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# Effect of suspension kinematic on 14 DOF vehicle model

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**Abstract.** Computer simulations play a major role in shaping modern science and engineering. They reduce time and resource consumption in new studies and designs. Vehicle simulations have been studied extensively to achieve a vehicle model used in minimum lap time solution. Simulation result accuracy depends on the abilities of these models to represent real phenomenon. Vehicles models with 7 degrees of freedom (DOF), 10 DOF and 14 DOF are normally used in optimal control to solve for minimum lap time. However, suspension kinematics are always neglected on these models. Suspension kinematics are defined as wheel movements with respect to the vehicle body. Tire forces are expressed as a function of wheel slip and wheel position. Therefore, the suspension kinematic relation is appended to the 14 DOF vehicle model to investigate its effects on the accuracy of simulate trajectory. Classical 14 DOF vehicle model is chosen as baseline model. Experiment data is collected from formula student style car test runs as baseline data for simulation and comparison between baseline model and model with suspension kinematic. Results show that in a single long turn there is an accumulated trajectory error in baseline model compared to model with suspension kinematic. While in short alternate turns, the trajectory error is much smaller. These results show that suspension kinematic had an effect on the trajectory simulation of vehicle. Which optimal control that use baseline model will result in inaccuracy control scheme.

## 1. Introduction

Computer simulations have made a significant impact in modern science and engineering. They are used to reduce time and computation resources on study and design. Accuracy of simulation depends on mathematical model that is implemented. Highly sophisticated model is able to represent natural phenomena but the simulation requires longer time to complete. Therefore, models with sufficient accuracy for a specific task is a better choice in simulation. Road vehicle simulations have been extensively studied with mathematical models representing dynamics of vehicles ranging from single track bicycle models to multibody models. These models are developed for their specific tasks that their requirements are different. For race car applications, optimal control is often coupled with the vehicle dynamic model to determine its control scheme to yield the minimum lap. Standard four-wheel vehicle models that are often used in this task are 7 DOF in [1] and [2], 10 DOF in [3], and 14 DOF in [3] and [4]. The standard 14 DOF model incorporate 6 DOF in body motion depicting surge, sway, heave, roll, pitch and yaw, with additional 8 DOF from the wheels. There are 2 DOF per wheel, which represent wheel-ground distance and wheel spin. The 10 DOF model neglects the wheel ground distance while the 7 DOF further neglects the following motions: body roll, body pitch and body. Even though the 14 DOF model is one of the most accurate models for minimum lap time optimization purpose, it still lacks one significant aspect of the vehicle that is suspension kinematic.



Suspension kinematic refers to the relative motion between the wheels and body whose movement is defined by linkage in between. The wheel angle with respect to body such as toe angle, camber angle can be expressed as functions of the suspension travel. These relative motions are required in the simulation because tire forces predicted by Pajaceka's magic formula tire model [5] uses the slip angle and vertical force as independent variables for tire force function.

The aim of this work is to identify the significance of suspension kinematic on the accuracy of trajectory in simulation results. The classical 14 DOF without suspension kinematic model is chosen as baseline model for comparison. The suspension kinematic will be added to the baseline vehicle model to study its heading and trajectory deviations from the experimental data.

## 2. Vehicle modeling

The baseline vehicle model is developed base on [3] and [4]. These models incorporate 6 DOF of body motion including surge, sway, heave, roll, pitch and yaw. Additional 8 DOF represent the four wheels with 2 DOF from each wheel denoting wheel height from ground and wheel spin. The wheel height from ground is represented by suspension spring compression displacement. This change in the generalized coordinates is made to simplify the incorporation of suspension kinematic into the baseline model. The tire model used in this study is taken from Pajaceka's magic formula [5] which is widely used in literature [1], [6] and [7].

### 2.1. Suspension kinematic

Suspension kinematic is a geometrical problem of linkages between the car body and the wheels. The double wishbone suspension type with pushrod actuated shock absorber is used in the derivation of these relations. Geometric problems are solved using numerical methods described in [8]. Solution for the roots of the equations encounter some instability, therefore, numerical relation result is fitted with polynomial equations to ensure there always exists a solution within the boundaries.

