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Anti-buckling design of variable stiffness composite cylinder under combined loading based on the multi-objective optimization method

Y Zheng[1,2,3] and J Chen[1,2,3]

[1]Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China
[2]Hubei Key Laboratory for Engineering Structural Analysis and Safety Assessment, Wuhan 430074, China

E-mail: zyc276565153@126.com/ jpchen@hust.edu.cn.

Abstract. Variable stiffness composite structures take full advantages of composite's design ability. An enlarged design space will make the structure's performance more excellent. Through an optimal design of a variable stiffness cylinder, the buckling capacity of the cylinder will be increased as compared with its constant stiffness counterpart. In this paper, variable stiffness composite cylinders sustaining combined loadings are considered, and the optimization is conducted based on the multi-objective optimization method. The results indicate that variable stiffness cylinder's loading capacity is increased significantly as compared with the constant stiffness, especially when an inhomogeneous loading is considered.

1. Introduction

Due to the excellent mechanical property and lightweight feature, composites are widely used in different engineering fields, such as the offshore platform in the ocean, the pipeline in deep sea, the pressure vessels in industry, the rocket launchers and space vehicles, and the aircraft cabin and wings, etc. Their large design freedom provides large design potential. In order to take full advantages of composite structures, composite structures can be continuous tailoring and with variable thickness or fiber angles, different stacking sequences, resulting in the variable stiffness (VS) composite structures. The design of VS composite structure has attracted many attentions in recent years [1-4]. Having been reported since the late 1980s, it is shown that automated fiber placement (AFP) technology can not only reduce labor costs and scrap materials, but also improve its quality and performance [5-10]. Continuously variable fiber orientations are steered by AFP machines, resulting in VS composite structures and giving the designers large designability to extend the design space and fully exploit the directional properties of composite materials [11]. As a result, VS design can offer a significantly improved performance compared with constant stiffness (CS) counterparts at the same weight.

In 1994, Hyer *et al* [12] optimized the fiber path of a flat plate by reducing the variation of the angle between adjacent elements. The laminated plate was fabricated and the buckling loading was verified by experiments. Tatting [13] is the first one who studied design optimization of VS composite cylinders for buckling. Rouhi [14-16] *et al* studied the optimization of maximum buckling load of composite cylinders subject to bending moment, the buckling behavior of a VS composite cylinder under axial compression and a test about VS composite has been done. These studies only considered buckling of structures under single loading.

There have been some studies conducted for multi-objective optimization of laminated composite structures. For example, Lee *et al* [17] presented a work which aimed to minimize the weight of multilayered composite plates and minimize their maximum displacement. The design variables include the type of fiber, thickness and the fiber orientations of each layer. Vosoughi and Nikoo [18] developed a hybrid method for maximizing the fundamental natural frequency and thermal buckling temperature of laminated composite plates.

The kind of combined loadings is the common situation in engineering, which should be involved in the optimization of a composite cylinder. In general, the VS composite shell structures are prone to buckling and their anti-buckling ability is crucial. In this paper, two kinds of combined loading, i.e., the bending and external pressure, and the bending and torque are considered respectively for the optimal design of VS composite cylinders. Multi-objective particle swarm optimization is utilized for obtaining the final Pareto-optimal solutions, which is the optimal compromise between the two loadings.

2. Design of a VS composite cylinder

The common composite material cylinder is the filament winding composite cylinder in which fiber is wrapped in a certain winding angle to make the stiffness constant along the cylindrical surface. The AFP technique can be used for manufacturing VS composite structures, leading to the increase of interest on the optimal design of composite structures with VS. VS composite structures can be formed in three ways: the curved fiber ply, the non-even thickness ply and the ply with different fiber volume fractions. The AFP technique changes the fiber-laying of composite structures, through the curve lay, thus greatly influencing the material's properties. The following figure shows a VS composite cylinder.

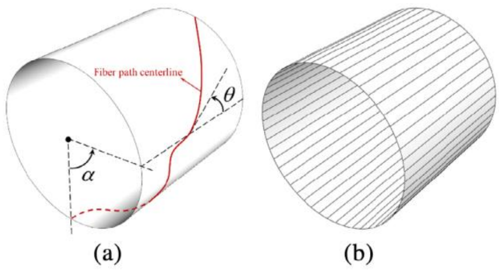


Figure 1. A VS composite cylinder, (a) Curve fiber path and (b) The cylinder is divided into narrow strips.

