IOP Conference Series: Materials Science and Engineering

PAPER • **OPEN ACCESS**

System Analysis by Mapping a Fault-tree into a Bayesian-network

To cite this article: B Sheng *et al* 2018 *IOP Conf. Ser.: Mater. Sci. Eng.* **362** 012025

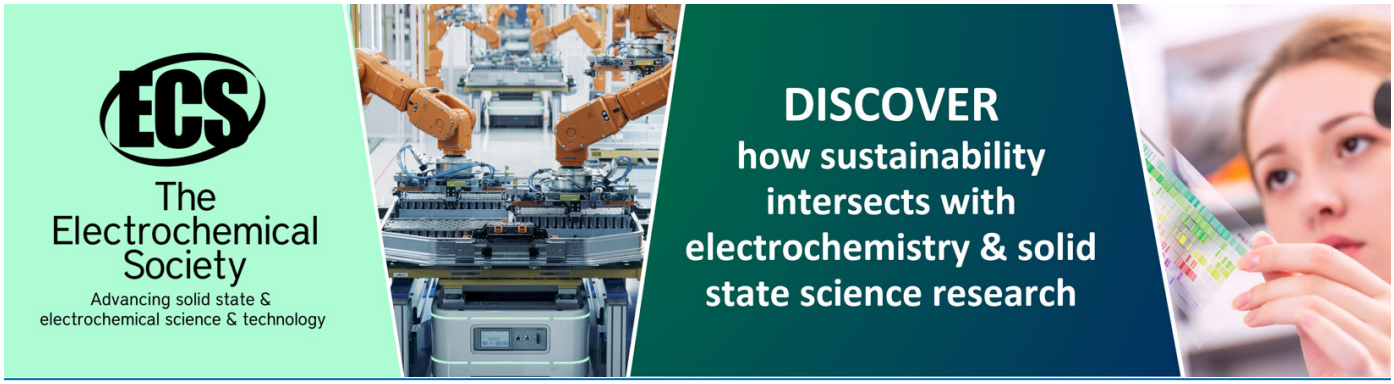
View the article online for updates and enhancements.

You may also like

- Low-Temperature Self-Limiting Growth of Cubic Boron Nitride Via Hollow-Cathode Plasma-Enhanced Atomic Layer Deposition
Adnan Mohammad, Krishna Joshi, Saidjafarzoda Ilhom et al.

- Boron Nitride - Gold As a Novel Electrocatalyst for Oxygen Reduction Reaction
Kohei Uosaki, Ganesan Elumalai, Hidenori Noguchi et al.

- Field Emission Characteristics of BN Films with Cubic-BN Phase
Gu Guang-Rui, Wu Bao-Jia, Jin Zhe et al.



This content was downloaded from IP address 3.19.211.134 on 04/05/2024 at 09:49

System Analysis by Mapping a Fault-tree into a Bayesian-network

B Sheng[1], C Deng[2], Y H Wang[3] and L H Tang[1*]

1 Department of Mechanical Engineering, The University of Auckland, Auckland, New Zealand
2 School of Mechanical Science & Engineering, Huazhong University of Science & Technology, Wuhan, China
3 China Electronic Product Reliability and Environmental Testing Research Institute, Guangzhou, China

[*]E-mail: l.tang@auckland.ac.nz

Abstract. In view of the limitations of fault tree analysis in reliability assessment, Bayesian Network (BN) has been studied as an alternative technology. After a brief introduction to the method for mapping a Fault Tree (FT) into an equivalent BN, equations used to calculate the structure importance degree, the probability importance degree and the critical importance degree are presented. Furthermore, the correctness of these equations is proved mathematically. Combining with an aircraft landing gear's FT, an equivalent BN is developed and analysed. The results show that richer and more accurate information have been achieved through the BN method than the FT, which demonstrates that the BN is a superior technique in both reliability assessment and fault diagnosis.

1. Introduction

From the view of reasoning and fault status description, the BN method developed in recent years is better than the FT. Not only the BN method has an ability to describe the polymorphism and the non-determinism of events, but also the bottom-event is not limit to the independent faults, which is more appropriate to analyse the security and reliability of complex systems [1-3].

A large number of researches have been carried out on the transformation from the FT into the BN [4-6]. Zhou [7] proposed a BN method for calculating the minimal cut/path sets, and the importance degrees for structure, probability and critical, while the correctness of this method was not proved. Due to this shortcoming, this paper aims to prove the correctness of the BN method by verifying equations of three importance degrees described above. Then a fault case of an aircraft landing gear was processed by the BN method to study the application in the reliability assessment and fault diagnosis.