In front suspensions, there are two independent variables including steering rack displacement (component of control vector,  $\mathbf{u}$ ) and suspension travel (component of state vector,  $\mathbf{x}$ ). These 2 variables define the position and angle of the wheel on front suspension (front suspension kinematic relation vector,  $\mathbf{S}_f$ ). While the rear suspension has only one independent variable which is the suspension travel. These are expressed as equation (1) and equation (2) for front suspension and rear suspension respectively. Suspension kinematic relation vector are a combination of functions where each function represents the position or the angle of wheel with respect to body.

$$\mathbf{S}_f = f(\mathbf{x}, \mathbf{u}) \quad (1)$$

$$\mathbf{S}_r = f(\mathbf{x}) \quad (2)$$

### 2.2. Integration of suspension kinematic in 14 DOF model

Energy equations in Lagrangian system of vehicle model are modified for suspension kinematic function. These equations are used in the Euler-Lagrange equation to derive the left-hand side of the equation of motion. Equation (3) shows the original front left tire spring potential energy equation where  $k$  denotes the tire vertical stiffness,  $x_{ss,fl}$  denotes front left spring deflection,  $r_t$  denotes the tire radius. While equation (4) shows a modified tire spring potential energy equation where  $S_{f,mo}$  denotes distance of front tire to ground which is component of equation (1). Deflection in the original equation changes to suspension kinematic relation of motion ratio between spring deflection and vertical movement of wheel. Another important modification is in the tire force generation function where camber and slip angles now depend on spring compression length and steering rack travel.

$$V_{st,fl} = \frac{1}{2} k (x_{ss,fl} - r_t)^2 \quad (3)$$

$$V_{st,fl} = \frac{1}{2} k (S_{f,mo})^2 \quad (4)$$

## 3. Verification and simulation

An experiment is carried out with a formula student style car to verify the vehicle model. The car weighs 280 kg with driver. It has a double wishbone suspension with front track width of 1.260 m, rear track width of 1.190 m, wheelbase of 1.550 m and a maximum suspension travel length of 0.05 m. The front spring stiffness is 43,781.7 N/m and rear spring stiffness is 52,538 N/m. Ten inch wheels and tires are selected.

Testing is performed in two cases. First, a 180-degree constant radius turn with a track inner radius of 5 m and outer radius of 9 m. The second test is a single lane change at constant. These two methods are chosen because they generate lateral acceleration during turning so that differences in left and right suspension travel can be detected. Longitudinal acceleration affects left and right suspension travel equally, hence it is not of our interest. These tests are carried out at constant speed of 40 kph. The sensors collect data from driver input i.e. steering rack travel and vehicle heading, position, speed, suspension travel as experiment data.

