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A comparison of different optimization techniques for variation propagation control in mechanical assembly

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Abstract. Variation propagation control is one of the procedures used to determine product quality in the manufacturing assembly process. The quality of a product assembly is also greatly dependent on the product type and the optimization criteria employed in the assembly. This paper presents three procedures for optimizing the assembly of component stacks by controlling variation propagation. The procedures considered are: (i) straight-build assembly by minimizing the distances from the centres of components to table axis; (ii) parallelism-build assembly by minimizing the angular errors between actual and nominal planes; (iii) target-axis-build assembly by minimizing the distances from the centres of components to a target axis. Simulation results are presented for the assembly of four cylindrical components. The results show that the variation can be reduced significantly by optimization; for example, Procedure 1 can reduce the variation by 48%, Procedure 2 by 63% and Procedure 3 by 35%, compared to assembly without the minimization. The results also show that the 3 proposed assembly methods have reasonable consistency, the maximum of the standard deviation is 0.0726 mm in Procedure 1, 3.5×10^{-4} rad in Procedure 2, and 0.029 mm in Procedure 3. These demonstrate that the three assembly techniques are fully functional on their own. Thus, the relevant assembly techniques need to be selected carefully in accordance with the particular industrial application.

1. Introduction

Variations are inherent to any manufacturing and assembly process due to the variability of the parts, fixtures and assembly methods. They cause small deviations in parts from the nominal geometry [1]. A different locating scheme or a different assembly sequence can also affect the probability of successfully assembling the parts or achieving final assembly dimensions to specification. Variation propagation control is one of the procedures used to determine product quality in the manufacturing assembly process. Due to the complexity of a manufacturing process, an effective method for the assembly would be highly desirable [2]. Furthermore, the quality of a product assembly is greatly dependent on optimization criteria of the assembly and the type of the product. For example, in a multistage radial flow submersible pump assembly, an axial play between the impeller and the volute casing is required to have a tolerance of ±0.5mm, to prevent the rubbing action [3]. For the assembly of aero-engine components, such as compressor stages, a precise alignment and clamping device of rotationally-symmetric parts is required [4]. The concentricity deviation of the part must be within a 2.5μm tolerance [5]. A key characteristic here is to give the best 'straight line' between the centres of

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the parts for the assembly process. Traditional methods, such as worst-case and the root-sum-square tolerance analysis, are often used to calculate the variation propagation. However, these methods do not take into account the practical assembly procedures, resulting in conservative and expensive solutions [6]. This paper presents three optimization techniques for controlling variation propagation in assembly of component stacks, as follows:

- Straight-build assembly by minimizing the distances from the centres of components to table axis.
- Parallelism-build assembly by minimizing the angular errors between actual and nominal planes.
- Target-axis-build assembly by minimizing the distances from the centres of components to the target axis.

The paper also considers the practical assembly operations to provide an economical and effective solution. During the assembly, variation propagation is minimized stage-by-stage by rotating the component about its central axis, according to available number of orientations. To validate the three assembly methods, Monte Carlo simulations are performed, based on randomly and normally distributed variables. Details of the three assembly methods are given in Section 2. To evaluate the three assembly methods, a fourth assembly method is introduced, which is simply assembly without minimization. Section 3 compares the results obtained using the four assembly methods for a practical four-cylindrical-component assembly. The conclusions from the study are presented in Section 4.

2. Three assembly techniques

The three proposed assembly techniques are: (i) straight-build assembly, (ii) parallelism-build assembly, and (iii) target-axis-build assembly. The techniques are defined as follows:

- **Procedure 1:** minimize table axis error from the component centre in a stage-by-stage, as shown in Figure 1(a), i.e. straight-build assembly.
- **Procedure 2:** minimize angular error between actual and nominal planes in a stage-by-stage, as shown in Figure 1(b), i.e. parallelism-build assembly.
- **Procedure 3:** minimize distance from the centres of component to the target axis, as shown in Figure 1(c), i.e. target-axis-build assembly.

As mentioned in Section 1, to evaluate the performance of the three assembly methods, the paper also presents a fourth assembly method, (iv) simply build without minimization. This is defined below.

- **Procedure 4:** direct assembly without minimization.