2. The Bayesian network

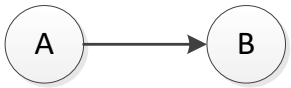
Definition 1： a BN is a directed acyclic graph, which is constituted by points (represent variables) and directed edges (connect these points). The directed edge is represented as a single arrow-line in the BN, and the direction of this arrow-line is from a parent point to a child point [8]. A simple BN is shown in Fig.1.

Moreover, the BN can be explained by qualitation and quantitation:

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

- Qualitation: the BN takes a directed acyclic graph to describe the dependence/independence relationships between the variables.
- Quantitation: the BN uses the conditional probability distribution to present the dependence relationships between the variables and their parent points. That is, each point in the network corresponds to a conditional probability table (CPT). For example, the CPT of a point X_i can be presented as $P(X_i \mid \pi(X_i))$, here $\pi(X_i)$ is the parent point set of the point X_i.

If there is a BN¥, its arbitrary variable set is $\boldsymbol{N} = \{X_1 \mathrm{L}\ X_n\}$, and the corresponding joint probability table is $\Pr(X_1 \mathrm{K}\ X_n)$, so its joint probability distribution can be obtained by multiplying each variable with its corresponding joint probability table [9,10], that is $\Pr(X_1 X_2 \mathrm{L}\ X_n) = \prod_{i=1}^{n} P(X_i \mid \pi(X_i))$.



A	θ_A
0	.3
1	.7

A	B	$\theta_{B\|A}$
0	0	.1
0	1	.9
1	0	.8
1	1	.2

Figure 1. A simple BN.

2.1. Mapping method

The FT and the BN are all the expressions for systems' causal logic. The logic gate in the FT represents the determined relationship between two events, while the CPT in the BN expresses the undetermined relationship between points. Obviously, setting the value of the CPT is the key for mapping a FT into an equivalent BN [4]. A brief description of the basic mapping steps is as follows.

- Each event (bottom-event, middle-event and top-event) in a FT will be presented as a parent point or a child point in an equivalent BN, and named as the event name. Moreover, for the repeated events, there will be a same point for them in the equivalent BN [5]. Furthermore, in the equivalent BN, the top-event in the FT will be treated as the point T, and the bottom-event x_i in the FT will be treated as the point X_i which should be 0 or 1(0 means normal, 1 means fault).
- Establish connections for points in the equivalent BN according to relationships of events in the FT.
- Determine the priori probability of the bottom-point in the equivalent BN according to the failure probability of the corresponding bottom-event in the FT.
- Set the CPT of points in the equivalent BN according to the corresponding middle events and top events in the FT [7, 11].

2.2. Bi-directional reasoning

The reasoning for a BN can be summarized as: solving issues of the posterior probability of interested variables based on evidence variables and network structures. The mathematical description of a BN is as follows.

For any BN¥, the variable set $\boldsymbol{N}$ can be divided into three categories: the evidence variable $\boldsymbol{E}=\boldsymbol{e}$, the query variable $\boldsymbol{Q}$ (the interested variable) and the other variables $\boldsymbol{S} = \boldsymbol{N} \setminus (\boldsymbol{Q} \cup \boldsymbol{M})$. That is, calculating the probability distribution $\Pr(\boldsymbol{Q} \mid \boldsymbol{E}=\boldsymbol{e})$ can solve posterior probability issues. Moreover, solutions of the probability distribution can be the same solutions of the Eq. (1) and the Eq. (2) based on the conditional independence, the chain rule and the Bayes theorem.

$$\Pr(\boldsymbol{Q}, \boldsymbol{E} = \boldsymbol{e}) = \sum_{\boldsymbol{S}} \prod_{i=1}^{n} P(X_i \mid pai(X_i)) \tag{1}$$

$$\Pr(\boldsymbol{Q} \mid \boldsymbol{E} = \boldsymbol{e}) = \frac{\Pr(\boldsymbol{Q}, \boldsymbol{E} = \boldsymbol{e})}{\sum_{\boldsymbol{Q}} \Pr(\boldsymbol{Q}, \boldsymbol{E} = \boldsymbol{e})} \tag{2}$$

A BN has an ability of bi-directional reasoning, which contains a bottom-up causal reasoning and a top-down diagnostic reasoning [2, 5]. As described above, the points of the bottom-event and the top-event in the equivalent BN are X_i and T, respectively. So the bottom-up causal reasoning can be expressed as calculating the probability of the top-event when the bottom-event is fault (Eq. (3)) and the top-down diagnostic reasoning is just the reverse (Eq. (4)).