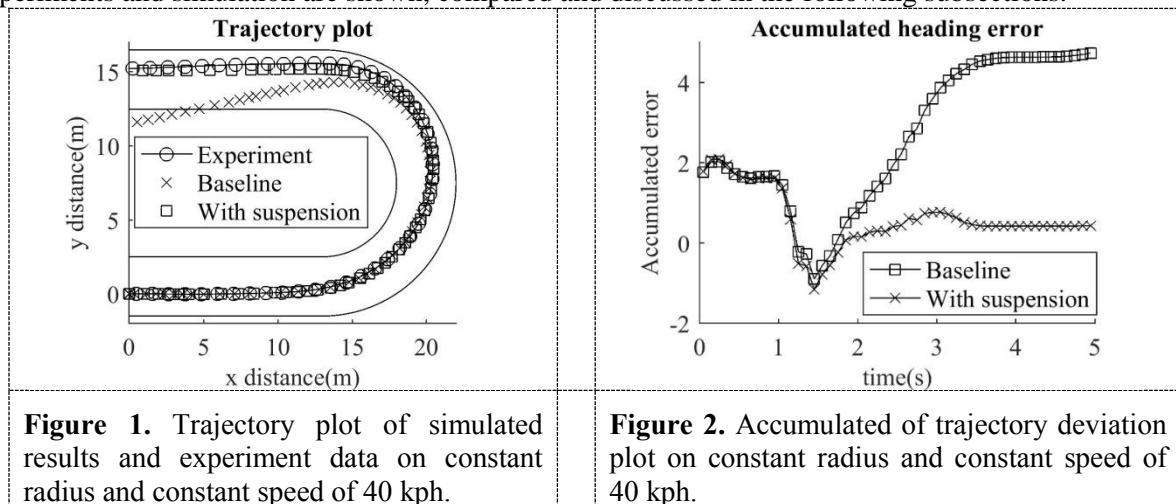
Both baseline and suspension kinematic model vehicle simulation use the steering rack travel as driver input. The simulation results will show comparison of suspension travel, trajectory and heading differences between the baseline model, suspension kinematic model and experiment data. The total trajectory and total heading errors are calculated using equations (5) and (6), respectively. Where  $x_{e,i}$  is the trajectory error in x direction,  $y_{e,i}$  is the trajectory error in y direction  $\phi_{e,i}$  is yaw deviation  $\omega_i$  is the yaw rate. The suspension travel error is calculated using the root mean square method.

$$Err_{traj} = \left\{ \sum_{i=1}^n [(x_{e,i}^2 + y_{e,i}^2) |\omega_i|] (\sum_{i=1}^n |\omega_i|)^{-1} \right\}^{1/2} \quad (5)$$

$$Err_{head} = \sum_{i=1}^n [(\phi_{e,i}) |\omega_i|] (\sum_{i=1}^n |\omega_i|)^{-1} \quad (6)$$

#### 4. Result and Discussion

Experiment data is collected from actual tests on a formula student car on a dry track with 35°C ambient temperature. Driver's steering controls are recorded and used as simulation inputs for both vehicle models. Two experiments as described in Section 3 are conducted. The results of the experiments and simulation are shown, compared and discussed in the following subsections.

