The Enactment of Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge in Teaching Practice of Ratio and Proportion: A Case of Two Primary

To cite this article: R Ekawati et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 288 012122

View the article online for updates and enhancements.
The Enactment of Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge in Teaching Practice of Ratio and Proportion: A Case of Two Primary

R Ekawati¹*, F Lin² and K Yang²

¹Department of Mathematics, Universitas Negeri Surabaya, Indonesia
²Department of Mathematics, National Taiwan Normal University

*rooselynaekawati@unesa.ac.id

Abstract. This study draws upon a research in investigating the enactment of primary teachers’ mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) in practice. The teaching on ratio and proportion of two teachers with different level MCK and MPCK were observed within video observation framework which involves qualitative research through case study. The result indicated that all factors were presented by Good MCK and MPCK teacher (GG) teacher appropriately but not for Low MCK and MPCK teacher (LL) teacher. LL ignored Figural representation factor and showed some errors in her teaching. MCK and MPCK are very essential for mathematics teaching. The evidence leads to the opportunity to learn for in-service primary teachers.

1. Introduction

Increased attention has been paid to teachers Mathematics content knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) as key resources for the work of mathematics teaching. Some studies suggest the nature, depth and organization of teacher knowledge influences teachers’ presentation ideas, flexibility in responding to students' questions, and capacity for helping students connect mathematical ideas [1][2]. An observational study on teacher knowledge and teacher’s practice showed that teachers with strong mathematical knowledge made fewer errors and provided rich examples of mathematics [3]. Furthermore, teachers with lower level content knowledge could exhibit some of their characteristics in their instruction, but it was not consistent across their lesson. Teacher with deep understanding of mathematical concepts may enable teachers to access a broad repertoire of strategies for explaining and representing mathematical content to their students [4]. The educators of teacher’s disciplinary training institution in general believe that the teachers who teach mathematics certainly need mathematical knowledge more than the mathematical knowledge being taught [5]. In terms of knowledge related to pedagogy, several studies considered the development of Pedagogical Content Knowledge in teaching such as Goodnough & Nolan [6]. The report showed that several aspects of PCK were engaged such as orientation to the teaching, knowledge and beliefs about curriculum, student learning, instruction and assessment. However, those were not specific on the mathematics subject matter. Therefore, MPCK is used in this study to represent the blending and complex interaction between mathematics content and pedagogy to build an understanding how particular content organized and presented to the learners.
In this study, we consider to point on one specific mathematics content such as ratio and proportion. Ratio and proportion regarded as content which complex and difficult for teachers to teach and for students to learn [7][8]. The blending of MCK and MPCK play an important role in mathematics teaching. It can be observed from how teacher select task for students as well as their teaching strategy. In addition, MPCK also includes the teachers understanding about students learning ratio and proportion, the error and misconception they had and the guidance teachers need to give to avoid misunderstanding. In this field, The need of study that focus on teachers who works in similar context but vary in the mathematics knowledge [9]. This regards the variance of different teachers with different knowledge categories with respect to MCK and MPCK on ratio and proportion in paper pencil instrument test. Some studies on ratio and proportion were used to developed framework for MCK and MPCK instrument item. Teachers’ struggle on ratio and proportion described in the study of Livy & Herbert [10] that shared second year pre-service Teachers demonstrated a lack of knowledge of multiplicative thinking, in particular, where multiplication and division were required within the items.

We explored teachers’ MCK and MPCK on ratio and proportion through paper and pencil test. By exploratory factor analysis on teachers’ response on item instrument, there were three factors of both MCK and MPCK on ratio and proportion. The MCK factors were the meaning of proportional and non-proportional situations; number structure in situation; figural representation [11]. Furthermore, the MPCK factors were Knowing student; Ratio and proportion task level; Teaching strategy of ratio and proportion. Besides, the big attention is currently given to the unique of teaching mathematics. It considered much on the exact knowledge, both content and pedagogical, that teachers need to effectively teach the subject matter [12].