$$\Pr(T = 1 \mid X_i = 1) = \frac{\Pr(T = 1, X_i = 1)}{\Pr(X_i = 1)} \tag{3}$$

$$\Pr(X_i = 1 \mid T = 1) = \frac{\Pr(T = 1, X_i = 1)}{\Pr(T = 1)} \tag{4}$$

It can be seen from comparisons of the Eq. (3) and the Eq. (2), and of the Eq. (4) and the Eq. (2), the bottom-up causal reasoning and the top-down diagnostic reasoning are both belong to the posterior probability issue. This issue can be solved by reasoning algorithms such as the tree propagation algorithm, the variable elimination algorithm and the Aho–Corasick algorithm [8, 12, 13].

3. Three importance degrees

For a FT, the first step of a qualitative analysis is to get an expression of the probability function $g(q)$ of an accident tree [14, 15]. Furthermore, the occurrence probability of a top-event in a FT can be obtained through calculating the marginal probability $\Pr(T = 1)$ of an equivalent BN (Eq. (5)) [5].

$$\Pr(T = 1) = g(q) \tag{5}$$

Setting a FT as a $\overline{FT}$, an equivalent BN as a $\overline{BN}$ and the number of bottom-events as n. Zhou [7] proposed a $\overline{BN}$ method to compute three importance degrees (structure, probability and critical). However, he did not provide any evidence to support his method. So the following content will prove the correctness of his method based on the Eq. (5).

3.1. The Structure Importance Degree

Top-event is the structure function $\phi(x) = \phi(x_1, x_2, \mathrm{L}\ x_n)$ of the bottom-event. The value of $\phi(x)$ represents the value of the bottom-event x_i, which could be 0 or 1. Furthermore, the structure importance degree of the bottom-event x_i in $\overline{FT}$ and the corresponding point X_i in $\overline{BN}$ could be calculated by the Eq. (6) [14] and the Eq. (7) [7], respectively.

$$\begin{aligned} I_{\phi(i)} = \frac{1}{2^{n-1}} \sum_{k_1, k_2, \mathrm{L}\ , k_{i-1}, k_{i+1}, \mathrm{L}\ , k_n = 0}^{1} & [\phi(x_i = 1, x_1 = k_1, \mathrm{L}\ , x_n = k_n) \\ & -\phi(x_i = 0, x_1 = k_1, \mathrm{L}\ , x_n = k_n)] \end{aligned} \tag{6}$$

Note in the Eq. (6), the $\sum$ means traverse the values of all bottom-events as 0 or 1(expect the bottom-event x_i).

$$I_{\phi(i)} = \Pr(T = 1 \mid X_i = 1) - \Pr(T = 1 \mid X_i = 0) \tag{7}$$

Moreover, reset the conditional probability distribution of each bottom-event as $\Pr(X_j = 0) = 1/2, \Pr(X_j = 1) = 1/2$, $j \neq i$. The proof is as follows.

The structure importance degree is not related to the failure probability of a certain bottom-event. In this work, the failure probability of the bottom-event x_j 's was set as $1/2$ and $j \neq i$. So, the conditional probability distribution of the point X_j (equal to the bottom-event x_j in the FT) in the equivalent BN is $\Pr(X_j = 0) = 1/2$ and $\Pr(X_j = 1) = 1/2$.

Then the Eq. (8) and the Eq. (9) can be obtained due to the independence of $X_1, X_2, \mathrm{L}\ , X_n$.

$$\begin{aligned}&\Pr(X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n) = \\ &\Pr(X_1)\cdot\Pr(X_2)\cdot\ldots\cdot\Pr(X_{i-1})\cdot\Pr(X_i)\cdot\ldots\cdot\Pr(X_n) = \frac{1}{2^{n-1}}\end{aligned} \tag{8}$$

$$\begin{aligned}&\Pr(X_1, X_2, \ldots, X_n) = \Pr(X_1)\cdot\Pr(X_2)\cdot\ldots\cdot\Pr(X_n) \\ &= \frac{1}{2^{n-1}}\cdot\Pr(X_i)\end{aligned} \tag{9}$$

In a FT, the meaning of $\phi(x_1 = k_1, \ldots, x_n = k_n) = 0$ is that all values of bottom-events x are 0, the top-event would not occur definitely. In an equivalent BN, it means that when the status (values) of all points X are given (as 0), the probability of point T gets the status of 1 is zero. Similarly, the meaning of $\phi(x_1 = k_1, \ldots, x_n = k_n) = 1$ is that all values of bottom-events x are 1, the top-event would occur definitely. In an equivalent BN, it means that when the states of all points X are given (as 1), the probability of point T gets the status of 1 is one. So the Eq. (10) can be obtained.

