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Characteristics of Crushing Energy and Fractal of Magnetite Ore under Uniaxial Compression

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Abstract. The crushing mechanism of magnetite ore is a critical theoretical problem on the controlling of energy dissipation and machine crushing quality in ore material processing. Uniaxial crushing tests were carried out to research the deformation mechanism and the laws of the energy evolution, based on which the crushing mechanism of magnetite ore was explored. The compaction stage and plasticity and damage stage are two main compression deformation stages, the main transitional forms from inner damage to fracture are plastic deformation and stick-slip. In the process of crushing, plasticity and damage stage is the key link on energy absorption for that the specimen tends to saturate energy state approaching to the peak stress. The characteristics of specimen deformation and energy dissipation can synthetically reply the state of existed defects inner raw magnetite ore and the damage process during loading period. The fast releasing of elastic energy and the work done by the press machine commonly make raw magnetite ore thoroughly broken after peak stress. Magnetite ore fragments have statistical self-similarity and size threshold of fractal characteristics under uniaxial squeezing crushing. The larger ratio of releasable elastic energy and dissipation energy and the faster energy change rate is the better fractal properties and crushing quality magnetite ore has under uniaxial crushing.

1. Introduction

Magnetite ore is an important raw material in iron and steel industry, because of the low grade of iron ore in many countries in the world, the ore can be used for smelting after being crushed and grinded and chose to become concentrate. According to the law of thermodynamics, material failure is driven by energy fundamentally [1]. Under the influence of the complex action from geologic activity and ore blasting, the internal structure of the magnetite ore is discontinuous and the mechanical property has obvious non-linear characteristic, the traditional rock mechanics method is difficult to explain the essence of the crushing process. Energy dissipation promotes damage evolution and strength reduction, and energy release leads to failure of specimen. Therefore, it is closer to the actual situation to study the breaking law of magnetite in the aspect of energy. Based on the energy principle, many scholars use statics test method to explore the energy law and failure criterion of rock destabilization failure [3-11].

Crushing is a critical process for magnetite ore processing, the rock fragmentation has the self-similarity of geometrical shape, and its distribution is a fractal structure [12-16]. The energy and material are exchanged with the ones in outside world during the crushing process of magnetite ore, and the distribution of fragmentation has an important influence on the subsequent grinding and monomer dissociation, and clarifying the energy and fractal characteristics in the crushing process of

magnetite ores can help to reveal the breaking law and solve the problem of high energy consumption in magnetite grinding, which has important theoretical value and engineering significance. However, the literature on the mechanism of magnetic iron ore fragmentation is still rare. One of the main functions of the mechanical crushing of magnetite is uniaxial crushing. Therefore, on the basis of the physical characteristics of magnetite ore, standard cuboid specimens were processed to carry out uniaxial crushing tests to analyse the essential laws of energy absorption and transformation during the crushing process of magnetite ore, and study the mechanism of energy evolution during crushing, fractal characteristics and their related relationships.

2. Uniaxial crushing test scheme

The random sampling was carried out at a blasting site in Shui Chang Iron Mine which is affiliated to Shou Gang Group Mining Company in China. The average density of magnetite ore is $3.26 \times 10^3 \text{kg/m}^3$ in natural state, the grade is 28%. It can be known that the main components of magnetite ore are O, Si and Fe, while the magnetite iron ore contains small proportions of Mg and Al and a lot of existed fissures and pores through scanning electron microscope tests. The magnetite ore with the same block size of the same mine pile were selected for being cut, polished and processed to be specimens with 70mm width, 70mm length and 35mm thickness. The size accuracy of the specimen meets the requirements of the Rock Mechanics Test Code (SL264-2001) of Hydraulic and Hydroelectric Engineering in China, and the uniaxial crushing tests of the standard cuboid specimens were carried out by using ATW-3000 electro-hydraulic servo test machine.

