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Kinematically Admissible Failure Mechanisms for Plane Trusses

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Abstract. The reliability models of statically indeterminate steel trusses are analysed in this paper. The plane trusses are considered. For analysed structures types of reliability model are determined. Identification of system reliability is based on studies on the transformation from the safe structural system into the geometrically variable system (mechanism). These researches intended to determine the kinematically admissible failure mechanisms which contain minimal critical sets of elements. For analysed truss the formula setting out the number of mechanisms for any number of repeatable sections is determined. To identify the mechanism, spectral analysis of the linear stiffness matrix is used. This stage of the study, in conjunction with consideration of static loads, allows to determine the minimum critical set of rods corresponding to the most probable scenario of the structural damage. For these analysis the computer program based on the finite element method has been created within the *Mathematica* environment.

1. Introduction

The safety and reliability of structures are very important in the design process and on the operation stage during the lifetime of a structure. Steel trusses among the most important and the simplest structures. The most dangerous situation for trusses is a damage of rods converting truss into mechanism causing a global damage of the system.

The subject of reliability analysis is to determine the probability of failure, which is understood as exceeding ultimate limit state [1-5]. System analysis in ultimate limit state of rod structures, and consequently identification of reliability system, is based on studies on the transformation the safe structural system into the mechanism. These research are intended to determine kinematically admissible failure mechanisms (KAFM) which contain minimal critical sets of elements (MCSE) [1,4,6-8]. MCSE is a collection which is characterized by the property that if only one element is operable, the entire system is operable – the structure is able to transfer acting loads. Exhaustion of the load of all the elements included in the causative MCSE makes the structure is converted into the geometrically variable system.

Reliability analysis of structural system consists of identification of reliability models and estimation of failure probabilities of individual modes and the overall system. In this paper the reliability models of statically indeterminate plane trusses are constructed. This theme is very important because the



probability of failure depends on the reliability model of structures. For these analysis the computer program based on the finite element method has been created within the Mathematica environment. This stage of the study, in conjunction with consideration of static loads, allows to determine the minimum critical set of rods corresponding to the most probable scenario of the structural damage.

2. Methods and Theory

2.1. Kinematically admissible failure mechanisms

To evaluate the reliability of the system one of three types of reliability model (Figure 1): serial, parallel and mixed (parallel-serial or serial-parallel) system is used [1,3]. A system that is functioning if and only if all of its components are functioning is called a serial system (Figure 1a). For such structures the higher number of members, the lower load bearing capacity and reliability. The reliability of the serial system is calculated as follows:

$$R_s = \prod_{i=1}^k R_i \quad (1)$$

where R_i is reliability of single element and k is the number of elements. The serial system is appropriate for structures that are statistically determinate. In this case number of minimal critical sets of elements is k (k MCSE) and the number of causative elements MCSE is $l=1$.

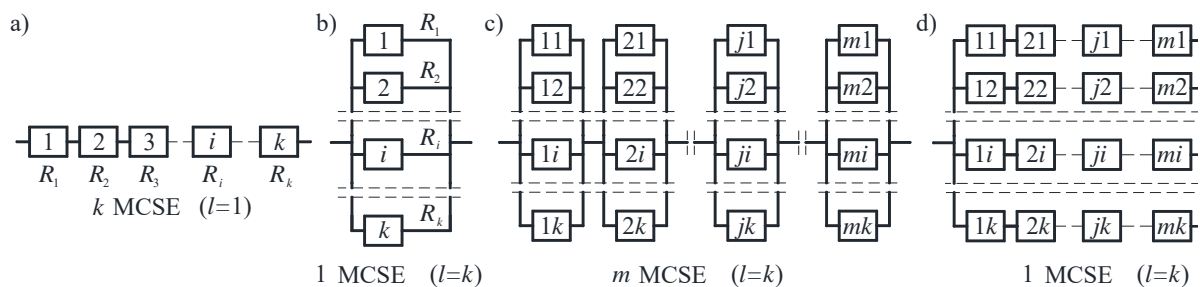


Figure 1. Types of reliability model: a) serial system, b) parallel system, c) parallel-serial system, d) serial-parallel system