The radial variation propagation from the table axis at the k'th stage is calculated by

$$Table-axis_{error}(k) = \sqrt{(dp_k^x)^2 + (dp_k^y)^2} \tag{1}$$

whist the angular error is given by:

$$Angular_{error}(k) = |d\theta_k| \tag{2}$$

where $d\,p_k^x$ and $d\,p_k^y$ are translational variations in x- and y-direction, and $d\theta_k$ is an angular error between actual and nominal planes.

In a similar way, the target-axis error at the k'th stage, which is the distance from the k'th component centre to the line passing through the centres of the first component base and the (k-1)'th component top, can be calculated, and will not be reiterated here.

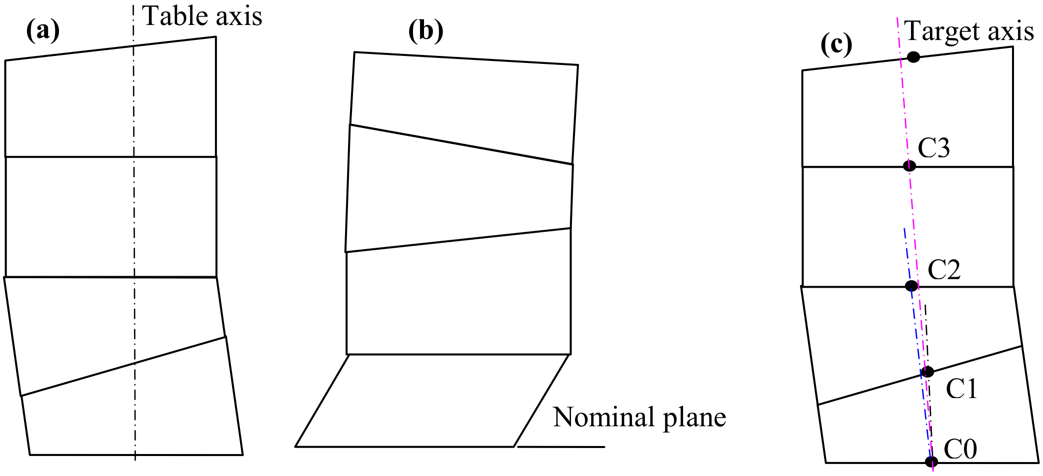

Figure 1. Different assembly procedures ((a) Procedure 1, (b) Procedure 2, (c) Procedure 3).

3. An assembly example of four cylindrical components

To compare the four assembly procedures, a practical four-cylindrical-component assembly is considered. The cylindrical component has a height of H (70mm) with tolerance h (0.1mm), and a diameter of Φ (100mm) with tolerance φ (0.1mm), as shown in Figure 2. Each component is assumed to have 4 possible orientations. The tolerances for the diameter and height of the cylindrical component are assumed to be normally distributed. Each assembly procedure is simulated 10,000 times using standard Monte Carlo methods. For simplicity, the following assumptions are made:

- The top and base surfaces are flat.
- Frames F1 and F1' are attached to the assembly features.
- Base centre (0,0,0) is the origin in the global coordinate system.
- Points 1,2,3 and 4 are uniformly distributed in XOY plane with centre O, and lie on the circumference of the cylinder on the OX and OY axes.
- The top centre lies at (dx,dy,dz+H).
- Points 1', 2', 3' and 4' are uniformly distributed on the top surface with centre O', and lie on the circumference of the cylinder on the $O'X'$ and $O'Y'$ axes
- Line O'1' has an angular error $d\theta_z$ about the z-axis with respect to line O1.
- The top surface has angular errors of $d\theta_x$ and $d\theta_y$ about the x- and y-axis, respectively, relative to base surface.
- F1' related to F1 has a translation of (0,0,H) with a translation error of (dx,dy,dz) and an angular error of ($d\theta_x$, $d\theta_y$, $d\theta_z$).

The dx,dy,dz, $d\theta_x$, $d\theta_y$ and $d\theta_z$ are normal distributed, and are constrained to lie within the shaded "bounded" regions shown in Figures 2(b), 2(c) and 2(d). Details of the generated variables can be found in [7].

