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Optimization of Thermal Object Nonlinear Control Systems by Energy Efficiency Criterion.

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Abstract. This article presents the results of thermal object functioning control analysis (heat exchanger, dryer, heat treatment chamber, etc.). The results were used to determine a mathematical model of the generalized thermal control object. The appropriate optimality criterion was chosen to make the control more energy-efficient. The mathematical programming task was formulated based on the chosen optimality criterion, control object mathematical model and technological constraints. The "maximum energy efficiency" criterion helped avoid solving a system of nonlinear differential equations and solve the formulated problem of mathematical programming in an analytical way. It should be noted that in the case under review the search for optimal control and optimal trajectory reduces to solving an algebraic system of equations. In addition, it is shown that the optimal trajectory does not depend on the dynamic characteristics of the control object.

1. Introduction

Heat treatment is the most energy-intensive stage in the production of construction materials and products [1, 2]. A process should be automated so we can control it effectively. The quality of automatic control systems (ACS) applied will determine to a wide extent how successful automation will be. In turn, the quality of ACS is determined by the assessment of its quality, i.e. by the optimality criterion adopted at the design stage.

Today, there are many optimality criteria developed through practice or based on the "minimal action" principles of theoretical mechanics. In this regard, linear and quadratic optimality criteria are distinguished [2-5]. The application of linear criteria requires a numerical solution of the optimal control problem. Using integral quadratic criteria allows us to design regulators in an analytical way [4].

The authors have developed an integral quadratic energy efficiency criterion, the mathematical expression of which includes controllable and control variables with coefficients that are parameters of the control object mathematical model. This approach helps determine the control device parameters, optimal control of the optimal control system and motion trajectory in an analytical way.

2. Research methods

When designing an energy-efficient control system, the following Modern Control Theory methods were applied [3].

The optimal control problem was solved with respect to thermal control objects.

In our case, the optimal control problem can be formulated as a mathematical programming problem max $J = \int_0^T \{\left[AX + B(x)U\right]' \cdot \left[AX + B(x)U\right]\} d\tau$ with constraints $\frac{dX}{d\tau} = AX + B(x)U$;

$$X(0) = 0; X(T) = X_T . \tag{1}$$

The auxiliary functional takes the following form

$$H = \frac{\left[AX + B(x)U\right]' \cdot \left[AX + B(x)U\right]}{2} + \Lambda \cdot \left[AX + B(x)U\right]. \tag{2}$$

The optimal control and trajectory can be calculated from the following Euler-Lagrange differential equations

$$\frac{\partial H}{\partial U} = 0 = \frac{\partial\left[AX + B(x)U\right]'}{\partial U} \cdot \left[AX + B(x)U\right] + \frac{\partial\left[AX + B(x)U\right]'}{\partial U} \cdot \Lambda; \tag{3}$$

$$-\frac{\partial H}{\partial X} = \frac{d\Lambda}{d\tau} = -\frac{\partial\left[AX + B(x)U\right]'}{\partial X} \cdot \left[AX + B(x)U\right] + \frac{\partial\left[AX + B(x)U\right]'}{\partial X} \cdot \Lambda; \tag{4}$$

$$\frac{\partial H}{\partial \Lambda} = \frac{dX}{d\tau} = AX + B(x)U. \tag{5}$$

After certain algebraic manipulations, the system of equations (3)–(5) takes the following form

$$B'(x) \cdot [AX + B(x)U + \Lambda] = 0; \tag{6}$$

$$\frac{d\Lambda}{d\tau} = -\frac{\partial\left[AX + B(x)U\right]'}{\partial X} \cdot \left[AX + B(x)U + \Lambda\right]; \tag{7}$$

$$\frac{dX}{d\tau} = AX + B(x)U \tag{8}$$

According to equations (6) and (7) it follows that $\frac{d\Lambda}{d\tau} = 0$, since

$$[AX + B(x)U + \Lambda] = 0 .$$

Then it can be argued that $\Lambda = const$.

According to equations (8) and (9), we can obtain the dependence of the speed of the object (1) moving along the optimal trajectory on the value Λ $\frac{dX}{d\tau} = -\Lambda$.

Hence, we can conclude that the object (1) should move along the optimal trajectory at a constant speed $\frac{dX}{d\tau} = const$. Since the initial $X(0)$ and final $X(T)$ states of the control object are given, the optimal speed can be calculated from the formula

$$\frac{dX}{d\tau} = \frac{\left[X_T - X_0\right]}{T} \tag{9}$$

and the optimal trajectory $\rightarrow X^0(\tau) = X_0 + \frac{\tau \cdot \left[X_T - X_0\right]}{T}$.

