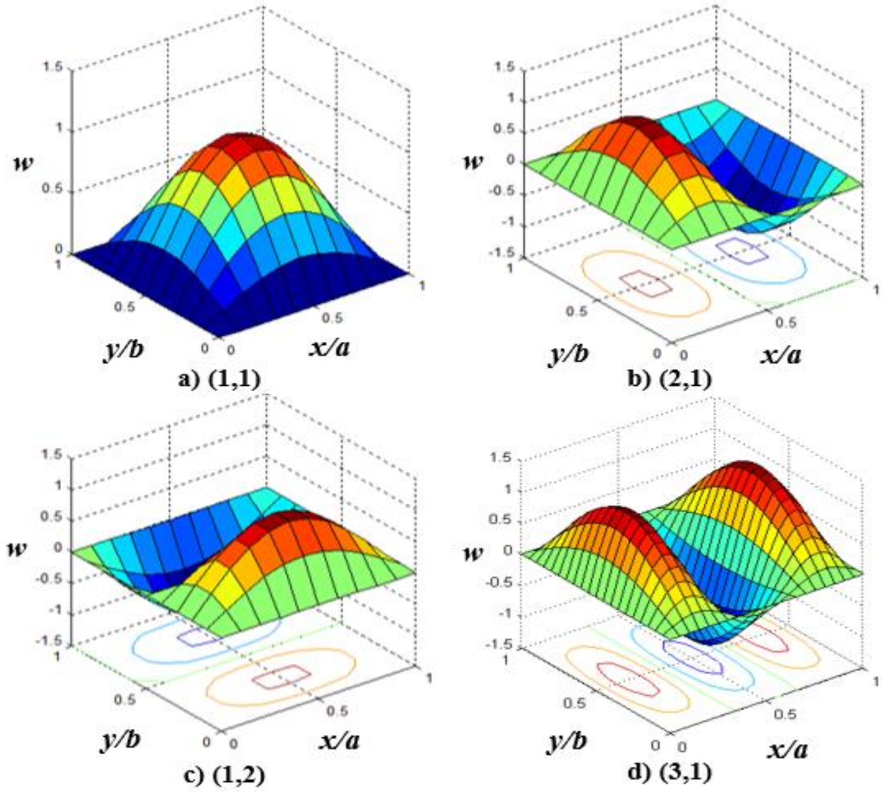

Figure 7. Different mode shapes of the plate.

5. References

[1] Wagner H D, Lourie O, and Y. Feldman 1997 *Appl. Phys. Lett.* **72** 188-190.

[2] Yas M H and Tahouneh V 2012 *Acta Mech.* **223** 43-62.

[3] Tahouneh V, Yas M H, Tourang H and Kabirian M 2013 *Meccanica* **48** 1313-1336.

[4] Tahouneh V 2016 *Steel and Composite Structures, An Internatinal Journal* **20** 623-649.

[5] Tahouneh V and Naei M H 2015 *J. Sandw. Struct. Mater*. **18**, 263-293.

[6] Tahouneh V and Naei M H 2013 *Meccanica* **49** 91-109.

[7] Cheng Z Q and Batra R C 2000 *J. Sound Vib.* **229** 879-895.

[8] Zenkour A M 2005 *Inter. J. Solid Struc*. **42** 5243-5258.

[9] Cheng Z Q and Batra R C 2000 *Archive Mech.* **52** 143-158.

[10] Reddy J N 2000 *Int. J. Numerical Methods Eng.* **47** 663-684.

[11] Matsunaga H 2008 *Compos. Struct.* **82** 499-512.

[12] Shi D L, Feng X Q, Yonggang Y H, Hwang K C and Gao H 2004 *J. Eng. Mater. and Technol.* **126** 250-257.

[13] Prylutskyy Y I, Durov S S, Ogloblya O V, Buzaneva E V and Scharff P 2000 *Comput. Mater. Sci*. **17** 352-355.