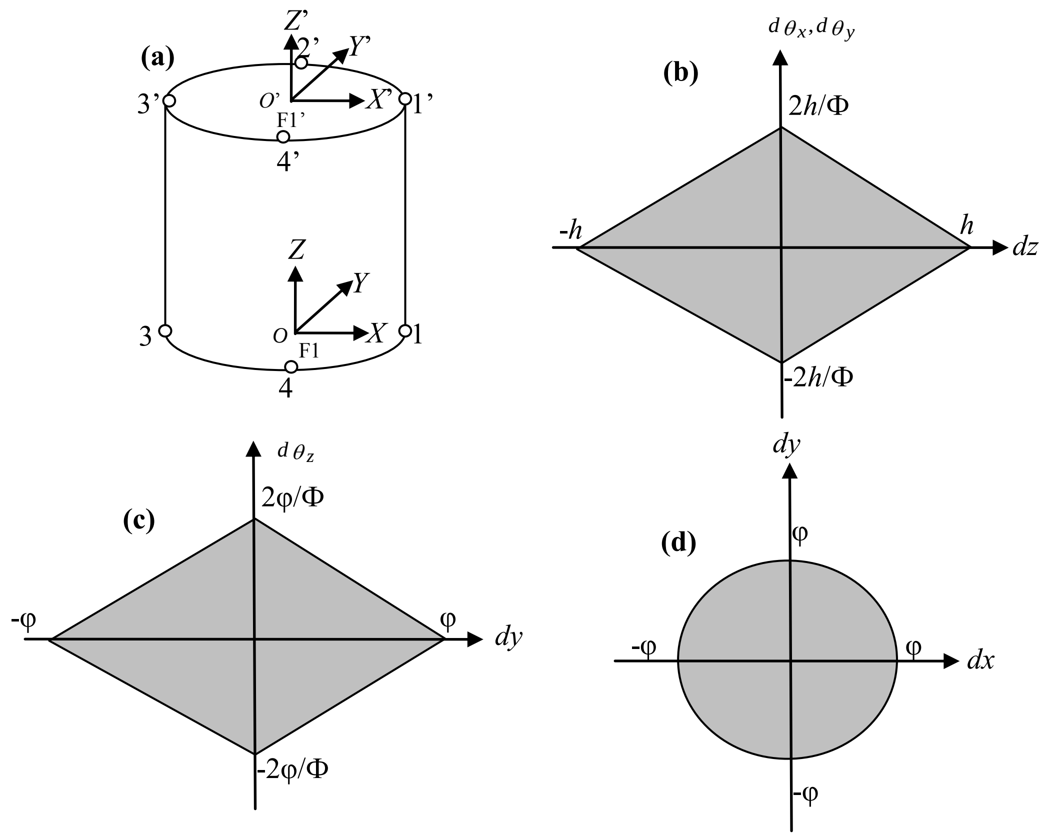

Figure 2. A cylindrical component, and the bounds used to specific kinematic parameter space ((a) cylindrical component, (b), (c) and (d) define the constrained regions).

3.1. Radial variation propagation from the table axis

The radial variation propagation from the table axis is calculated using equation (1). The average stage-by-stage radial variation from the table axis for the four different procedures is shown in Figure 3. The mean and standard deviations of the variations are provided in Table 1, together with the maximum and minimum values. The statistical distributions of the radial variations for each procedure are shown in Figure 4.

From Figure 3 and Table 1, the average radial variations tend to increase with the stage for each procedure – there is a slight decrease for Procedure 1 between Stages 1 and 2. Procedures 1-3 all have lower errors than Procedure 4, which is a direct assembly without minimization. For example, Procedure 1 reduces the average variation by 48%, compared to Procedure 4. Comparing Procedures 1-3, it can be seen that Procedure 1 produces the smallest error, whilst Procedure 3 produces the highest. It is very interesting to notice that the variation increases rapidly after Stage 2 for Procedures 1, 3 and 4, but the variation changes much more gently for Procedure 2.

From Table 1, it is found that the standard deviation increases with the stage as well for each procedure. This indicates that consistency will decrease with the assembly process. Comparing all results, Procedure 1 produces the smallest error, with a maximum standard deviation of 0.0726 mm, and reasonable consistency.

From Figures 4(a)-(d), Procedure 1 produces the smallest levels of radial variation, with most values occurring in the range 0 to 0.1 mm. These results further demonstrate that Procedure 1 provides reasonable consistency for the straight-build assembly.