Figure 1(a) schematically shows the curve fiber path. Fiber angle is changing along the circumferential direction while maintaining the same along the axial direction. The cylinder structure is formed by finding the fiber angle in each narrow strip as shown in figure 1(b). In order to describe the variation of the orientation angle, 4 steps from the keel to the crown of the cylinder is defined. Each segment is divided into M narrow bands in the finite element model. All elements in one narrow band possess the same orientation angle in a ply. The symmetry about the vertical axis is assumed so that five design variables per VS ply: φ_1, φ_2, φ_3, φ_4 and φ_5, should be considered for 4 segments, as shown in figure 2. Interpolation between five design variables approaches with satisfaction. More steps will take more computational cost and the results will remain the same. In each segment, the orientation angle of the fiber paths are linear variation along the circumferential direction:

$$\theta_{i,k} = \varphi_i + \frac{\varphi_{i+1} - \varphi_i}{\alpha_{i+1} - \alpha_i}(\alpha_k - \alpha_i) \quad i = 1,2,3,4 \quad k = 1,2,\cdots,M \,. \tag{1}$$

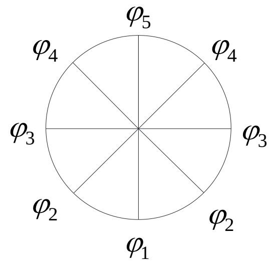


Figure 2. Circular cylindrical design nodes.

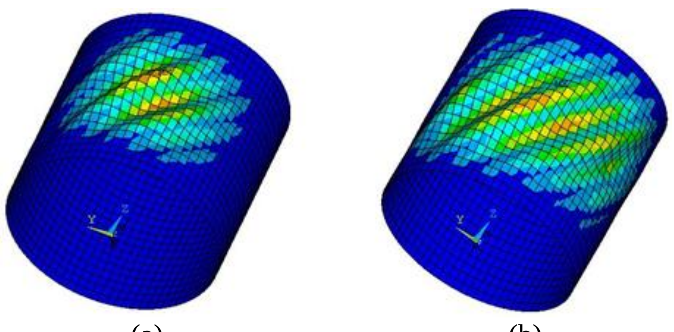
Where $[\alpha_i, \alpha_{i+1}]$ denotes the lower boundary and the upper boundary of the *i*-th segment, $\theta_{i,k}$ is the fiber orientation angle in the *k*-th narrow band, α_k is the circumferential angle of the *k*-th narrow band, φ_i is the fiber orientation angle of the *i*-th segment lower boundary, and φ_{i+1} is the upper boundary.

A composite cylinder is studied in this section whose diameter is 0.457 m and length is 0.457 m. Sixteen layers are arranged in the form of $[0°/+\theta/90°/-\theta/-\theta/90°/+\theta/0°]_s$, where $\theta \in [0°, 90°]$ is the ply angle that varies in the circumferential direction. The thickness of each layer is 0.127 mm. The plies are made of AS4D/9310 carbon/epoxy materials for which the material properties are given in table 1.

Table 1. AS4D/9310 Material properties of carbon fiber-epoxy material.

Property	AS4D/9310
E_1 (GPa)	134
E_2 (GPa)	7.71
$G_{12} = G_{13}$ (GPa)	4.31
G_{23} (GPa)	2.76
$\upsilon_{12} = \upsilon_{13}$	0.301
υ_{23}	0.396
V_f	0.55
Thickness (mm)	0.127

When two sides of the composite cylinder are simply supported under bending loading, buckling will take place and the mode. Two cylinders' buckling modes which are quasi-isotropic (QI) composite and VS composite are shown in the figure 3.



(a) (b)

Figure 3. The buckling modes, (a) Quasi-isotropic cylinder and (b) Variable stiffness cylinder.

It indicates that buckling failure occurs in the compression area. The difference between two cylinders is the area of compression. Compression area in VS cylinder's mode is larger than QI composite because multiple design orientation angles make the cylinder sustaining loading uniformly after optimized. Its critical bending moment promotes from 1.058e5 N*m to 1.330e5 N*m. More elements sustain compression resulting in critical bending moment's promoting.

3. Design optimization of VS composite cylinders under combined loadings

In engineering, a common situation is that the cylinder is under a combined loading condition. Considering the buckling of cylinders, most emphasis is placed on axial compressive force and bending. How to design a cylinder under a variety of load conditions to ensure the performance such as buckling and the critical strength is of important. In order to enhance the different kinds of cylinders' loading capacities simultaneously, designs of VS cylinders under two kinds of combined loadings are considered. One is bending and external pressure combination and the other one is bending and torsion. The cylinder may be failure when sustaining different combined loadings. The critical loadings are chosen as the objectives. The five design variables are those mentioned in section 2. To solve such a multi-objective optimization problem, the multi-objective particle swarm optimization (MOPSO) is applied to obtain the final Pareto-optimal solutions.