With respect to the MCK and MPCK item on ratio and proportion, teacher’s knowledge was varied within the factors. Some studies showed the relation between teachers’ knowledge and teaching practice with variance of content. However, there is a lack of literature that explored knowledge in practice that focus on one specific content such as ratio and proportion. Regarding these phenomena, in this study, we investigated the enactment of primary in-service teachers’ MCK and MPCK in teaching practice. We assigned teachers participants into three different understanding categories on MCK and MPCK by factor and cluster analysis method namely Good, Middle and Low [11]. Hence, we aimed at reporting the attempt of two primary teachers with varied MCK and MPCK factors to observe the enactment of knowledge in teaching practice via video vignettes. By providing some evidence, it would support the validity of the test of knowledge in practice in which it can help policy maker to determine the appropriate in-service primary Teacher Professional Development.

2. Principle for video observational system and the framework, framework for analyzing MCK and MPCK on teaching video observation

2.1. Principle for video observational system and the framework

Several studies regard the use of video as the basis for assessing teachers’ teaching practice within classroom situations [13] [14]. Furthermore, it attempts to predict teachers’ Mathematics teaching practice from their MCK as well as MPCK captured from paper and pencil test result. The aim of the observational system concerned on the description and explanation of teachers’ major activities within the classroom and their interaction with students. Mathematics teaching occurs in complex situation which could make it impossible to observe everything. Since the focus of this study was examining the impact of knowledge within teaching practice, the feature of the mathematics classroom observation system concerned on the component of MCK and MPCK adapted from Mathematics Quality of Instruction (MQI) framework and several research result with categories of classroom activity such as Radical Monologue (RM), Simple Monologue (SM), Supervised practice (SP) and Dialogue (D). The focus process of classroom interaction consists of activity, communication and negotiation in the observational system.
2.2. Framework for analyzing MCK and MPCK on teaching video observation
The three MCK and MPCK factors resulted from Exploratory Factor Analysis on paper and pencil test were the main component for observation. Those two knowledge domains are possible to be activated and presented in mathematics teaching or not. However, the presentation could be appropriate and inappropriate with regards to several resources in relation to ratio and proportion teaching. Below is two-dimensional framework with the criteria for presentation of MCK and MPCK in teaching ratio and proportion. With its presentation, the overview of teachers with different categories could clearly observe. The framework for analyzing video teaching with regard MCK and MPCK consist of three presentations such as Presented and Appropriate; Presented and Inappropriate as well as Not Presented. Three factors of MCK are Number structures in situation, Meaning of proportional situation and Figural representation. In addition, the three factors of MPCK are ratio and proportion task level feature; Teaching problem solving strategy; Knowing students’ conceptual understanding.

3. Methods
The method of this study involves qualitative research where researcher wants to understand the phenomena of the implementation of knowledge in practice through case study. A case study was chosen to be a way of investigating an empirical topic by following a set of pre-specified procedures [15]. Researchers chose two in-service primary teachers from 271 teachers’ participants of paper and pencil test with result of Good MCK and Good MPCK here after called as GG teacher and another one teacher with Low MCK and Low MPCK called LL teacher. We observed, take notes and also videotaped the teachers’ teaching. The analysis of the enactment of MCK and MPCK in teaching practice is based on the principle for video observational system and the framework adapted from Mathematics Quality of Instruction (MQI) framework with regards MCK and MPCK factors resulted from paper and pencil test.