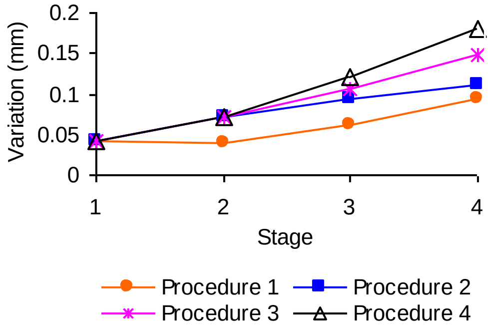

Figure 3. Average stage-by-stage radial variation from the table axis for the different procedures.

Table 1. Radial variation propagation from the table axis for the different procedures

Procedure	Stage	Max variation (mm)	Min variation (mm)	Average variation (mm)	Standard deviation of variation (mm)
Procedure 1	1	0.0999	4.0078×10^{-4}	0.0412	0.0208
	2	0.1782	3.3417×10^{-4}	0.0405	0.0249
	3	0.3069	6.1712×10^{-4}	0.0605	0.0439
	4	0.5117	3.4981×10^{-4}	0.0946	0.0726
Procedure 2	1	0.0999	4.0078×10^{-4}	0.0412	0.0208
	2	0.2589	3.3417×10^{-4}	0.0721	0.0376
	3	0.3343	9.6801×10^{-4}	0.0949	0.0529
	4	0.4487	3.4981×10^{-4}	0.1109	0.0626
Procedure 3	1	0.0999	4.0078×10^{-4}	0.0412	0.0208
	2	0.2241	0.0015	0.0720	0.0365
	3	0.3286	0.0017	0.1064	0.0554
	4	0.5460	0.0014	0.1487	0.0788
Procedure 4	1	0.0999	4.0078×10^{-4}	0.0412	0.0208
	2	0.2481	7.7686×10^{-4}	0.0727	0.0380
	3	0.4434	0.0018	0.1198	0.0643
	4	0.6599	5.9583×10^{-4}	0.1813	0.0978

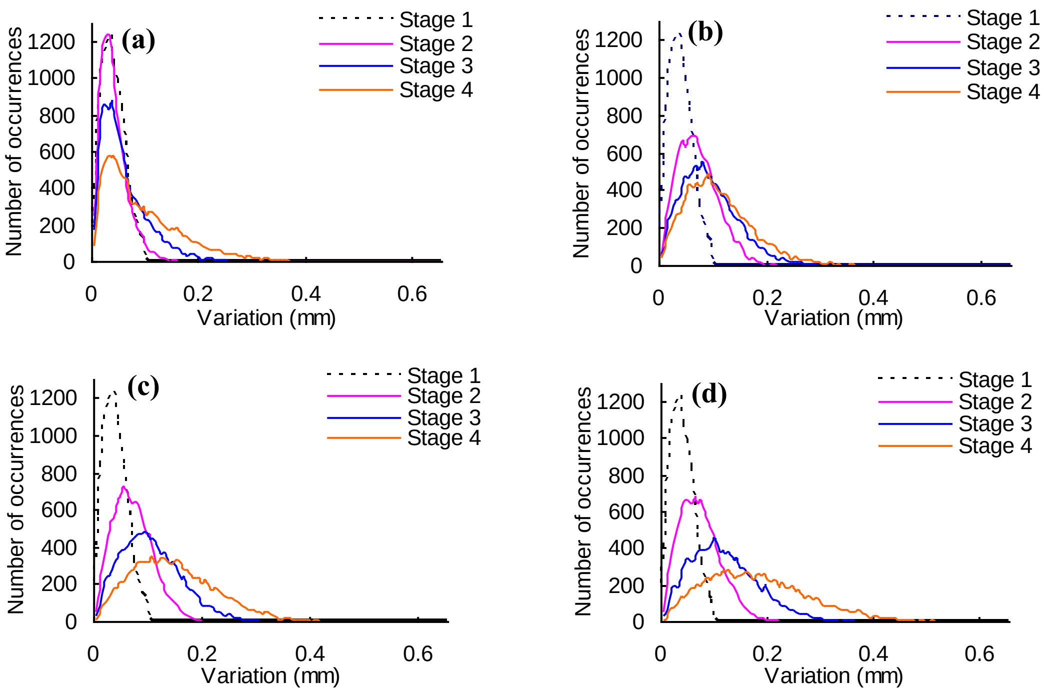

Figure 4. Histogram of radial variation propagation from the table axis ((a) Procedure 1, (b) Procedure 2, (c) Procedure 3, (d) Procedure 4).