A system that is functioning if at least one of its components is functioning is called a parallel (Figure 1b). This system is appropriate for some structures that are statically indeterminate and the reliability is calculated as follows:

$$R_s = 1 - \prod_{i=1}^k (1 - R_i). \quad (2)$$

For parallel system there is one minimal critical set of elements (1 MCSE) and number of causative elements MCSE is $l=k$.

In the case of complex structures, there is usually a need to identify mixed systems, which are a combination of parallel and serial systems [4,8]. There are two basic systems as parallel-serial (Figure 1c) and serial-parallel (Figure 1d). The reliability for the first system is calculated as:

$$R_s = \prod_{j=1}^m \left[1 - \prod_{i=1}^k (1 - R_{ji}) \right], \quad (3)$$

where m is the number of columns and k is the number of elements in columns. The reliability for the second system is calculated as:

$$R_s = 1 - \prod_{i=1}^k \left[1 - \prod_{j=1}^m R_{ji} \right]. \quad (4)$$

where k is the number of rows and m is the number of elements in rows. These systems are characterized by multiple MCSE associated with KAFM. However in real structures, mixed systems are usually more complicated [6,9]. Some examples of the mixed systems are presented further in the paper.

2.2. Spectral analysis of linear stiffness matrix

The spectral analysis is widely used in many areas of human investigations. Between others it is one of the most popular techniques in structural health monitoring procedures [10]. To identify mechanism of truss the spectral analysis of the linear stiffness matrix is used:

$$(\mathbf{K}_L - \lambda \mathbf{I}) \mathbf{q} = \mathbf{0} \quad (5)$$

where \mathbf{K}_L is the linear stiffness matrix and \mathbf{q} is displacement vector. The eigenvalues λ of linear stiffness matrix describe the energy states of the model, while the eigenvectors describe the form of its own deformation. In the case when all the eigenvalues are greater than zero there are no movements. Zero eigenvalues are related to the finite or infinitesimal mechanisms, but in general the information from the null-space analysis alone does not suffice to establish the difference between them. The mechanism can be considered as an eigenvector related to zero eigenvalue. To establish if the mechanism is infinitesimal it is necessary to apply the nonlinear analysis with the use of geometric stiffness matrix [11].

The program based on the finite element method application was created in Mathematica environment to identify possible mechanisms in the analyzed structure. The main functionality of the program was carry out spectral analysis for all possible trusses having removed given number of rods. To do that all possible combinations of given number of rods were generated and linear stiffness matrix was aggregated. Next the eigenvalues of created matrix were searched and zero eigenvalues was identified. If zero eigenvalue was found the structure was classified as the mechanism. As a result of calculations all possible not repeatable combinations of removed rods transforming truss to mechanism were generated.

3. Examples

The subject of further qualitative analysis is supported, statically indeterminate plane truss which is composed of n repeatable sections (Figure 2). For this type of truss, the reliability and damage analysis are considered in some papers [3,12-15]. In the paper five types of trusses, of varying number of: repeatable sections n , nodes ln and rods le , are considered:

- $n=1, ln=4, le=6$,
- $n=2, ln=6, le=11$,
- $n=3, ln=8, le=16$,
- $n=4, ln=10, le=21$,
- $n=5, ln=12, le=26$.

Kinematically admissible failure mechanisms (KAFM) which contain minimal critical sets of elements (MCSE) are determined for each case. The reliability R_s of studied systems is calculated assuming the same reliability of all the rods $R_i=R$.