The uniaxial compressive test of magnetite is operated in strictly accordance with the rules of rock mechanics test. The preload 3KN is applied firstly, then the flexible displacement control is used, and the loading rate is 0.0025mm/s. The servo control machine records the load, displacement, loading time and so on, and simultaneously draws the load-displacement curve. When the specimen breaks down, the loading stopped, the states of the specimen before and after loading are shown in figure 1.
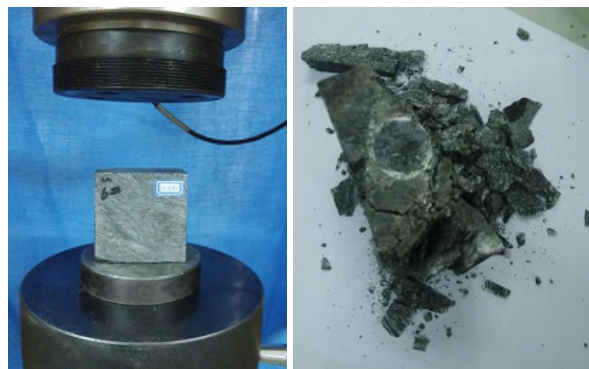

Figure 1. The states of specimen before and after loading.

3. Deformation characteristics during uniaxial crushing process

The mechanical properties have a certain difference from the original ore mass because of the effect of blasting shock wave and high-speed impact. According to the recorded results of the load displacement, the stress-strain curves of the crushing of magnetite were plotted, as shown in figure 2. According to the modified Hock Brunn Criteria of jointed rock mass studied by E HOEK and D WOOD [17], and the GE X R's division on uniaxial stress-strain curve [18], the stress-strain curve of the uniaxial crushing of the magnetite ore is divided into the compaction stage (OA), elastic stage (AB), plasticity and damage stage (BC), unstable crushing stage (CD) based on the variation of stress-strain curve slope. O is the starting point of the curve, the slope of the OA segment is always increasing, the slope of AB remains unchanged, and the stress and strain are linearly changing. The curve slope becomes smaller after B, and the whole curve of BC segment is concave. C is the curve vertex, CD segment drops rapidly, D is the end point of loading. The proportions of loading time in each stage are OA: 45.96%~53.83%, AB: 5.88%~10 69%, BC: 62%~10.69%, CD: 7.03% ~ 8.87%. The average value of the strain range in compaction phage is 53.74% in total strain, and the ratio of

the maximum stress to the peak stress is 46.55%. The elastic stage range is smaller than that of ordinary hard brittle rock, but the ratio of maximum stress in elastic stage to peak stress is much greater than that of ordinary hard brittle rock [19]. The time ratios in each stage are corresponding to strain ratios.
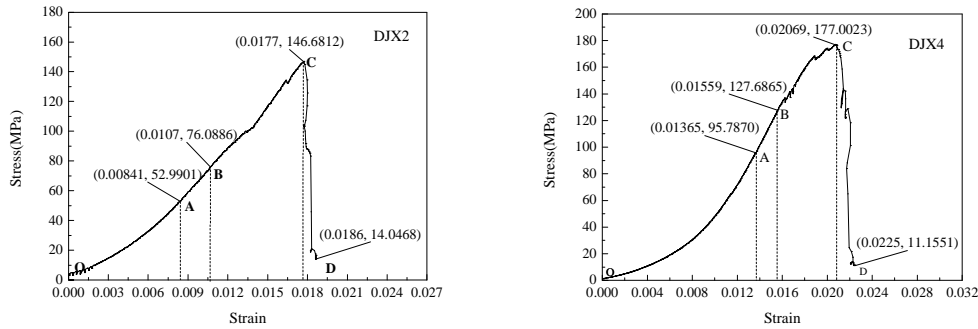

Figure 2. Strain-stress curves of magnetite ore in uniaxial crushing.

The compaction stage and the plasticity and damage stage account for the main part of the crushing process of the uniaxial crushing of the magnetite ore. The scope of compaction stage shows that the cracks in the magnetite ores are abundant. The curves recovery plasticity rises after significant stress damage, and the sticky-slip phenomenon occurs in the plastic and damage state [20]. It is indicated that the new crack initiation is significant and the pore expands after the elastic stage, plastic deformation and sticky slip are the main transitional forms of the crushing process of magnetite ores. With the increase of the stress, the slip degree of ore particle and rock particle and the expansion degree of porosity and fissure increase, the stress concentration of pores and fissures reaches the limit at the peak stress, and a large number of pores and fissures are rapidly connected to form shear and tensile failure inner magnetite ore, and then the stress drops quickly.