$$\begin{aligned}&\phi(x_1 = k_1, \ldots, x_n = k_n) \\ &= \Pr(T = 1 \mid X_1 = k_1, \ldots, X_n = k_n)\end{aligned} \tag{10}$$

The Eq. (11) can be got after putting the Eq. (10) into the Eq. (9):

$$\begin{aligned}I_{\phi(i)} = \frac{1}{2^{n-1}} \sum_{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n = 0}^{1} &[\Pr(T = 1 \mid X_i = 1, X_1 = k_1, \ldots, X_n = k_n) \\ &- \Pr(T = 1 \mid X_i = 0, X_1 = k_1, \ldots, X_n = k_n)]\end{aligned} \tag{11}$$

Moreover, the first half of the Eq. (11) can be expressed as:

$$\begin{aligned}&\Pr(T = 1 \mid X_i = 1, X_1 = k_1, \ldots, X_n = k_n) = \\ &\frac{\Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1, X_1 = k_1, \ldots, X_n = k_n)} = \\ &\frac{\Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1)\cdot\Pr(X_1 = k_1, \ldots, X_n = k_n)} = \\ &\frac{\Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1)}\cdot 2^{n-1}\end{aligned} \tag{12}$$

Similarly, the last half of the Eq. (11) can be expressed as:

$$\begin{aligned}&\Pr(T = 1 \mid X_i = 0, X_1 = k_1, \ldots, X_n = k_n) = \\ &\frac{\Pr(T = 1, X_i = 0, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 0)}\cdot 2^{n-1}\end{aligned} \tag{13}$$

Then put the Eq. (12) and the Eq. (13) into the Eq. (11), the result is the same as Eq. (7).

$$\begin{aligned}I_{\phi(i)} &= \frac{1}{2^{n-1}} \sum_{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n = 0}^{1} \Big[\frac{\Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1)}\cdot 2^{n-1} \\ &- \frac{\Pr(T = 1, X_i = 0, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 0)}\cdot 2^{n-1}\Big] \\ &= \sum_{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n = 0}^{1} \Big[\frac{\Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1)} \\ &- \frac{\Pr(T = 1, X_i = 0, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 0)}\Big] \\ &= \frac{\sum_{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n = 0}^{1} \Pr(T = 1, X_i = 1, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 1)} \\ &- \frac{\sum_{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n = 0}^{1} \Pr(T = 1, X_i = 0, X_1 = k_1, \ldots, X_n = k_n)}{\Pr(X_i = 0)} \\ &= \frac{\Pr(T = 1, X_i = 1)}{\Pr(X_i = 1)} - \frac{\Pr(T = 1, X_i = 0)}{\Pr(X_i = 0)} \\ &= \Pr(T = 1 \mid X_i = 1) - \Pr(T = 1 \mid X_i = 0)\end{aligned}$$

3.2. The Probability Importance Degree

In a $\overline{FT}$, the probability importance degree of bottom-event x_i can be calculated by the Eq. (14) [14].

$$I_g(i) = \frac{\partial g(q)}{\partial q_i} \tag{14}$$

Here q_i represents the occurrence probability of the ith bottom-event.

In an equivalent $\overline{BN}$, the probability importance degree of the point X_i can be calculated by the Eq. (15) [7].

$$I_g(i) = \Pr(T=1 \mid X_i = 1) - \Pr(T=1 \mid X_i = 0) \tag{15}$$

Furthermore, for any BN, if its points' values are all binary, this BN can be expressed as a multivariate linear equation (MLE) [8,16,17], which is a foundation of the Eq. (15). At the same time, each MLE contains two types of parameters: evidence indicators and network parameters. The meaning of the evidence indicator is that in a BN, any variable X corresponds to an evidence indicator λ_x. The meaning of the network parameter is that in a BN, any variable X's family group XU corresponds to a parameters set $\theta_{x|\boldsymbol{u}}$, which is the probability parameter in the CPT.

Definition 2 gives an accurate definition of the MLE in a BN as follow [8].