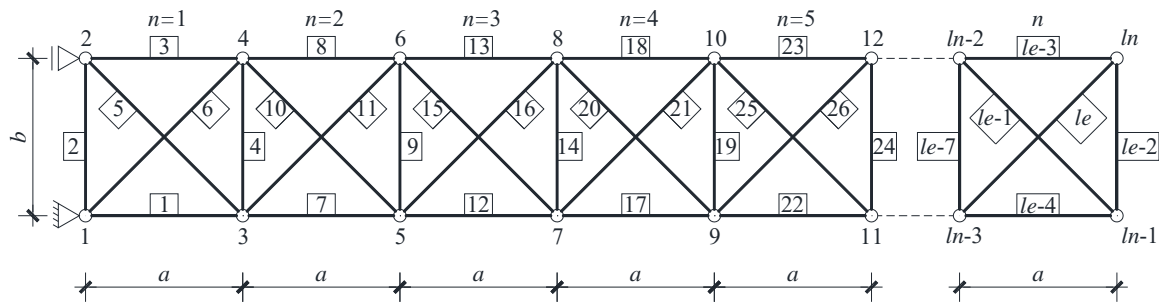


Figure 2. Plane truss composed of n repeatable sections with ln nodes and le elements

For a truss which is composed of one repeatable section ($n=1$) two (I, II) kinematically admissible failure mechanisms are determined (Figure 3) [3]. The I KAFM consists of seven MCSE with two causative elements ($l=2$) and it is divided into two KAFM: IA and IB. For IA KAFM the failure of any pairs of elements 1-4 results in the failure of the whole structure and for IB KAFM the structure failure occurs as the result of the failure of both cross-braces (5,6). The reliability ($R_t = R_{IA} \cdot R_{IB}$) for this mechanism is calculated as for the parallel-serial system (3). The II KAFM consists of one MCSE with two causative elements like I KAFM but it is a mixed system. The structural failure is caused by the failure of one of cross-braces (5,6) and one of the elements 1-4. Reliability is calculated as for the serial-parallel system (4). All KAFMs are connected in a serial way. Therefore, the reliability of the whole truss is calculated as follows:

$$R_{T1} = R_I \cdot R_{II}$$

where:

$$R_I = [1 - (1 - R)^2]^7; R_{II} = 1 - (1 - R^2)(1 - R^4)$$

For this truss number of mechanisms is 15 and all mechanisms consist of two causative elements.

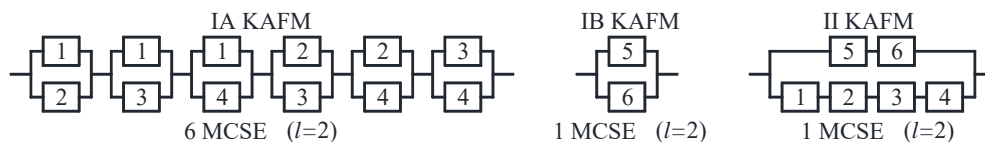


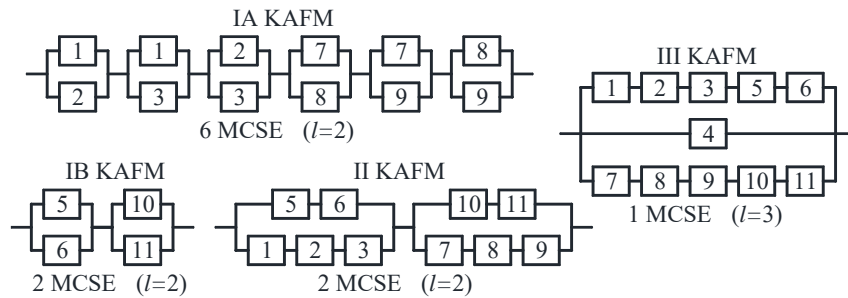
Figure 3. Kinematically admissible failure mechanisms for $n=1$