4. Results and discussion

4.1. GG and LL teachers’ classroom activities
Analysis is carried out on two participants that is GG and LL teachers. GG and LL teachers applied simple monologue in their teaching in which teachers posed questions or problems that either answered by themselves after it was responded by the whole class in addition to the radical monologue. These could be interpreted that the common teaching strategy used by teachers was more on the direct instruction model in which teachers frequently taught students with task specific action sequence and model in the given task context. Those two teachers employed students’ group discussion that working on task given. The different appears in exercise phase. GG teacher considered supervised practice which she guided students and provided opportunity to have verbal transaction with students. Besides, she presented more supervised practice activity in terms of leading students in their work time in sharing their problem solution compared to LL.GG teacher always asked students to pay attention to her verbal prompt related to content. In addition, both teachers shared similar tasks to all students.

4.2. The Potential of MCK and MPCK factors in GG’s Teaching
Three meetings of Good MCK and Good MPCK teacher (GG teacher) were observed. With regard number structures in situation, she started with introducing easy integer number structure in ratio such as 1:2, 1:3 and 1:6. She presented the changes of number structure within from 3:6 to 2:3 in which it considered as from easy to more difficult number structures. In addition, inappropriate number structures were presented in the students’ work time with the context of the data of students in classroom. For example, based on the real data of students in classroom, the ratio of boys to all students in group AB was 2/11 and girls to all students in group AB was 9/11. These number structures were unintentionally appeared from the real situation in the classroom. In the third meetings, she also tried to show big integer multiple number structures for ratio as scale topics such as 1:4500000 that can be considered as inappropriate for primary level. She showed this number structures due to its relation to the real map scale available in the classroom and students familiar with.
Therefore, these inappropriate number structure presented were reasonable appear and could be accepted. In terms of number structures presented appropriately, GG teacher shared integer multiple and non-integer multiple number structure. Specifically, she presented hierarchical number structure in teaching practice such as in first meeting started with easy number structure (number structure of multiplication by 2 and multiplication by 6) and move to the second level in which rate easy to find or can be obtained by taking an amount then half as much again (e.g. 4: 6). The hierarchical number structures presented by GG teacher was in line with number structure level.

In terms of MCK factor of meaning of proportional situations, within all teaching stages, GG teacher did not only present situations that appear in the textbook which consider ratio as comparison of quantities. She also presented measurement situation to introduce scale and also enlargement situation for proportional problem in opening stage in the second and third meeting respectively. These situations are appropriate for young child as mention by Norberg et.al [16] in which psychologically in general they are able to recognize similar objects or situation notwithstanding scaling and/or picturing on different sizes. The shared proportional situation started with ratio as comparing quantities situation and continued with situation that incorporated geometrical properties. These could be interpreted that the used situation was from the easy to the more complicated ones and regarded as appropriate presentation of the factor of meaning of proportional and non-proportional situation.

Another finding for the figural representation factor was shown in the student work time in all three meetings. Those were presented in the form of ratio table for teaching to help students to investigate ratio relation of two quantities and to explore the scale of measurement. Besides, it was also shown in the form of drawing enlargement rectangular figure as in the problem of paper and pencil test. It aimed at to bring students make sense of multiplication relation of enlarged rectangular size. To sum up, GG teacher presented all MCK factors in her teaching in addition to emphasizing between ratios in her teaching.