Definition 2: for any BN, if its variable set is W; and for any variable X's family group XU, if its set is W as well. The BN's corresponding MLE can be defined as follow.

$$f \overset{def}{=} \sum_{\boldsymbol{z}} \prod_{\theta_{x|\boldsymbol{u}}:\boldsymbol{z}} \theta_{x|\boldsymbol{u}} \prod_{x:\boldsymbol{z}} \lambda_x$$

In definition 2, $\theta_{x|\boldsymbol{u}}:\boldsymbol{z}$ represents the values of $x\boldsymbol{u}$ and z are compatible, $x:\boldsymbol{z}$ represents the values of x and $\boldsymbol{z}$ are compatible. So, according to definition 2, the BN in Fig.1 can be expressed as follow.

$$f = \lambda_a\lambda_b\theta_a\theta_{b|a} + \lambda_a\lambda_{\bar{b}}\theta_a\theta_{\bar{b}|a} + \lambda_{\bar{a}}\lambda_b\theta_{\bar{a}}\theta_{b|\bar{a}} + \lambda_{\bar{a}}\lambda_{\bar{b}}\theta_{\bar{a}}\theta_{\bar{b}|\bar{a}}$$

Definition 3: for each BN, given the evidence variable e, set the value of evidence indicator λ_x as 1, if it is compatible with e; set the value of evidence indicator λ_x as 0, if it is not compatible with e. Then, the following equations can be obtained [8]:

$$f \overset{def}{=} f(\boldsymbol{e}) \text{ and } f(\boldsymbol{e}) = \Pr(\boldsymbol{e})$$

As the same example in Fig.1, if the evidence variable e=a, set $\lambda_a = 1, \lambda_{\bar{a}} = 0, \lambda_b = 1, \lambda_{\bar{b}} = 1$ in MLE, then the $f(a) = \theta_a\theta_{b|a} + \theta_a\theta_{\bar{b}|a} = \Pr(a)$ can be obtained.

So a proof for the Eq. (15) is as follows based on the knowledge as described above:

The Eq. (16) is correct according to the MLE as described above. Then the Eq. (17) can be obtained by calculating the partial derivative for the parameter $\theta_{y|\boldsymbol{v}}$.

$$\Pr(\boldsymbol{e}) = f(\boldsymbol{e}) = \sum_{\boldsymbol{z}:\boldsymbol{e}} \prod_{\theta_{x|\boldsymbol{u}}:\boldsymbol{z}} \theta_{x|\boldsymbol{u}} \tag{16}$$

$$\frac{\partial f(\boldsymbol{e})}{\partial \theta_{y|\boldsymbol{v}}} = \sum_{\substack{\boldsymbol{z}:y\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \prod_{\substack{\theta_{x|\boldsymbol{u}}:\boldsymbol{z} \\ x\boldsymbol{u}\neq y\boldsymbol{v}}} \theta_{x|\boldsymbol{u}} - \sum_{\substack{\boldsymbol{z}:\bar{y}\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \prod_{\substack{\theta_{x|\boldsymbol{u}}:\boldsymbol{z} \\ x\boldsymbol{u}\neq \bar{y}\boldsymbol{v}}} \theta_{x|\boldsymbol{u}} \tag{17}$$

Moreover, two sub-equations in right side of the Eq. (17) can be translated to the Eq. (18) and the Eq. (19), respectively.

$$\sum_{\substack{\boldsymbol{z}:y\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \prod_{\substack{\theta_{x|\boldsymbol{u}}:\boldsymbol{z} \\ x\boldsymbol{u}\neq y\boldsymbol{v}}} \theta_{x|\boldsymbol{u}} = \sum_{\substack{\boldsymbol{z}:y\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \left(\prod_{\theta_{x|\boldsymbol{u}}:\boldsymbol{z}} \theta_{x|\boldsymbol{u}} / \theta_{y|\boldsymbol{v}}\right) = \sum_{\substack{\boldsymbol{z}:y\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \frac{\Pr(\boldsymbol{z})}{\theta_{y|\boldsymbol{v}}} \tag{18}$$

$$\sum_{\substack{\boldsymbol{z}:\bar{y}\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \prod_{\substack{\theta_{x|\boldsymbol{u}}:\boldsymbol{z} \\ x\boldsymbol{u}\neq \bar{y}\boldsymbol{v}}} \theta_{x|\boldsymbol{u}} = \sum_{\substack{\boldsymbol{z}:\bar{y}\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \left(\prod_{\theta_{x|\boldsymbol{u}}:\boldsymbol{z}} \theta_{x|\boldsymbol{u}} / \theta_{\bar{y}|\boldsymbol{v}}\right) = \sum_{\substack{\boldsymbol{z}:\bar{y}\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \frac{\Pr(\boldsymbol{z})}{\theta_{\bar{y}|\boldsymbol{v}}} \tag{19}$$

Then put the Eq. (18) and the Eq. (19) into the Eq. (17) to get the Eq. (20).