For a truss which is composed of two repeatable sections ($n=2$) three kinematically admissible failure mechanisms are determined (Figure 4). Minimal critical sets of elements (MCSE) are presented in the Figure 4. In this case there are 45 mechanisms: 20 – which consist of two causative elements ($l=2$) (I-II KAFMs) and 25 – containing three causative elements ($l=3$) (III KAFM). The reliability is calculated in the same way as at the first example:

$$R_{T2} = R_I \cdot R_{II} \cdot R_{III}$$

where:

$$R_I = [1 - (1 - R)^2]^8; R_{II} = [1 - (1 - R^2)(1 - R^3)]^2; R_{III} = 1 - (1 - R^5)(1 - R).$$

Figure 4. Kinematically admissible failure mechanisms for $n=2$

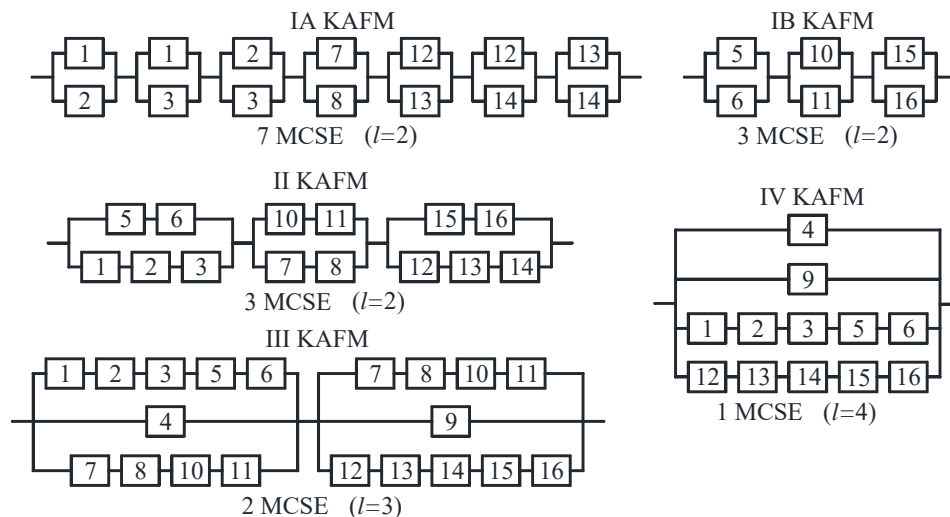
For a truss which is composed of three repeatable sections ($n=3$) four kinematically admissible failure mechanisms are determined (Figure 5). Minimal critical sets of elements (MCSE) are shown in the Figure 5. In this case there are 91 mechanisms: 26 – which consist of two causative elements ($l=2$) (I-II KAFMs), 40 – consisting of three causative elements ($l=3$) (III KAFM) and 25 – which consist of four causative elements ($l=4$) (IV KAFM). The reliability of the whole truss is calculated as follows:

$$R_{T3} = R_I \cdot R_{II} \cdot R_{III} \cdot R_{IV}$$

where:

$$R_I = [1 - (1 - R)^2]^{10}; \quad R_{II} = [1 - (1 - R^2)(1 - R^3)]^2 [1 - (1 - R^2)^2];$$

$$R_{III} = [1 - (1 - R^5)(1 - R)(1 - R^4)]^2; \quad R_{IV} = 1 - (1 - R)^2(1 - R^5)^2.$$

Figure 5. Kinematically admissible failure mechanisms for $n=3$

For a truss which is composed of four repeatable sections ($n=4$) five kinematically admissible failure mechanisms are determined (Figure 6). Minimal critical sets of elements (MCSE) are shown in the Figure 6. In this case there are 153 mechanisms: 32 – which consist of two causative elements ($l=2$) (I-II KAFMs), 56 – consisting of three causative elements ($l=3$) (III KAFM), 40 – which consist of four causative elements ($l=4$) (IV KAFM) and 25 – consisting of five causative elements ($l=5$) (V KAFM). The reliability of the whole truss is calculated as follows:

$$R_{T4} = R_I \cdot R_{II} \cdot R_{III} \cdot R_{IV} \cdot R_V$$

where:

$$R_I = [1 - (1 - R)^2]^{12}; R_{II} = [1 - (1 - R^2)(1 - R^3)]^2 [1 - (1 - R^2)^2]^2;$$

$$R_{III} = [1 - (1 - R^5)(1 - R)(1 - R^4)]^2 [1 - (1 - R^4)^2(1 - R)]^2; R_{IV} = [1 - (1 - R)^2(1 - R^5)(1 - R^4)]^2;$$

$$R_V = 1 - (1 - R)^3(1 - R^5)^2.$$

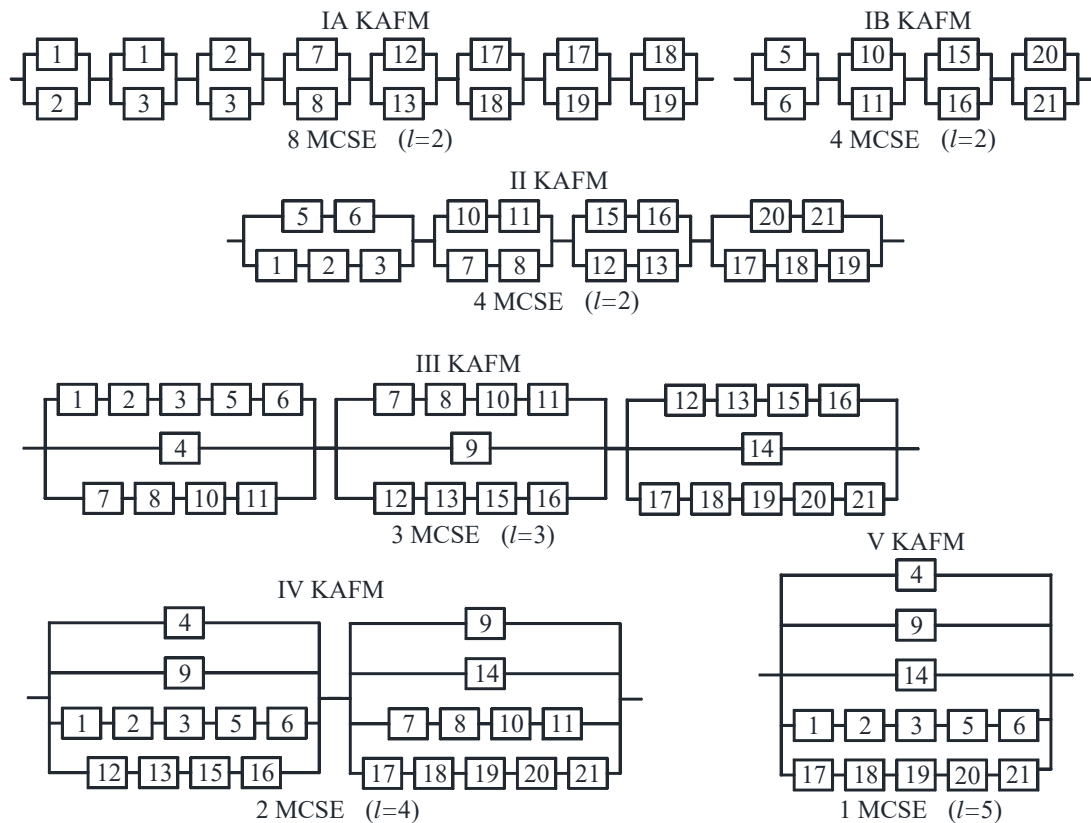


Figure 6. Kinematically admissible failure mechanisms for $n=4$

For a truss which is composed of five repeatable sections ($n=5$) six kinematically admissible failure mechanisms are determined (Figure 7). Minimal critical sets of elements (MCSE) are shown in the Figure 7. In this case there are 231 mechanisms: 38 – which consist of two causative elements ($l=2$) (I-II KAFMs), 72 – consisting of three causative elements ($l=3$) (III KAFM), 56 – which consist of four causative elements ($l=4$) (IV KAFM), 40 – consisting of five causative elements ($l=5$) (V KAFM) and 25 – which consist of six causative elements ($l=6$) (IV KAFM). The reliability of the whole truss is calculated as follows:

$$R_{T5} = R_I \cdot R_{II} \cdot R_{III} \cdot R_{IV} \cdot R_V \cdot R_{VI}$$

where:

$$R_I = [1 - (1 - R)^2]^{14}; R_{II} = [1 - (1 - R^2)(1 - R^3)]^2 [1 - (1 - R^2)^2]^3;$$

$$R_{III} = [1 - (1 - R^5)(1 - R)(1 - R^4)]^2 [1 - (1 - R^4)^2(1 - R)]^2;$$

$$R_{IV} = \left[1 - (1-R)^2(1-R^5)(1-R^4) \right]^2 \left[1 - (1-R)^2(1-R^4)^2 \right]; \quad R_V = \left[1 - (1-R)^3(1-R^5)(1-R^4) \right]^2;$$

$$R_{VI} = 1 - (1-R)^4(1-R^5)^2.$$

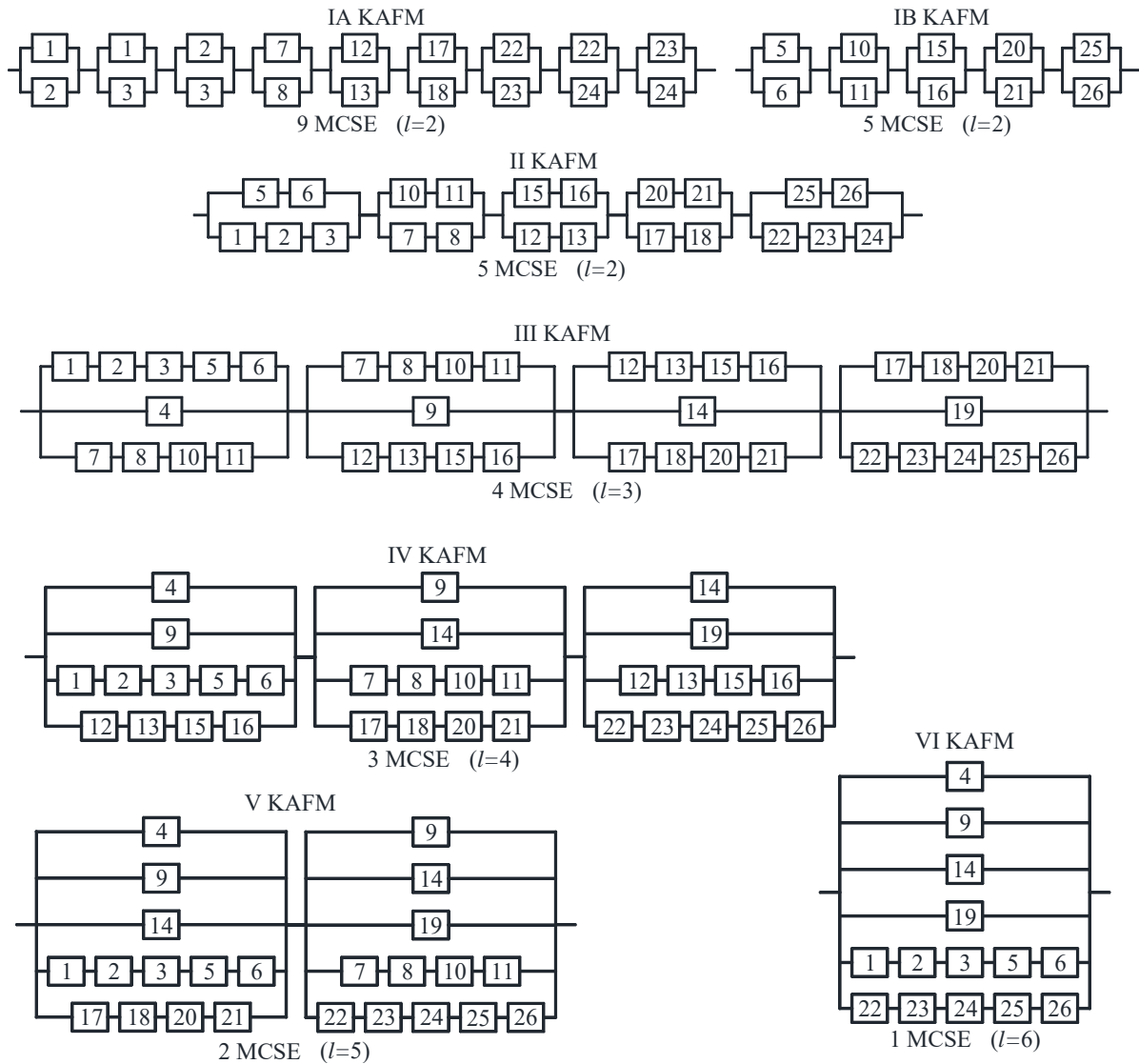


Figure 7. Kinematically admissible failure mechanisms for $n=5$

4. Results and discussions

The considerations performed in the paper allows to establish relations for every truss composed of n repeatable sections for $n \geq 2$. The number of nodes ln and rods le is calculated as:

$$\begin{aligned} ln &= 4 + 2(n-1), \\ le &= 6 + 5(n-1). \end{aligned} \quad (6)$$

The minimum number of failure rods converting structure into the geometrically variable system is $l=2$ and maximal – $l=n+1$. The number of kinematically admissible failure mechanisms is $n+1$. The number

of minimal critical sets of elements for II KAFM is n , for III – $n+1$, for IV – $n+2$ etc., but the number of MCSE for I KAFM is calculated as:

$$\text{I KAFM} \rightarrow l_{\text{MCSE}} = le - 3(n-1). \quad (7)$$

The formula setting out the number of mechanisms is computed as:

$$lm = lm_2 + lm_n + lm_{n+1} \quad (8)$$