3.2 Angular error propagation

The angular error is calculated using equation (2). The average angular error for the four different procedures is shown in Figure 5. The mean and standard deviations of the errors are provided in Table 2, together with the maximum and minimum values. The statistical distributions of the angular errors for each procedure are shown in Figure 6.

From Figure 5 and Table 2, the average angular error increases with the stage for Procedures 1, 3 and 4 while it decreases with the stage for Procedure 2. Furthermore, Procedure 2 has a lower average angular error than the other procedures at all stages. For example, the angular error at the final stage is reduced by 63%, when compared to Procedure 4.

Table 2 indicates that the standard deviation of the angular error increases with the stage for Procedures 1, 3 and 4, while it is much smaller for Procedure 2. These results confirm that Procedure 2 has the best performance for the parallelism-build assembly.

From Figures 6(a)-(d), Procedure 2 produces the smallest levels of angular error, with most values occurring in the range 0 to 0.001 rad. These results further demonstrate that Procedure 2 produces the best parallelism-build assembly.

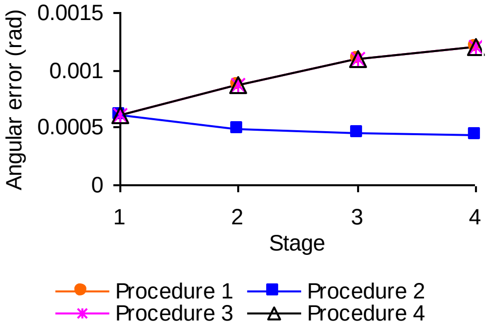

Figure 5. Average stage-by-stage angular error for the different procedures.

Table 2. Angular error for the different procedures

Procedure	Stage	Max error (rad)	Min error (rad)	Average error (rad)	Standard deviation of error (rad)
Procedure 1	1	0.0020	3.6311×10^{-7}	6.0901×10^{-4}	3.5427×10^{-4}
	2	0.0030	4.2030×10^{-6}	8.7187×10^{-4}	4.7413×10^{-4}
	3	0.0039	1.8941×10^{-5}	0.0011	5.7701×10^{-4}
	4	0.0047	9.4271×10^{-6}	0.0012	6.6464×10^{-4}
Procedure 2	1	0.0020	3.6311×10^{-7}	6.0901×10^{-4}	3.5427×10^{-4}
	2	0.0018	4.2030×10^{-6}	4.9176×10^{-4}	2.9455×10^{-4}
	3	0.0018	4.1063×10^{-6}	4.5436×10^{-4}	2.8031×10^{-4}
	4	0.0018	5.4738×10^{-6}	4.4375×10^{-4}	2.8361×10^{-4}
Procedure 3	1	0.0020	3.6311×10^{-7}	6.0901×10^{-4}	3.5427×10^{-4}
	2	0.0031	1.2025×10^{-5}	8.7998×10^{-4}	4.7900×10^{-4}
	3	0.0040	6.1769×10^{-6}	0.0011	5.8068×10^{-4}
	4	0.0046	2.1399×10^{-5}	0.0012	6.7198×10^{-4}
Procedure 4	1	0.0020	3.6311×10^{-7}	6.0901×10^{-4}	3.5427×10^{-4}
	2	0.0030	4.2030×10^{-6}	8.7466×10^{-4}	4.7417×10^{-4}
	3	0.0044	1.8471×10^{-5}	0.0011	5.7669×10^{-4}
	4	0.0045	1.0520×10^{-5}	0.0012	6.6198×10^{-4}