4. Energy evolution analysis of crushing process

4.1. Energy mechanism of uniaxial compression
The energy absorption, transformation and dissipation occur during the uniaxial crushing process of magnetite specimen. During the loading process, the test machine has done the mechanical work to the magnetite, which causes the specimen deformed, damage, destroy and break. Assuming that the test machine loading system is a closed-energy system, the mechanical work done by the press machine can be regarded as the total energy absorbed by the specimen. According to the first law of thermodynamics, in the case of no heat exchange with the outside world, the total energy absorbed by the specimen U equals the sum of dissipation energy U_d and the releasable elastic strain energy U_e, namely:

$$U = U_d + U_e \tag{1}$$

$$U = \int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1 + \int_0^{\varepsilon_2} \sigma_2 d\varepsilon_2 + \int_0^{\varepsilon_3} \sigma_3 d\varepsilon_3 \tag{2}$$

$$U^e = \frac{1}{2E_0}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right] \tag{3}$$

In uniaxial compression tests, confining pressure $\sigma_2=0$ and $\sigma_3=0$, the total absorbed energy and the releasable elastic strain energy of the specimen under uniaxial compression are respectively calculated as follows [1]:

$$U = \int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1 = \sum_{t=0}^{n} \frac{1}{2}(\varepsilon_{i+1} - \varepsilon_i)(\sigma_{i+1} + \sigma_i) \tag{4}$$

$$U^e = \frac{\sigma_i^2}{2E_O} \tag{5}$$

Where ε_i, σ_i are the stress value and strain value of specimen at a time in uniaxial compression process, according to rock strength and overall failure criterion based on the principle of energy dissipation and release, E_0 is initial elastic modulus of cell without damage for simplified computing, the elastic modulus at the end of the elasticity stage is the initial elastic modulus [2, 4].

4.2. Energy evolution analysis of crushing process

The energy curves during uniaxial compression of magnetite ore are drawn, as shown in figure 3. The peak stresses of magnetite ore were counted, as well as absolute absorption energy, releasable elastic energy and dissipation energy of critical points (A, B and C) between deformation stages, and the absolute absorption energy, the residual releasable elastic energy and the maximum dissipated energy at the end of loading (D). Absolute absorption energy is the value of each point in absorption energy curve. The ratios of the change of the absorption energy (ΔU), the change ratios of the releasable energy (ΔU_e) and dissipation energy (ΔU_d) to the total absorption energy U_0 were calculated in the four deformation stages as shown in table 1. The average change rates of absorption energy, the releasable elastic energy and the dissipation energy are calculated in each stage, as shown in table 2, the positive change rates indicates the increase of energy, and the negative change rates indicates energy reduction. The ratios of releasable elastic energy (U_e), dissipation Energy (U_d) accounted for the absorption energy (U) were calculated at the transition points and the loading end moment, as shown in table 3.
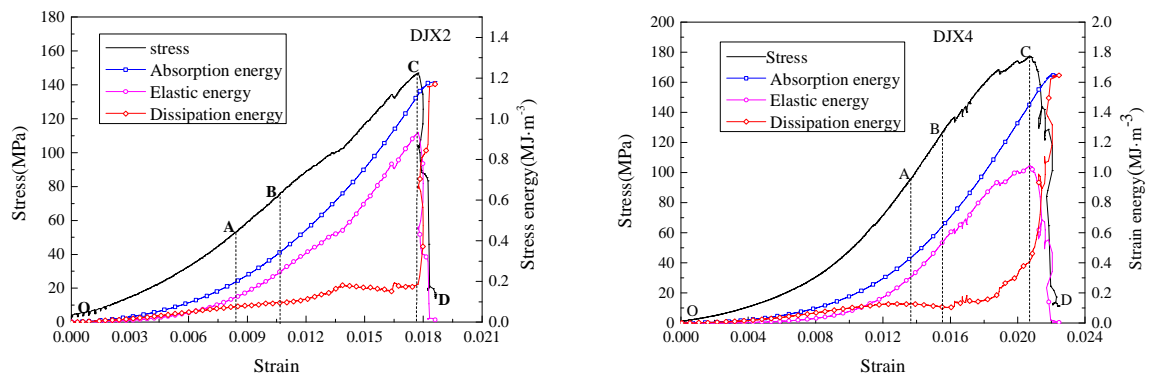