The optimal control can be determined from equations (8) and (9)

$$U^0(X) = \left[B(x)\right]^{[-1]} \cdot \left\{ \frac{\left[X_T - X_0\right]}{T} - AX \right\} \tag{10}$$

or

$$U(\tau) = \left[B(x)\right]^{[-1]} \cdot \left\{ \left[I - A \cdot \tau\right] \cdot \frac{\left[X_T - X_0\right]}{T} - AX_0 \right\}$$

The results of analytical work can be illustrated by a practical example. Let the control object is a dryer, in which the drying process is controlled by the temperature of the drying space environment. As is known, evaporation happens when heat is supplied to material in the drying process. To maintain a dryer's internal environment temperature at the desired level, the common practice is to utilize industrial regulators in order to implement positional, proportional and proportional-integral regulation laws. The above laws are intended only to ensure the specified accuracy of regulation. The efficiency of the use of thermal energy in the calculation of systems utilizing such regulators is not taken into account. To compare the energy efficiency of a traditional system with the optimal one by criterion (2), it is necessary to determine the mathematical model of the dryer. The calculated model of the dryer and the structural diagram of heat flows can be represented as in figure 1.

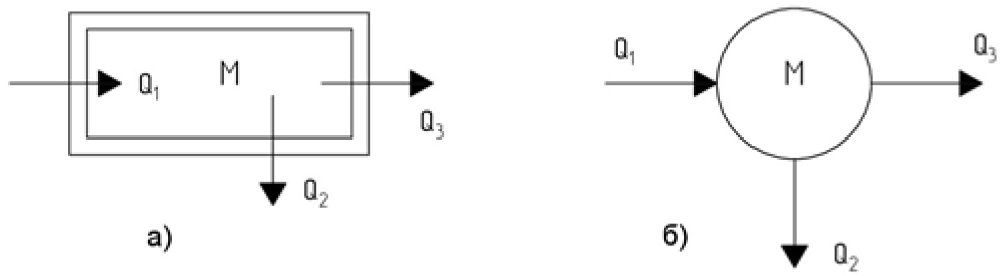

Figure 1. a) calculated model of the dryer; b) structural diagram of heat flows.

The following notations are used here: $Q_1 = cGt_1$ - heat flow supplied by the heat-transfer agent, $Q_2 = cGt_1$ - heat flow leaving with the spent heat-transfer agent, $Q_3 = FK(t_1 - t_{oc})$; $M = cmt_1$ - heat accumulation in the dryer's internal environment; *c*- heat-transfer agent specific heat capacity; *G*- heat-transfer agent flow rate; *t*- coolant temperature; t_1- temperature of the dryer's internal environment; F - dryer inner surface area; *m* = dryer's internal environment mass; *K* - coefficient of heat transfer from the dryer's internal environment to the external environment; t_{oc}- external environment temperature.

According to the structural diagram (figure 1(b)), the heat balance can be represented by a differential equation

$$\frac{dM}{d\tau} = Q_1 - Q_2 - Q_3$$

or

$$cm \cdot \frac{dt_1}{d\tau} = cGt - cGt_1 - FKt_1 - Fkt_{oc} \tag{11}$$

If the constant temperature (t_{oc}) of the environment is taken as a reference point, then equation (11) will be as follows (at the constant temperature (*t*) of the heat-transfer agent):

$$\frac{dx}{d\tau} = ax + (ex + f)u, \tag{12}$$

where: $x = t_1 - t_{oc}$; $a = -\dfrac{FK}{cm}$; $u = G$; $e = -\dfrac{1}{m}$; $f = \dfrac{t}{m}$.

If we put $(ex + f) = b(x)$, then equation (15) takes the following form

$\dfrac{dx}{d\tau} = ax + b(x)u,$ which is analogous to equation (1).

In this example, the criterion (2) is represented by the functional

$$J = \int_0^T \left[ax + b(x)u\right]^2 d\tau \tag{13}$$

Then the optimal equation problem can be formulated as follows

$$\min\ J = \int_0^T \left[ax + b(x)u\right]^2 d\tau \tag{14}$$

with the following constraints: $\dfrac{dx}{d\tau} = ax + b(x)u$; $x(0) = x_0$; $x(T) = x_T$.

The auxiliary functional is determined by the expression

$$H = \frac{\left[ax + b(x)u\right]^2}{2} + \left[ax + b(x)u\right]\Lambda. \tag{15}$$

The problem (17) can be tackled by Pontryagin's method

$$\frac{\partial H}{\partial u} = 0 = b(x)\left[ax + b(x)u\right]; \tag{16}$$

$$\frac{\partial H}{\partial x} = -\frac{d\Lambda}{d\tau} = \left\{\frac{\partial\left[ax + b(x)u\right]}{\partial x}\right\} \cdot \left[ax + b(x)u + a\Lambda\right] \tag{17}$$

$$\frac{\partial H}{\partial \Lambda} = \frac{\partial x}{\partial \tau} = ax + b(x)u. \tag{18}$$