3.1. MOPSO

In traditional particle swarm optimization (PSO) method, candidate solutions are represented by a number of particles. These particles are attracted by the global optimum position of the crowd $\boldsymbol{gbest}$ and the personal ones $\boldsymbol{pbest}_i$. In each step, the velocity and position of a particle will be updated by equation (2):

$$\begin{aligned} &\boldsymbol{v}_i(t+1)=\omega\boldsymbol{v}_i(t)+c_1 rand_1(\ )\left(\boldsymbol{pbest}_i-\boldsymbol{X}_i(t)\right)+c_2 rand_2(\ )\left(\boldsymbol{gbest}-\boldsymbol{X}_i(t)\right) \\ &\boldsymbol{X}_i(t+1)=\boldsymbol{X}_i(t)+\boldsymbol{v}_i(t+1) \end{aligned} . \tag{2}$$

where $\boldsymbol{v}_i(t)$ and $\boldsymbol{X}_i(t)$ are the velocity and position, respectively, of particle i at step t; ω is the inertia factor which is 0.9 at the beginning of the iteration and decreases to 0.4 along the iteration; c_1 and c_2 are acceleration coefficients which are set to 2.05; $rand_1(\ )$ and $rand_2(\ )$ are two independent random numbers following a uniform distribution $U(0,1)$. A maximum velocity value $v_{\max}$ is assigned for $\boldsymbol{v}_i(t)$. As the iteration proceeds, all particles tend to move towards $\boldsymbol{gbest}$ until the optimal is obtained.

In MOPSO, different updating rules are applied as compared to PSO, i.e., $\boldsymbol{pbest}_i$ and $\boldsymbol{gbest}$ are updated through a dominating principle. A solution $\boldsymbol{X}_{(1)}$ dominating another solution $\boldsymbol{X}_{(2)}$ means that it is not worse than $\boldsymbol{X}_{(2)}$ in all objectives, and strictly better than $\boldsymbol{X}_{(2)}$ for at least one of the objectives. So, three kinds of dominating cases exist: $\boldsymbol{X}_{(1)}$ dominates $\boldsymbol{X}_{(2)}$, $\boldsymbol{X}_{(2)}$ dominates $\boldsymbol{X}_{(1)}$, and $\boldsymbol{X}_{(1)}$ and $\boldsymbol{X}_{(2)}$ are non-dominated to each other. When $\boldsymbol{X}_{(1)}$ and $\boldsymbol{X}_{(2)}$ are non-dominated in a step of MOSPO, they should be saved and gathered in a set called an external archive. In the next step, $\boldsymbol{gbest}_i$ for particle i is selected randomly from the external archive. When updating $\boldsymbol{pbest}_i$, if $\boldsymbol{pbest}_i$ and $\boldsymbol{X}_i(t+1)$ are non-dominated to each other or $\boldsymbol{pbest}_i$ dominates $\boldsymbol{X}_i(t+1)$, $\boldsymbol{pbest}_i$ will be saved, otherwise $\boldsymbol{pbest}_i$ is replaced by $\boldsymbol{X}_i(t+1)$.

3.2. Formulation of VS composite cylinders' design under combined loading

In this paper, two kinds of combined loadings are considered. The first case is the combination of bending M and external pressure P. And the second case is bending M and torsion T. The five ply angles are the design variables. The multi-objective optimization problem can be described as:

$$\begin{aligned} &\max_{\varphi} \quad M_{cr}, F_{cr} \\ &s.t. \quad F.I. \le 1 \\ &\boldsymbol{\varphi} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\} \\ &F_{cr} = P_{cr} \; or \; T_{cr} \end{aligned}, \tag{3}$$

in which, M_{cr}, P_{cr} and T_{cr} represent the loading capacity for bending, external pressure and torsion respectively. $F.I.$ is the Tsai-Wu criterion coefficient and it is used to avoid strength failure.