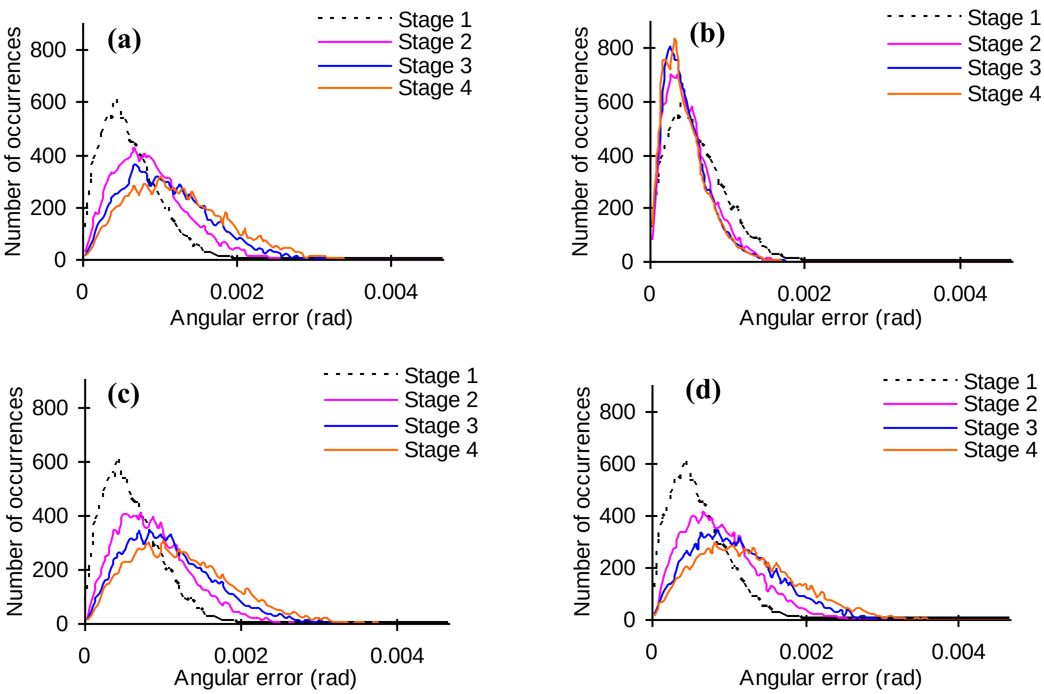

Figure 6. Histogram of angular error propagation ((a) Procedure 1, (b) Procedure 2, (c) Procedure 3, (d) Procedure 4).

3.3. Variation propagation from the target axis

The target-axis error at the *k*'th stage is calculated by the distance from the component centre to the line passing through the centres of the first component base and the (*k*-1)'th component top, as described in Section 2. The average variation propagation from the target axis for the four different procedures is shown in Figure 7. The statistical parameters of variation propagation from the target axis for each procedure are listed in Table 3. Histograms of variation propagation from the target axis for each procedure are shown in Figure 8.

Using Figure 7 and Table 3, and comparing Procedures 1-3 with 4 indicate that the average target axis error can be reduced using any of Procedures 1-3. It is also found that the variation propagation from the target axis increases with the stage for Procedure 3, and that Procedure 3 is much better than the other procedures. This indicates that Procedure 3 has much better eccentricity than the others.

From Table 3, Procedure 3 has a smaller standard deviation from the target axis than the others. It also show that the standard deviation from the target axis increase with the stage for Procedure 3. The maximum standard deviation for Procedure 3 is only approx. 0.029 mm, indicating that it has reasonable consistency for the target-axis-build assembly.

From Figure 8, Procedure 3 is more likely to produce assemblies with low variations from the target axis than the others for all stages. This indicates that Procedure 3 has improved eccentricity for all stages.

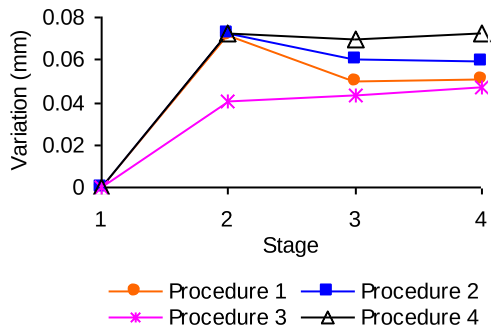

Figure 7. Average stage-by-stage variation from the target axis for the different procedures.