After certain algebraic manipulations, the equation (16) takes the following form

$$b(x) \cdot \left[ax + b(x)u + \Lambda\right] = 0. \tag{19}$$

Since $b(x) \neq 0$, equations (17) and (18) can be written down as follows

$$\left[ax + b(x)u\right] + \Lambda = 0; \tag{20}$$

$$\left[ax + b(x)u\right] - \frac{dx}{d\tau} = 0. \tag{21}$$

According to equations (20) and (21) it follows that $\dfrac{dx}{d\tau} = -\Lambda.$

The equation (17) can be replaced by the expression

$$-\frac{d\Lambda}{d\tau} = \left\{\frac{\partial\left[ax + b(x)u\right]}{\partial x}\right\} \cdot 0. \tag{22}$$

We can conclude from the equation (22), that

$$\frac{d\Lambda}{d\tau} = 0. \tag{23}$$

Then the conjugate variable vector has constant coordinates

$$\Lambda = const \tag{24}$$

and, consequently, the vector

$$\frac{dx}{d\tau} = const. \tag{25}$$

Based on the obtained results (24) and (25), as well as constraints of the original problem (14), it is possible to calculate the analytic expressions of the optimal trajectory

$$\left(\frac{dx}{d\tau}\right)^0 = \frac{[x_T - x_0]}{T}; \tag{26}$$

$$x^0(\tau) = x_0 + \frac{\tau[x_T - x_0]}{T}; \tag{27}$$

and control

$$u^0(x) = \frac{[x_T - x_0]}{[T \cdot b(x)]} - \frac{a}{b(x)}x; \tag{28}$$

$$u^0(\tau) = \frac{\left[(x_T - x_0)\left(\frac{1 - a\tau}{T} - ax_0\right)\right]}{\left[e\left(x_0 + \frac{\tau[x_T - x_0]}{T}\right) + f\right]}. \tag{29}$$

Let $x_0 = 0$; $x_T = 1$; $a = -1$; $b(x) = (ex + f)$; $e = -1$; $f = 1$; $T = 4$.

Then the behavior of the optimal system in the transition from the state $x_0 = 0$ to $x_T = 1$ can be illustrated by the following diagrams (see figure 2) obtained on the basis of the following expressions:

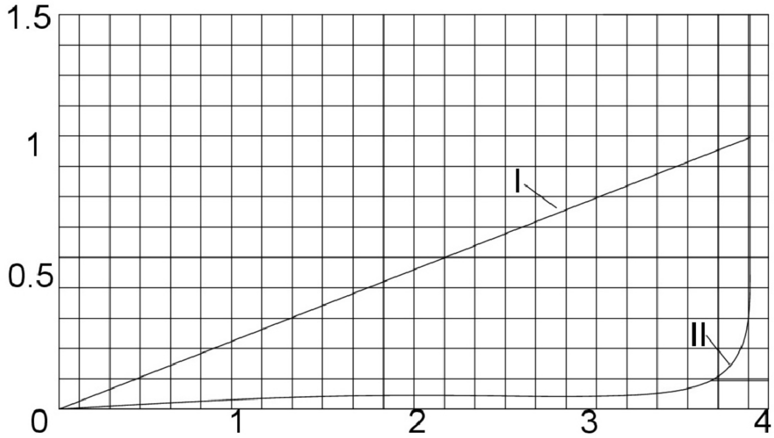

Figure 2. Optimal trajectory (I) and control (II) diagrams.

$$x^0(\tau) = \frac{\tau}{4}; \tag{30}$$

$$u^0(\tau) = \frac{(1+\tau)}{(4-\tau)}. \tag{31}$$

In the example above, the most unfavorable initial data were chosen, where at $\tau \to 4$ and $u(4) \to \infty$.

Therefore, the target x_T cannot be achieved within a given time period, because of an infinitely big control. Hence, the problem (14) is solved taking into account the constraints on control $\left[u^0(\tau) \le u_{\max}\right]$. Thus, the problem (14) is solved in two steps: first, we should solve the problem analytically without taking into account the constraints on control, and then change the duration of the control interval or boundary conditions to achieve the required accuracy of the value x_T.

In practice, the probability of such ratios is very small, since the value $f >> e$, i.e. $f/e = t$. Generally, the temperature of heat-transfer agents used in the drying of construction materials exceeds 100°C.

3. Conclusion

The design methodology developed simplifies the procedure for designing energy-efficient one-dimensional and multi-dimensional systems, because there is no need to solve systems of non-linear differential equations.

Since the mathematical expression of the proposed energy efficiency criterion includes controllable and control variables with coefficients that are parameters of the control object mathematical model, the problem of synthesizing the optimal automatic control system reduces to identifying the control object.

It should be noted that the optimal trajectory does not depend on the parameters of the mathematical model of the control object.

The practicality of the proposed method is proven by the above example of solving the optimization problem.

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