In problem (3), for a candidate solution $\boldsymbol{\varphi}$, there is a negative correlation between M_{cr} and F_{cr}. To determine the correlation between them, a number of M_{cr} or F_{cr} need to be considered, and one of them is regarded as the independent variable and the other one is the dependent variable. The problem (3) is transformed as:

$$\begin{aligned} &\max_{\varphi, F_{cr}} \quad M_{cr}, F_{cr} \\ &s.t. \quad F.I. \le 1 \\ &\boldsymbol{\varphi} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\} \\ &F_{cr} = P_{cr} \; or \; T_{cr} \end{aligned} \quad \text{or} \quad \begin{aligned} &\max_{\varphi, M_{cr}} \quad M_{cr}, F_{cr} \\ &s.t. \quad F.I. \le 1 \\ &\boldsymbol{\varphi} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\} \\ &F_{cr} = P_{cr} \; or \; T_{cr} \end{aligned}. \tag{4}$$

Problem (4) is solved by MOPSO mentioned in section 3.1.

4. Examples

In this section, the multi-objective optimization of VS composite cylinders under two kinds of combining loadings is calculated. For each condition, three kinds of fiber paths will be considered: QI composite, CS composite and VS composite. The differences among them are that $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5 = 45°$ for QI, $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5$ for CS and φ_i is independent to the others for VS.

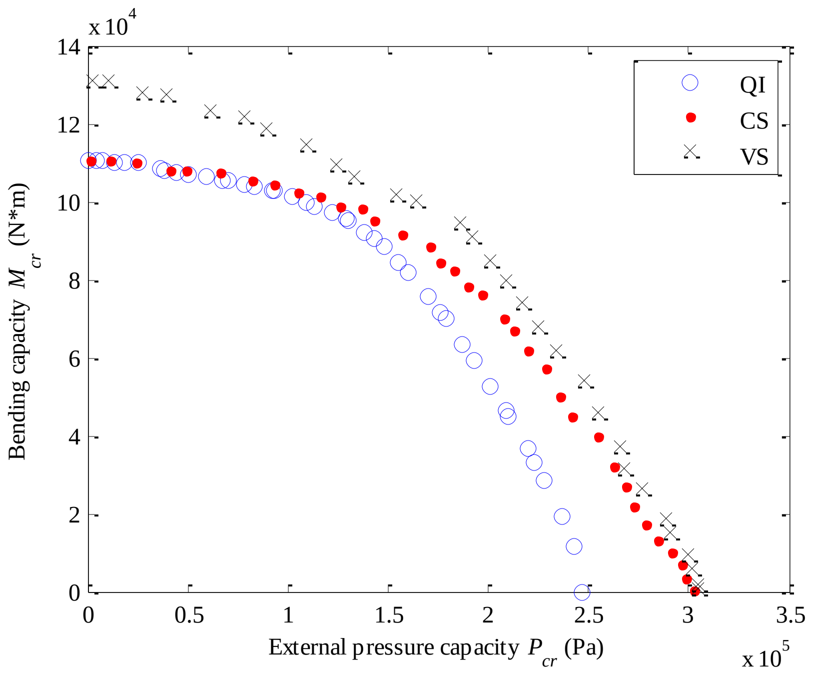


Figure 4. The optimal compromise between bending capacity and external pressure capacity.

4.1. Bending and external pressure case

The Pareto-optimal solutions for the combination of bending and external pressure are shown in figure 4.

It indicates that the Pareto front of VS is the best among QI, CS and VS, i.e., VS's loading capacity is superior to CS and QI when sustaining bending and external pressure simultaneously. Performance of CS is the middle, and that of QI is the worst. The enlarged design space enhances composite cylinder's combining loading capacity. In the single bending case, CS's bending capacity is almost the same as QI's. This is because that a QI composite cylinder possesses almost the best anti-bending performance under the linear fiber path. VS's bending capacity is superior to CS's and QI's because curve fiber path makes composite inhomogeneous and optimization of curve fiber path can increase VS composite cylinder's loading capacity. In the single external pressure case, the optimal solution of VS composite cylinder is close to CS, owing to the fact that external pressure is a kind of homogeneous loading and the curve fiber path is degenerated to the linear path.

4.2. Bending and torsion

The Pareto-optimal solutions are shown in figure 5 for the bending and torsion combined loading.