$$\frac{\partial f(\boldsymbol{e})}{\partial \theta_{y|\boldsymbol{v}}} = \sum_{\substack{\boldsymbol{z}:y\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \frac{\Pr(\boldsymbol{z})}{\theta_{y|\boldsymbol{v}}} - \sum_{\substack{\boldsymbol{z}:\bar{y}\boldsymbol{v} \\ \boldsymbol{z}:\boldsymbol{e}}} \frac{\Pr(\boldsymbol{z})}{\theta_{\bar{y}|\boldsymbol{v}}} = \frac{\Pr(y,\boldsymbol{v},\boldsymbol{e})}{\theta_{y|\boldsymbol{v}}} - \frac{\Pr(\bar{y},\boldsymbol{v},\boldsymbol{e})}{\theta_{\bar{y}|\boldsymbol{v}}} \tag{20}$$

Furthermore, in a $\overline{FT}$, the equation for probability importance degree is $I_g(i)=\partial g(q)/\partial q_i$; in an equivalent $\overline{BN}$, the $g(q)=\Pr(T=1)$ and the $q_i=\Pr(X_i=1)$; in a MLE, the $\Pr(X_i=1)=\theta_{x_i}$, the $\Pr(X_i=0)=\theta_{\overline{x}_i}$ and the $f(\boldsymbol{e})=f(T=1)=\Pr(T=1)$. So the Eq. (21) can be got as follow.

$$I_g(i)=\frac{\partial g(q)}{\partial q_i}=\frac{\partial \Pr(T=1)}{\partial \Pr(X_i=1)}=\frac{\partial f(\boldsymbol{e})}{\partial \theta_{x_i}} \tag{21}$$

According to the Eq. (20), the Eq. (21) can be translated as follow, which is the same as the Eq. (15).

$$\begin{aligned} I_g(i)&=\frac{\partial f(\boldsymbol{e})}{\partial \theta_{x_i}}=\frac{\Pr(x_i,\boldsymbol{e})}{\theta_{x_i}}-\frac{\Pr(\overline{x}_i,\boldsymbol{e})}{\theta_{\overline{x}_i}}=\frac{\Pr(x_i,\boldsymbol{e})}{\Pr(x_i)}-\frac{\Pr(\overline{x}_i,\boldsymbol{e})}{\Pr(\overline{x}_i)} \\ &=\Pr(\boldsymbol{e}\mid x_i)-\Pr(\boldsymbol{e}\mid \overline{x}_i) \\ &=\Pr(T=1\mid X_i=1)-\Pr(T=1\mid X_i=0) \end{aligned} \tag{22}$$

3.3. The Probability Importance Degree

In a $\overline{FT}$, the critical importance degree of the bottom-event x_i can be calculated by the Eq. (23) [14].

$$I_c(i)=I_g(i)\frac{q_i}{g(q)} \tag{23}$$

While an equivalent $\overline{BN}$, the critical importance degree of the X_i can be calculated by the Eq. (24) [7].

$$I_c(i)=\frac{\Pr(X_i=1)[\Pr(T=1\mid X_i=1)-\Pr(T=1\mid X_i=0)]}{\Pr(T=1)} \tag{24}$$

A proof for the Eq. (24) is as follow.

Put the Eq. (22) into the Eq. (23) to get the equation which is the same as the Eq. (24).

$$\begin{aligned} I_c(i)&=[\Pr(T=1\mid X_i=1)-\Pr(T=1\mid X_i=0)]g\frac{q_i}{g(q)} \\ &=[\Pr(T=1\mid X_i=1)-\Pr(T=1\mid X_i=0)]g\frac{\Pr(X_i=1)}{\Pr(T=1)} \end{aligned}$$

4. Further study

The Eq. (5), the Eq. (7), the Eq. (15) and the Eq. (24) are all belong to the prior/posterior probability issues of the BN. For solving the posterior probability issue, the prior probability has to be solved firstly (see the Eq. (3)). At the same time, the solutions of the prior/posterior probability issues are all belong to the solution of the marginal probability $\Pr(\boldsymbol{Y}=\boldsymbol{y})$, where $\boldsymbol{Y}$ represents some points' sets for a BN. So, a unified algorithm can be used by applying the BN method to analyse the FT, which is better than the normal FT methods. Because in the normal FT methods, different problems have to be solved by different methods rather than the unified algorithm.