Figure 3. Energy evolution curves of raw magnetite ore in uniaxial squeezing crushing.

Table 1. The change ratios of strain energy in total absorption energy in stages.

Number	P_{max} (MPa)	Compression stage			Elastic stage			Plasticity and damage stage			Crushing stage		
		$\Delta U/U_0$	$\Delta U_e/U_0$	$\Delta U_d/U_0$	$\Delta U/U_0$	$\Delta U_e/U_0$	$\Delta U_d/U_0$	$\Delta U/U_0$	$\Delta U_e/U_0$	$\Delta U_d/U_0$	$\Delta U/U_0$	$\Delta U_e/U_0$	$\Delta U_d/U_0$
DJX6	113.18	0.232	0.143	0.084	0.066	0.069	0.001	0.549	0.415	0.138	0.153	-0.608	0.760
DJX2	147.69	0.169	0.105	0.065	0.121	0.099	0.016	0.653	0.580	0.075	0.058	-0.776	0.833
DJX4	177.24	0.262	0.184	0.077	0.129	0.147	-0.013	0.488	0.298	0.184	0.121	-0.627	0.747
Average	146.04	0.22	0.14	0.08	0.11	0.11	0.00	0.56	0.43	0.13	0.11	-0.67	0.78

Table 2. The change rate of strain energy in stages (KPa·m^{-3}s^{-1})

Number	P_{max} (MPa)	Compression stage			Elastic stage			Plasticity and damage stage			Crushing stage		
		$\frac{\Delta U}{\Delta t}$	$\frac{\Delta U_e}{\Delta t}$	$\frac{\Delta U_d}{\Delta t}$	$\frac{\Delta U}{\Delta t}$	$\frac{\Delta U_e}{\Delta t}$	$\frac{\Delta U_d}{\Delta t}$	$\frac{\Delta U}{\Delta t}$	$\frac{\Delta U_e}{\Delta t}$	$\frac{\Delta U_d}{\Delta t}$	$\frac{\Delta U}{\Delta t}$	$\frac{\Delta U_e}{\Delta t}$	$\frac{\Delta U_d}{\Delta t}$
DJX6	113.18	0.76	0.47	0.28	1.98	2.09	0.02	3.10	2.34	0.78	3.06	-12.2	15.19
DJX2	147.69	0.81	0.50	0.31	2.49	2.05	0.33	3.92	3.48	0.45	1.82	-24.35	26.13
DJX4	177.24	1.13	0.80	0.33	3.97	4.52	0.41	5.66	3.46	2.14	3.92	-20.39	24.28
Average	146.04	0.90	0.59	0.31	2.81	2.89	0.25	4.23	3.09	1.12	2.93	-18.98	20.66

Table 3. The ratios of strain energy in absorption energy at the transition points and the end of loading

Number	P_{max} (MPa)	A		B		C		D	
		U_e/U	U_d/U	U_e/U	U_d/U	U_e/U	U_d/U	U_e/U	U_d/U
DJX6	113.18	0.618	0.363	0.715	0.285	0.742	0.264	0.020	0.983
DJX2	147.69	0.623	0.385	0.706	0.279	0.833	0.166	0.008	0.989
DJX4	177.24	0.704	0.295	0.848	0.163	0.716	0.282	0.003	0.995
Average	146.04	0.65	0.35	0.76	0.24	0.76	0.24	0.01	0.99

4.2.1. Analysis *on the change of absorption energy.* The results of the calculation in table 1 shows that the average ratios of absorption energy in each stage accounted for the total absorption energy indicate that the absorption energy fluctuation changes during the compression process, and the plasticity and damage stage is the main link of energy absorption. It can be seen from table 2 that the change rate of the absorption energy increases in turn in the compaction stage, the elastic stage, the plasticity and the damage stage, and the change rate of absorption energy in the crushing stage is reduced, indicating that the capacity of the specimen absorbing mechanical energy tends to saturation near the peak strength, and the breakage happens when the absorption energy reaches the limit in the specimen.