Table 3. Variation propagation from the target axis for the different procedures

Procedure	Stage	Max variation (mm)	Min variation (mm)	Average variation (mm)	Standard deviation of variation (mm)
Procedure 1	1	0	0	0	0
	2	0.2231	6.3218×10^{-4}	0.0719	0.0364
	3	0.1854	2.1404×10^{-4}	0.0501	0.0270
	4	0.2119	5.8908×10^{-4}	0.0512	0.0279
Procedure 2	1	0	0	0	0
	2	0.2322	6.3218×10^{-4}	0.0725	0.0379
	3	0.1981	8.2338×10^{-4}	0.0599	0.0318
	4	0.2074	4.3910×10^{-4}	0.0592	0.0313
Procedure 3	1	0	0	0	0
	2	0.1800	1.8356×10^{-4}	0.0404	0.0248
	3	0.1854	6.1643×10^{-4}	0.0432	0.0260
	4	0.1883	2.1247×10^{-4}	0.0474	0.0289
Procedure 4	1	0	0	0	0
	2	0.2439	0.0011	0.0723	0.0383
	3	0.2300	6.0637×10^{-4}	0.0692	0.0366
	4	0.2544	4.3910×10^{-4}	0.0728	0.0374

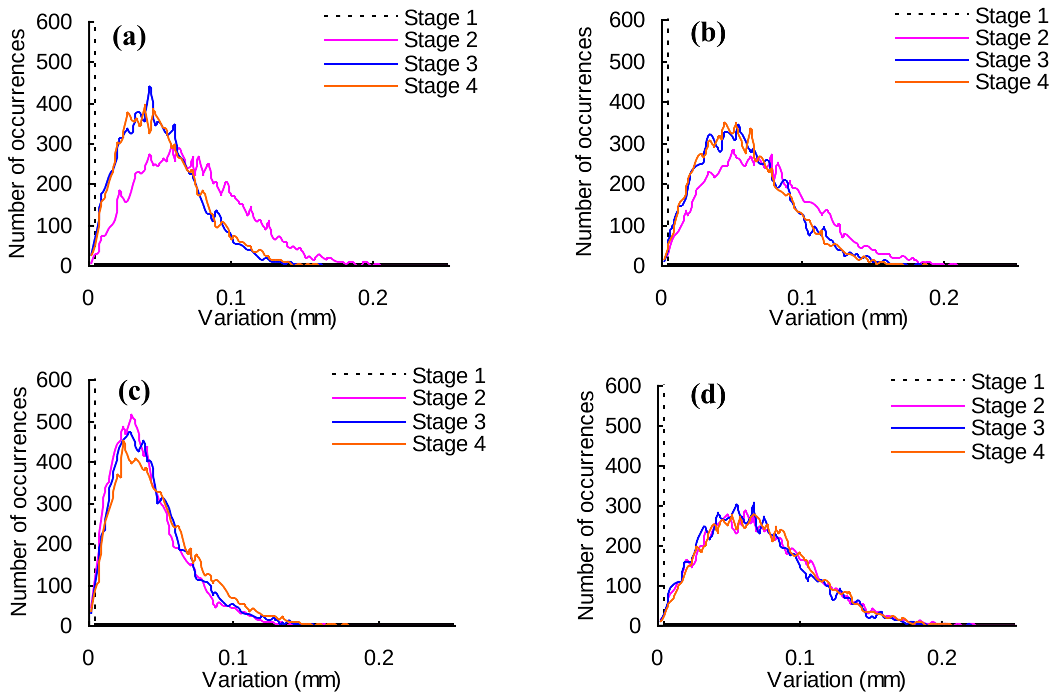

Figure 8. Histogram of variation propagation from the target axis ((a) Procedure 1, (b) Procedure 2, (c) Procedure 3, (d) Procedure 4).

4. Conclusions

The three proposed assembly methods have the potential to significantly reduce manufacturing assembly errors. For the example considered, the first method reduced the variation by 48%, the second by 63% and the third by 35%, when compared to assemblies without the minimization.

The first method has the best performance in straight-build assembly, the second method has the best performance in parallelism-build assembly, and the third method has the best performance in target-axis-build assembly.

The three proposed assembly methods have reasonable consistency, for instance, the maximum standard deviation is 0.0726 mm in the first method, 3.5×10^{-4} rad in the second method, and 0.029 mm in the third method, for the specific example.

The procedure of the three assembly methods is very simple, and can be easily implemented in manufacturing processes.

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