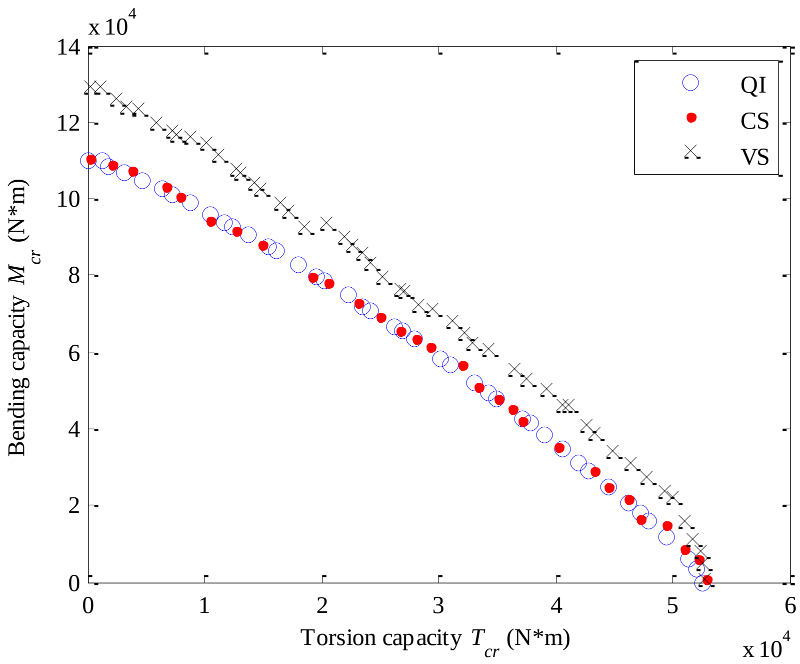


Figure 5. The optimal compromise between bending capacity and torsion capacity.

It is found that VS's combining loading capacity is superior to CS and QI. For the single torsion, solutions of VS, CS and QI are almost the same, implying that the three type structures provide the same anti-torsion performance. For the single bending or bending-torsion, VS structures' advantage is obvious.

5. Conclusions

Loading capacity of a VS composite cylinder under combined loading is studied by conducting multi-objective design optimization. It is shown that the VS composite cylinder whose fiber path is curve is superior to CS and QI ones. Due to the curve fiber path, a VS composite cylinder possesses inhomogeneous feature. When sustaining inhomogeneous loadings, its loading capacity will be enhanced greatly as compared with its CS or QI counterpart. For homogeneous loading condition, CS composite cylinder provides almost the best solution.

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References

[1] Jing Z, Fan X L and Sun Q 2015 *Compos Struct* **121** 225-36
[2] Jing Z, Fan X L and Sun Q 2015 *Compos B: Eng* **69** 181-90
[3] Jing Z, Sun Q and Silberschmidt V V 2016 *Compos Struct* **141** 240-52
[4] Jing Z, Sun Q and Silberschmidt V V 2016 *Compos Struct* **154** 106-28
[5] Pinckney R L 1991 *Proceedings of First NASA Advanced Composites Technology Conference* (NASA Langley Research Center) p 385-97
[6] Anderson R L and Gran C 1991 *Proceeding of First NASA Advanced Composites Technology Conference* (NASA Langley Research Center) p 817-30
[7] Kisch R A 2006 *Proceedings of International SAMPE Symposium and Exhibition* (Long Beach CA (USA)) p 1537-47
[8] Measom R and Sewell K 1996 *Proceedings of First NASA Advanced Helicopter Society* (American Helicopter Society 52nd Annual Forum) p 611-22
[9] Grant C and Benson V 1993 *Proceedings of Third NASA Advance Composites Technology Conference* (NASA Langley Research Center) p 648-52
[10] Buchanan D, Alexander R and Thomas J 1999 *Proceedings of the 40th AIAA/ASCE/AHS/ASC SDM Conference* (St. Louis, MO (USA)) p1999
[11] Sabido A, Bahamonde L, Harik R and Tooren M J L V 2017 *Compos Struct* **160** 804-12
[12] Hyer M W, Rust R J and Waters W A 1994 *Technical Report NASA-CR-197045*, NASA
[13] Tatting B F 1998 Analysis and design of variable stiffness composite cylinders (Virg. Tech, Blacksburg, Virginia)
[14] Rouhi M 2015 *Composite: Part B* **69** 249-55
[15] Rouhi M and Ghayoor H 2016 *Composite Structures* **150** 115-23
[16] Rouhi M, Ghayoor H, Fortin-Simpson J, Zacchia T T, Hoa S V and Hojjati M 2017 *Composite Structures* **184** 146-52
[17] Lee D S, Morillo C, Bugeda G, Oller S and Onate E 2012 *Compos Struct* **94** 1087-96
[18] Vosoughi A R and Nikoo M R 2015 *Thin-Walled Struct* **95** 408-15