4.2.2. Analysis *on the change of the releasable elastic energy.* The ratios of the amount of releasable elastic energy to the total absorption energy in the process of specimen compression and deformation were shown in table 1. Because of the mineral Fe_3O_4 and Al_2O_3 in the magnetite specimen in the test, the plastic deformation and damage occur in the plastic and damage stage, and the elastic deformation of the specimen was still in a certain degree at the same time. The absorption energy can be converted into the releasable elasticity energy, which is mainly caused by internal stress and elastic modulus of the specimen. The elastic strain energy is higher in the plastic and deformation stage than that in the elastic stage. The ratio of elastic strain energy released in the crushing stage to total absorption energy is 60.79% ~ 77.61% (average value 67%). The remaining absorption energy dissipated in the crack compaction, initiation, expansion, acoustic emission, infrared radiation before the peak stress, and so on. The concentrated releasing of elastic-strain energy is the main reason for the fragmentation of magnetite specimen. According to the calculation results of table 2, the absolute values of the average change rate of the releasable elastic energy in the crushing stage are respectively 32 times, 6.56 times and 6.14 times than the ones in former stages before crushing.

Table 3 shows that the absorption energy is stored in the form of the releasable elastic energy, and the less absorption can be consumed before the peak stress in the aspects of fissure and pore compaction, mineral particle movement, volume element deformation, internal local damage, specimen temperature change, thermal radiation, acoustic emission, etc. When the stress reaches the peak value, the energy storage ability of the specimen reaches limit, and the most of releasable elastic energy in the specimen is released and consumed by the crack extension and the block splitting. However, the specimen has been separated into blocks at the end of loading and the mechanical energy produced by the press machine can be directly consumed in the further broken of blocks. It should be noted that the average ratio of releasable elastic energy stored in the specimen accounts for 72.3% to the total absorption energy before the peak stress, which is higher than the average ratio (67%) of the releasable elastic energy consumed in specimen crushing to the total absorbed energy, indicating that elastic strain energy stored in the specimen before the peak stress can't be used in the specimen crushing thoroughly. A small portion of the stored elastic strain energy can be dissipated by the shape recovery of many crystals during unloading, which is the result of rich containing Fe_3O_4, Al_2O_3 and other minerals in specimens.

4.2.3. Analysis *on the change of dissipation energy.* Energy dissipation always happens from deformation beginning to the end of loading, the ratio of dissipation energy to the total absorption energy can reflect the rock deformation and damage degree [1, 14]. It can be seen from table 1 that the damage degree of the magnetite ore is very small in the elastic deformation stage, and the dislocation and slippage of mineral particles occur in the plastic and damage stage. After the peak stress, the dissipation energy ratio in the crushing stage is 78% to the total absorption energy, which is higher than released energy, whose ratio is 67% to the total absorption energy. The cause of this phenomenon is that, the deformation still occurs after the peak stress, the mechanical work from the press machine directly acts on the further crushing of the specimen, the dissipated energy of the crushing stage should be the sum of the releasable elastic energy and the partial mechanical energy.

The change rate of dissipation energy can reflect the activity from deformation to damage inner specimens. According to the calculation results of table 2, the average change rates of dissipation energy indicates that the fracture compression and less crack initiation happen in the compaction stage and the elastic stage, granular slip dislocation and large fissure initiation and expansion are active in the plastic and damage stage, the crack through the junction is very rapid after the peak stress.