As to the observation on MCK on teaching practice, we explored scene that revealed MPCK factors such as Ratio and proportion task Level Feature, teaching problem solving strategy and knowing students’ conceptual understanding. Regarding MPCK factor of “Ratio and proportion task level feature”, It was highlighted in the connection to the used number structures in that task. The hierarchical number structures appeared within task shared to students presented across the teaching stage. Students did not show any error in solving problem with easy number structure (i.e. doubling number structure). However, when the number structure within proportion problem changed into the harder one (i.e. 2: 3), it found that students felt challenges in solving the problem with equal fraction. GG teacher addressed her students challenged and stated that the non-integer multiplier would influence students’ error. Besides, different situations showed in the problem given also became factor that influences the difficulty of the problem. In the first meeting, she pointed on ratio as comparing quantities context which was easily for students to understand. Continue with ratio as scale in second meeting and ratio and proportion within sharing cakes and enlargement figure in third meeting. The enlargement context was Regarding MPCK factor of “Ratio and proportion task level feature”, It was highlighted in the connection to the used number structures in that task. The hierarchical number structures appeared within task shared to students presented across the teaching stage. Students did not show any error in solving problem with easy number structure (i.e. doubling number structure). However, when the number structure within proportion problem changed into the harder one (i.e. 2: 3), it found that students felt challenges in solving the problem with equal fraction. GG teacher addressed her students challenged and stated that the non-integer multiplier would influence students’ error. Besides, different situations showed in the problem given also became factor that influences the difficulty of the problem. In the first meeting, she pointed on ratio as comparing quantities context which was easily for students to understand. Continue with ratio as scale in second meeting and ratio and proportion within sharing cakes and enlargement figure in third meeting. The enlargement context was felt more difficult by students since they need to integrate the multiplicative thinking in similarities figures. These could be interpreted that teachers identify and used hierarchical task level difficulties in her teaching, though the teaching material (i.e. textbook) did not support the integration
of geometrical figure in proportional problem. The phenomena described above were included in the appropriate presented factor for Ratio and proportion task level feature.

For the MPCK factor of Teaching problem solving strategy, GG teacher presented this factor component in some lesson stages appropriately. The overview of appropriate teaching problem solving strategy factor components was described in this part. In the example stage, GG teacher led students’ discussion to formulate the meaning of ratio and scale for the first meeting and second meeting respectively. GG teacher also gave opportunity to students to explore the solution of exemplary problem such as comparing two proportional problem-solving strategies (i.e. cross multiplication and equivalence fraction). Those two strategies were shared by students themselves as shown in transcribed below

Student 1 : I found three is multiplied by 2 to be 6, so the numerator should be multiplied by 2 so I got 2 cakes (write on board 1 kuex23 anakx 2 = 26)
GG : Good. Who has different solution strategy to solve that problem?
Student 2 : (student wrote in figure 1)

Figure 1. photo of Students’ cross multiplication strategy

Teacher : What is the difference of those two strategies? Anyone try to see the difference.
Student : Cross multiplication
Teacher : I have another problem a thirty pages book could be read within 2 days, how many pages does Mrs. GG teacher read if she has 3 days, what do you think if I used this strategy (refer to equivalence fraction strategy)?
Students : it is more difficult
GG teacher : Yes, it is more difficult since the multiplier is non-integer or in the form of fraction.

In that scene, teacher created appropriate setting to develop the students’ interest in mathematics in which they were motivated to investigate different students’ strategies. Teacher did not directly evaluate and share her opinion to those two different solution strategy, instead, she formulated a new example proportion problem with non-integer multiple number structure and asked other students solution strategy. Afterwards she asked students to pay attention to the problem again and gave opportunity to students to investigate how if it is solved with equivalence fraction strategy as shown by student 2. The way GG teacher gave responses to two different students strategies showed her appropriate feedback and evaluation to help students aware of its differences. Besides, another event in the example stage of Meeting 3, GG teacher was intended to share precise used of language of unit to solve proportional problem. The use of language “for every” stated by teacher could be interpreted that teacher guide students to unitary strategy. Regarding these phenomena, in interview, GG teacher shared that the scene was what she expected in which students shared different solution strategies and all students recognize equivalence fraction strategy first as basic for cross multiplication and it helps students to avoid addition strategy in solving proportional problem.