Moreover, in addition to calculating three importance degrees and the failure probability of the top-event, the BN method can also calculate the minimal cut/path sets to analyse the FT [7]. Furthermore, the BN method can be used for causal reasoning (see the Eq. (3)) and fault diagnosis (see the Eq. (4)) due to the characteristics of the Bi-directional reasoning. At the same time, the Eq. (3) and the Eq. (4) can be transferred to the Eq. (25) and the Eq. (26), respectively, and $\boldsymbol{X}=\boldsymbol{x}$ represents arbitrary point set and its value in a BN. It means that the BN method not only can calculate the normal working probability of a network by given any one or more points, but also can calculate the failure probability of any one or more points to judge the weakness of the network when it is fault. That is, more information about the whole system could be acquired by the BN method, which is better than the FT method.

$$\Pr(T=1\mid \boldsymbol{X}=\boldsymbol{x})=\frac{\Pr(T=1,\boldsymbol{X}=\boldsymbol{x})}{\Pr(\boldsymbol{X}=\boldsymbol{x})} \tag{25}$$

$$\Pr(\boldsymbol{X}=\boldsymbol{x}\mid T=1)=\frac{\Pr(T=1,\boldsymbol{X}=\boldsymbol{x})}{\Pr(T=1)} \tag{26}$$

Finally, the FT method is only suitable for a system which has a determined fault mechanism and a clear causal logic. Moreover, the FT method is based on three hypotheses [6,18]: only two states (0/1) for an event; the determined causal logic between events (one event causes another event definitely or not); the fault independence of the bottom-event. While the application of the BN method can transform the FT into the BN to modify the network further to get rid of these three hypotheses as described above [18].At the same time, the BN method can also reconstruct a new network directly according to the causal logic of points in the network to free from these three hypotheses as well [12].

5. Case study

The Fig. 2(a) is a FT of an aircraft landing gear [14] and the Fig. 2(b) is an equivalent BN.

5.1. Results

The unreliability (the occurrence probability of the top-event) of system is $\Pr(T=1)=1.25047\times10^{-5}$ by the Eq. (5). The structure importance degree, the probability importance degree, the critical importance degree, the causal reasoning and the diagnostic reasoning are calculated by the Eq. (2), the Eq. (15) and the Eq. (24), the Eq. (3) and the Eq. (4), respectively. All results are shown in Table I.

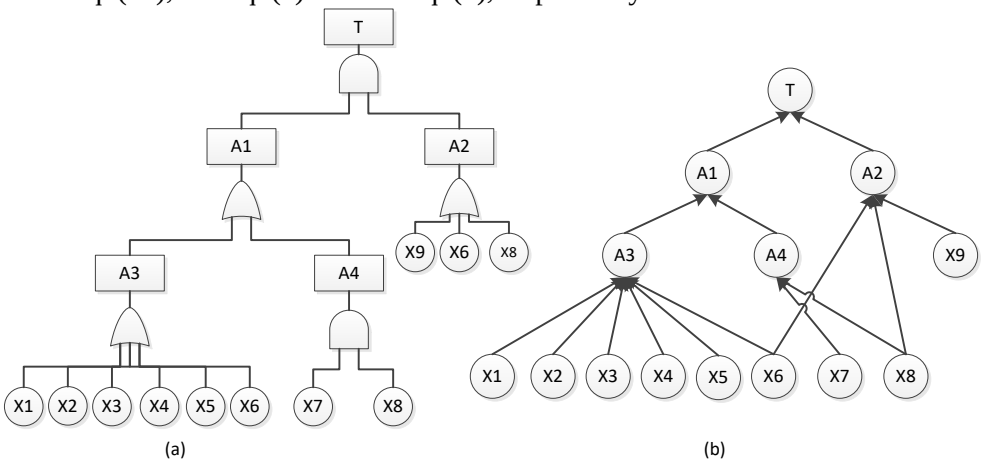


Figure 2. (a) The FT of an aircraft landing gear and (b) The equivalent BN.

Table 1. The arrangement of channels

B.	S.	P.	Cr.	Ca.	D.
X1	0.0156	6.3197e-5	1.2635e-5	6.3199e-5	1.2634e-4
X2	0.0156	6.3198e-5	6.3174e-5	6.3199e-5	6.3171e-5
X3	0.0156	6.3199e-5	5.0540e-13	6.3199e-5	5.0537e-13
X4	0.0156	6.3194e-5	3.6134e-4	6.3199e-5	3.6134e-4
X5	0.0156	6.3199e-5	1.0108e-12	6.3199e-5	1.0107e-12
X6	0.0156	0.9999	0.9996	0.9999	0.9996
X7	0.0078	2.7504e-5	4.3989e-8	2.7504e-5	4.3987e-8
X8	0.2500	1.2150e-4	1.4576e-4	1.2152e-4	1.4575e-4
X9	0.2422	1.2149e-4	3.4685e-4	1.2150e-4	3.4684e-4

B.=Bottom-event, S.=Structure importance degree, P.=Probability importance degree, Cr.=Critical importance degree, Ca.=Causal reasoning and D.=Diagnostic reasoning.