5. Fractal characteristics of uniaxial extrusion crushing of magnetite ore

5.1. Fractal theory of rock fragmentation

Self-similar characteristics of rock failure and fragmentation are usually described with fractal dimension theory [12, 21]. Fractal dimension is the main geometrical parameter describing the self-similarity of rock fragmentation statistics. At present, the method of double logarithm fitting of granularity-accumulative mass percent is widely used in the research of fragmentation fractal[18, 21, 22].

$$Log\frac{M(r)}{M}=bLogr \tag{6}$$

$$D=3-b \tag{7}$$

Where, *M(r)/M* is a percentage of the quality of rock masses with a block degree less than *r* in total crushing mass, *r* is feature size of fragment, *D* is fractal dimension of fragment size, *b* is the slope of the regression line in the double logarithm coordinate system.

5.2. Fractal characteristics of magnetite ore block size

The fragments of magnetite ore were sieved; the cumulative mass distribution curves were drawn. When the feature sizes of magnetite fragments are smaller than 20mm, the cumulative mass distribution curves show good linearity, however the quality of fragments whose feature sizes is larger than 20mm occupies main part in total fragment quality.
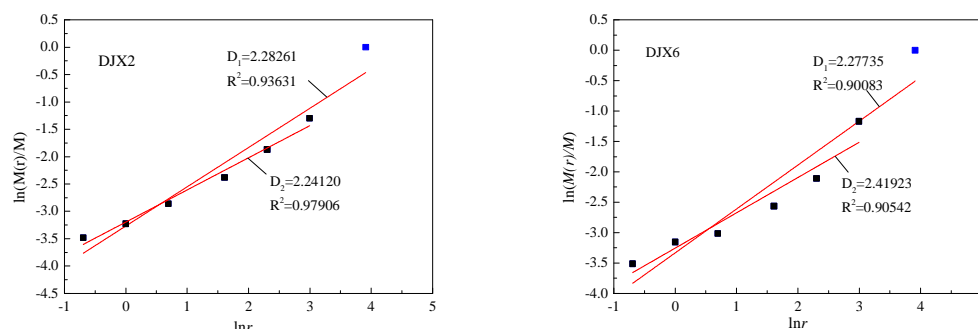

Figure 4. Double logarithm curves between quality percentage and granularity.

According to the sieving results of fragment, the double logarithm scatter plots between the cumulative mass percent (M(r)/m) and the granularity (r) are drawn, as shown in figure 4. The two-logarithm scatter graphs for all fragments and fragments smaller than 20mm were fitted on fractal dimension respectively to gain the fractal dimension of all fragments, ie D_1, and the fractal dimension of less than 20mm fragments, ie D_2, as shown in table 4. The results of fractal dimension calculation show that the uniaxial crushing of magnetite ore is statistically self-similar, and the statistical self-similarity in the small scale range is higher than that in all the fragmentation, which is consistent with fractal characteristics of rock fragmentation [12-15]. According to figure 4, the fitting results of fragmentation distribution and fractal change significantly at 20mm, which reflects that magnetite ore has the dual-scale self-similarity of under uniaxial compression. The ore fragments of magnetite have self-similar characteristics in the small size range and the whole size range, but the significance is different. According to the research results by HUANG D, et al [14], 20mm can be used as the size threshold of fractal dimension of magnetite ore under uniaxial crushing, and the fractal characteristic of magnetite iron ore was weakened when fragment sizes are larger than 20mm.

Table 4. Fractal dimensions of raw magnetite ore under uniaxial squeezing crushing.

Specimen number	Fractal dimension calculation results of all fragments			Fractal dimension calculation results of fragments smaller than 20mm		
	Fitting formulas	D_1	R^2	Fitting formulas	D_2	R^2
DJX2	y=0.71739x-3.26922	2.28261	0.93631	y=0.58802x-3.1974	2.41198	0.97906
DJX3	y=0.69117x-3.16396	2.30883	0.93233	y=0.56259x-3.09257	2.43741	0.97712
DJX4	y=0.72053x-3.34569	2.27947	0.90691	y=0.5732x-3.2639	2.4268	0.93498
DJX5	y=0.69444x-3.19903	2.30556	0.92849	y=0.55958x-3.12416	2.44042	0.97996
DJX6	y=0.72265x-3.33449	2.27735	0.90083	y=0.58077x-3.25573	2.41923	0.90542