For the factor of knowing students conceptual understanding, GG teacher created proportional problem that fit to primary level and were not taken from textbook such as investigating ratio as scale within measurement activity from easy number structure to the harder ones. The problem types were missing value problem and ratio comparison problem. In GG teacher’s classroom, there was no task
that can reveal students’ misconception. However, there was a moment in student’s work time stage of third meeting in which students shared errors in finding one missing size of enlargement figures problems. It was shown by two students with similar strategy and another student different solution in figure 2:

![Figure 2. photo of error that was showed by students](image_url)

After her two students showed errors, GG teacher tried to address that student’ errors or misconception and responded to those first two students’ thinking. She discussed student’s error to other students within the whole class instead of simply telling the students that it was wrong. She guided all students to aware of the misconception by giving chance to students to present different solution strategy. Besides the exploration of MPCK in teaching with regard the conceptual framework for teaching observation, the observation from the teaching was also done to explore the possible MPCK that was not in line with the factors. For instance, in every opening stage of his teaching, GG teacher always asked students about the previous related lesson. This could be interpreted that GG teacher presented the MPCK factors of activating students’ previous knowledge. In addition, GG teacher also provided students’ opportunity to do hands on activity such as counting and measuring so that students can actively ‘do mathematics’.

From the exploration of MCK and MPCK in GG teacher’s teaching, it could be summarized that all MCK and MPCK factors were presented and activated in her teaching. Most MCK and MPCK in GG teaching were presented. In the next part, I portrayed some finding on the relation among MCK and MPCK factors component that presented appropriately.

4.3. The Potential of MCK and MPCK factors in GG’s Teaching

The MCK and MPCK factors were also explored within the LL teaching practice. From the two meetings, there was no scene about figural representation factor. However, in some stages, LL showed appropriate factor of number structures in situation and meaning of proportional and non-proportional situation. In terms of number structures, LL shared appropriate number structures in the students’ work time of meeting one. She presented both integer multiple and non-integer multiple number structures in situation. In this stage, the first problem she shared was with non-integer multiple number structure (2:3) that was categorized in level two hierarchical number structure. It continued with a problem contains integer multiple number structures (i.e. multiple by three). Those number structures were not in hierarchical number structures. In addition, hard non-integer multiplier number structure in missing value problem which could be considered in level three and four of number structure to her students (i.e. 4/7, 5: 750, 5: 4). Therefore, LL teacher presented inappropriately the number structures in most of her teaching.

In terms of the MCK factor of meaning of proportional and non-proportional situation, LL regarded situation of ratio as comparing quantities. Most of them were directly taken from textbook such as ratio of boys and girls, ratio of red marbles and yellow marbles, proportional problem with age context, price context and money context. Those proportional situations were learned by students to develop their proportional understanding as suggested in teaching material (i.e. textbook). These situations were appropriate to lead to the development of proportional thinking. By this phenomenon,
number structure in situation and the meaning of proportional and non-proportional situations factor were factors that could be observed within LL’s teaching practice. In addition to it, the Low MCK influenced LL in showing misconception/error in her teaching. She posed a problem as follows:

“Ega’s money 3/7 Yoga’s money. Yoga’s money is Rp. 14000. How much Ega’s money?”

LL teacher made error in solving her problem due to in her teaching she mentioned and used terminology of proportion in addition so that the ratio numbers for Ega and Yoga in the problem were added. Error made by LL teacher can be see in figure 3

![Figure 3. Photo of error made by LL teacher](image)

The next important finding is regarding proportion problem, LL teacher shared missing value problem that required students to find one unknown value with given three values. In terms of ratio and proportion task level feature, LL teacher offered target task to students that enables for them to work productively. However, there was no scene that show students work productively to the task given. This was due to LL teacher took most of the instruction time and some of the problem were solved by her in radical monologue way. In addition, the structure of the problem was similar across lesson, only some changes were on the context situation and no hierarchical number structures used. Based on this, there was no scene that represented the task level difficulty for students to experience. Therefore, this could be regarded as inappropriately presented. The factor of Teaching problem solving strategy was presented appropriately in which there were some scenes where she introduced unitary method for solving proportional problem.

LL teacher: I gave you another problem such as buy 5 kg rice for Rp. 30000. How much to buy 8 kg rice?

LL teacher directly answer by wrote (Answer: 30000: 5 = 6000 and 8 x Rp 6000 = Rp 48000.