5.2. Discussion

- In order to avoid the complex disjoint method, Deng used a direct calculation method to get the occurrence probability of the top-event approximately [14]. In contrast, the BN method can acquire more accurate results, which is simple and easy.
- Table I shows more accurate results of the structure importance degree, while in Deng's paper [14], the structure importance degree was roughly obtained by the minimal cut/path sets. That

is why two results are slightly different. Compared with the traditional FT method, the BN method can get more accurate results, and the calculation of the structure importance degree is much simpler.

- For the complex FT, the probability function is difficult to obtain, which not only leads to the inaccurate reliability, but also cause the inaccurate probability importance degree and critical importance degree of the bottom-event. While there is no such shortage for the BN method, and more accurate results of the probability importance degree and critical importance degree can be obtained through the Eq. (15) and the Eq. (24).
- The BN method not only can obtain the results that calculated by the traditional FT method, but also can use the causal reasoning and diagnostic reasoning to analyse the reliability of the system. Because the causal reasoning can be used to determine the influence for the system caused by the bottom-event, and the diagnostic reasoning can diagnose the system to identify the weak points (for a system, the higher probability of Pr(X=1|T=1), the weaker point it is).

6. Conclusion

Combined with the characteristics of the BN method, this paper systematically study the reasoning processes for calculating the structure importance degree, the probability importance degree and the critical importance degree. A case study has demonstrated that the BN method is better than the conventional FT method. The conclusions for the BN method are as follows.

- The BN method can use a unified algorithm to get richer information and more accurate results, which is important for the fault diagnosis and the reliability assessment.
- The BN method overcomes the shortcomings of the FT method since it does not rely on the three hypotheses. That is, the BN method is more comprehensive and accurate for modelling complex systems, and it also can use an original reasoning method to analyse and process complex systems as well.

7. References

[1] Wang H, Zhou J L and He Z Y 2004 *J. Compt. Integr. Manuf. Syst.* **10(2)** 230-234
[2] Yi X W, Qian W X and Xie L Y 2008 *J. Acta Aeronautica et Astronautica Sinica* **29(6)** 1482-1489
[3] Torres-toledano J G and Sucar L E 1998 in *Proc. 1998 IBERAMIA Conf.* p 195-206
[4] Khakzad N, Khan F and Amyotte P 2011 *J. Reliab. Eng. Syst. Saf.* **96(8)** 925-932
[5] Mahadevan S, Zhang R and Smith N 2001 *J. Struct. Saf.* **23(3)** 231-251
[6] Portinale L and Bobbio A 1999 in *Proc. 1999 UAI Conf.* p 551-558
[7] Zhou Z B, Dong D D and Zhou J L 2006 *J. Syst. Eng. Theory. Pract.* **26(6)** 95-100
[8] Darwiche A 2012 *Modeling and Reasoning with Bayesian Networks* (New York: Cambridge University Press) p 53-306
[9] Shi Z F and Zhang A 2012 *Theory of Bayesian network and its application in military system* (Beijing: National Defense Industry Press) p 78-104
[10] Jensen F V 1996 *An Introduction to Bayesian Networks* (New York: Springer-Verlag) p 1-178
[11] Bobbio A, Portinale L and Minichino M 2001 *J. Reliab. Eng. Syst. Saf.* **71(3)** 249-260
[12] Mengshoel O J, Chavira M and Cascio K 2010 *J. IEEE Trans Syst. Man Cybern. A* **40(5)** 874-885
[13] Huang C and Darwiche A 1996 *J. Int. J. Approx. Reason.* **15(3)** 225-263
[14] Deng Q 2009 *Safety system engineering* (Xi'an: Northwest Industrial University press) p 95-97
[15] Song B F 2006 *Reliability engineering of aircraft* (Xi'an: Northwest Industrial University press) p 77-93
[16] Chavira M and Darwiche A 2007 in *Proc. 2007 IJCAI Conf.* p 2443-2449
[17] Chavira M and Darwiche A 2005 in *Proc. 2005 IJCAI Conf.* p 1306-1312
[18] Langseth H and Portinale L 2007 *J. Reliab. Eng. Syst. Saf.* **92(1)** 92-108