5.3. Correlation analysis of fractal features and energy

The fragmentation of materials is the result of energy evolution, and fractal dimension is the quantitative describe of fragmentation characteristics. Therefore, the fractal characteristics of magnetite ore fragments after uniaxial crushing are related to the energy characteristics. The magnetite ore are broken down integrally under compression in dressing, combining the energy characteristic data of table 1 and table 2, the relation curves between fractal dimension of all fragments (D_1) and strain energy ratios ($\Delta U_e/U_0$ and $\Delta U_d/U$)accounting for total absorption energy in the crushing stage were plotted, the relation curves between the fractal dimension of all fragments and the strain energy change rate ($\Delta U_e/\Delta t$ and $\Delta U_d/\Delta t$) in the crushing stage were plotted as well, as shown in figure 5. It can be seen that in the crushing stage, the ratio of the elastic energy reduction to the absorption energy and the absolute value of the reducing rate increase with the increase of the integral fractal dimension, and the dissipation energy ratio to the absorption energy and the change rate are basically increase with the increase of the integral fractal dimension.
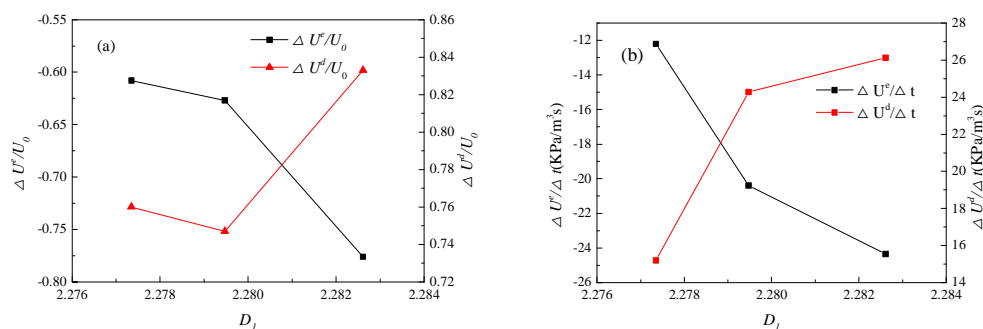

Figure 5. Relational curves between whole fractal dimension and strain energy.

The release value and release rate of elastic energy affect the fragmentation degree, as well as dissipation energy in the crushing process of magnetite ores. The higher ratios of energy release and

dissipation to the total energy and the faster the change rates of energy are, the more obvious the fractal characteristics of all magnetite ore is, and the higher crushing degree can be gained.

6. Conclusions

Following conclusions can be obtained according to the present study:

(1) During the magnetite ore crushing process under uniaxial compression, the compaction stage and plasticity and damage stage are the main deformation stages, plastic deformation and visco-slip are the main transitional forms from elastic deformation stage to plasticity and damage stage, the proportion of strain range in different stages corresponds well with time ratio.

(2) The plasticity and damage stage is the main stage on energy absorption and transformation inner specimen. The absorption energy value varies with the loading process. The average ratio of releasable elastic energy stored in the specimen before the peak stress is higher than that consumed in specimen crushing of the total absorbed energy. After the peak stress, the press machine continues to work to make some blocks much more broken, the average ratio of the energy consumed in specimen crushing accounting for the total absorption energy is 78%. The damage evolution of magnetite can be realized by energy dissipation, and the rapidly release of elastic energy in the crushing stage is the fundamental reason of the disintegration of magnetite.

(3) The uniaxial crushing of the magnetite ore is characterized by dual-scale fractal characteristics, and the fractal characteristics of fragments smaller than 20mm are more obvious than that of all fragments. The uniaxial compression crushing of magnetite ore has a size threshold of fractal dimension. The crushing energy of magnetite ore is related with the fractal dimension of all blocks. The more the releasable elastic energy ratio and release rate, dissipation energy ratio and dissipation rate are, the higher the fractal dimension is , the better fragmentation effect of magnetite ore can be get.

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