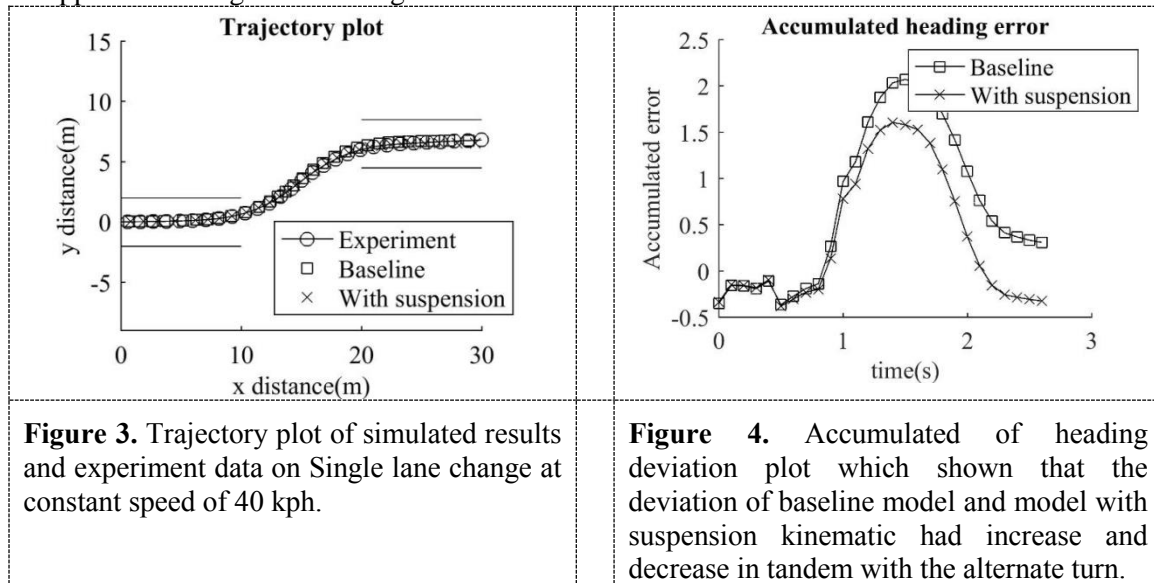


##### 4.1. Turning at constant radius and constant speed

The car makes a constant radius turn at a constant speed of 40 kph. The car trajectory from experiment data and simulation results are illustrated on figure 1 to show a deviation of baseline path and 14DOF with suspension kinematic path in comparison to experiment data. The large deviation at the end of the turn of baseline trajectory is a result of accumulated heading error over the turn. The 14 DOF with suspension kinematic trajectory displays a small deviation at the end of the turn but its heading is in the same direction as the experiment data, therefore the trajectory error is significantly smaller compared to baseline simulation. The accumulated error of heading over the track is shown in figure 2.

#### 4.2. Single lane change at constant speed

In the single lane change at constant speed of 40 kph test, the simulation results show that the deviations of trajectory of baseline model and model with suspension kinematic are minimal as shown in figure 3. The heading accumulated error is investigated further to identify the difference. The heading error as shown in figure 4 begins to accumulate after 0.8 sec when the driver begins to steer left, but the accumulated error decreases after 1.45 sec when the steering is reversed to turn into the new lane. The suspension kinematic effect seems to be reduced because the error cancel itself out due to the opposite steering maneuvering.



#### 4.3. Discussion

**Table 1.** An error of simulation result compare to experiment data on constant radius track and single lane change. Suspension travel error of each wheel, heading error and trajectory deviation of the car was shown.

	Constant radius		Single lane change	
	Baseline	With suspension kinematic	Baseline	With suspension kinematic
Front left suspension travel (m)	0.0014	0.0012	0.0021	0.0009
Front right suspension travel (m)	0.0032	0.0021	0.0021	0.0009
Rear left suspension travel (m)	0.0012	0.0011	0.0021	0.0010
Rear right suspension travel (m)	0.0041	0.0029	0.0020	0.0013
Heading (degree)	4.7356	0.4432	0.3119	-0.3221
Trajectory (m)	0.8491	0.1892	0.2609	0.1878

These two experiments and simulation results show that deviation of trajectory also depend on the track. Suspension kinematic effects accumulate over the track especially on tracks with long turn. Tracks with many short alternate turns will have minimum effect on trajectory as described in the single lane change simulation. The heading errors are caused by the compression steer which further affects the wheel angles. In baseline model, these dynamic changes due to spring compression are neglected. Hence, these effects become more pronounced long turns but in short alternate turns they quickly cancel themselves out. Another indicator is the suspension travel data in Table 1, which also shows that the errors in baseline model are higher in long turn than in short alternate turn.

Although, the suspension kinematic is added to the 14 DOF baseline model there is no additional computational cost. This is because the suspension kinematic relation is integrated into baseline model. However, the major disadvantage of model with suspension kinematic is acquiring accurate vehicle data for the suspension kinematic simulation.

## 5. Conclusion

In this work the 14 DOF vehicle model with suspension kinematic has been successfully developed and investigated its accuracy in comparison to a baseline 14 DOF vehicle model without suspension kinematic and experiment data on formula student style car. The model with suspension kinematic exhibits higher accuracy compared to baseline model in case of constant radius turn with constant speed. However, under single lane change with constant speed situation, these two models show comparable accuracy. To complete a lap around a closed circuit race track, the car usually has to turn one way more than the other, therefore this can potentially result in significant accumulated heading errors when predicted with the baseline model. The error is caused by the neglect of suspension kinematic which affects the wheel position, wheel angle and tire force. Optimal controls with the use of suspension kinematic model yield high accuracy of control scheme. This is useful in minimum lap time optimization problem.

An important aspect of this work depends on the range of suspension travel and suspension kinematic of such car. This aspect should be further investigated to define level of impact on accuracy of vehicle model. Thus, define the level of impact which depend on type of car or range of suspension travel that need vehicle model with suspension kinematic.

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