In another scene, a proportional problem with context of buying pencil also shared such as “4 pencils are Rp 25000. How much you pay if you buy 1 dozen pencil?”. She also described unitary method to solve this problem in radical monologue way. Beside the unitary strategy shared by LL teacher, she also shared equivalence fraction strategy for solving the problem.

From the phenomena above, it could be interpreted that LL teacher shared variety of solving proportional problem that should be applied by students including the consideration of unitary teaching method which considered as appropriate. It was due to the unitary method that she shared was the most primitive strategy for proportional problem and suitable for primary level.

Another crucial finding is that in LL teacher’s teaching, there was no opportunity for students to share their different ideas to teacher. Students were supported to follow teacher’s strategy and apply formula to the problem. Furthermore, most of the exemplary problem and problem in students work time were solved by LL teacher with her own strategy. When a student shared error in solving the problem, she did not give any feedback to her. Therefore, there was no appropriate feedback given to students for their error. In addition, in another scene of second meeting, LL teacher asked students to apply and memorize the formula of scale without giving more explanation of the formula as follow:
LL teacher: Before we solve the problem on scale, we should memorize the formula. How to find the scale? Check your book.

Student 2: the distance on map divided by the real distance

LL teacher: From the formula, if you want to find the real distance, you can multiply the distance on map multiplied by the scale. To find the distance on map, multiplied the scale to the real distance.

The teaching of scale above showed that LL teacher led students to solve the scale problem by the formula she shared. Overall for the factor of teaching problem solving strategy, LL teacher showed some appropriate components and some inappropriate components.

In terms of knowing students’ conceptual understanding, some components in this factor were presented inappropriately. For example, in the section of teacher gave hints to students in solving proportional problem and teacher missed the point of students error. In her teaching, LL teacher always gave hints about how to solve the proportional problem. The hints were about the strategy to solve proportional problem so that it could limit students to work productively. One scene showed that LL teacher gave unclear hint about a manipulative calculation to find solution of age problem as shown below

LL teacher: We could take a half of Ira’s age and added to Ira’s age. Because father’s age is older than Ira.

Other hints were about finding unit and the scalar multiplier for solving proportional problem and equivalence fraction respectively. In addition, in the students’ work time of first meeting, it found students made error in solving proportional problem. However, LL teacher ignored it and directly shared her own solution to all students. Overall, the factor of knowing students conceptual understanding was presented inappropriately.

5. Conclusion
With regard the potential of MCK and MPCK factors, there are two different direct transformations of MCK and MPCK into their natural teaching practice (GG and LL teachers). All MCK and MPCK factors were activated by GG teacher in her teaching appropriately differ with LL teacher. LL teacher did not consider figural representation which differs with GG teacher who connect proportional situation to the geometry aspect such as enlargement. In terms of MPCK, all factors were presented by GG teachers appropriately but not for LL teacher. For instance, in the task level, LL teacher did not enable students to work productively due to those were not presented in hierarchical form. Some errors were also appeared in LL teaching and she missed some points in her students’ errors. It showed that Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge is very essential for mathematics teaching. Besides, this study also highlighted the lack of mathematics connectedness in ratio and proportion teaching for teacher with Low MCK. Teacher with Low MCK and MPCK tend to follow the textbook and need more opportunity to learn several aspects related to the content such as ratio and proportion.

References


[8] Lamon S J 2007 Rational and proportional reasoning: Toward a theoretical framework for research Second handbook of research on mathematics teaching and learning 1 629-668 Information Age Publishing


[10] Livy S and Herbert S 2013 Second-Year Pre-Service Teachers’ Responses to Proportional Reasoning Test Items Australian Journal of Teacher Education 38 11


[16] Norberg R A and Aldrin B S W 2010 Scaling for stress similarity and distorted-shape similarity in bending and torsion under maximal muscle forces concurs with geometric similarity among different-sized animals The journal of Experimental Biology 213 2873-2888