where lm_2 is the number of mechanisms consisting of two causative elements ($l=2$), lm_n is the number of mechanisms which consist of n causative elements ($l=n$) and lm_{n+1} is the number of mechanisms consisting of $n+1$ causative elements ($l=n+1$):

$$\begin{aligned} l = 2 & \Rightarrow lm_2 = 20 + 6(n-2), \\ l = 3, 4, \dots, n & \Rightarrow lm_n = \sum_{l=3}^n [40 + 16(n-l)], \\ l = n+1 & \Rightarrow lm_{n+1} = 25. \end{aligned} \quad (9)$$

It is noticed that the equation (9)₂ is not taken into account in the case of a truss which is composed of two repeatable sections ($n=2$). The first (I) and the second (II) KAFM consist of mechanisms determined by (9)₁. I KAFM is always the parallel-serial system of two causative elements. II KAFM is the serial-parallel system in which the failure of one of cross-braces n repeatable sections and one of the another elements causes the failure of the whole structure. The n -th KAFM consists of mechanisms determined by (9)₂ and $(n+1)$ -th KAFM by (9)₃.

To illustrate proposed approach a truss which is composed of eight repeatable sections ($n=8$) is considered. The number of rods and nodes is respectively $le=41$ and $ln=18$. Nine kinematically admissible failure mechanisms which consist of 561 mechanisms are determined for this structure (Table 1). The reliability of this truss may be calculated using the analysis presented for previous trusses. There is a certain repeatability which allows to determine reliability as:

$$R_{T8} = R_I \cdot R_{II} \cdot R_{III} \cdot R_{IV} \cdot R_V \cdot R_{VI} \cdot R_{VII} \cdot R_{VIII} \cdot R_{IX}$$

where:

$$\begin{aligned} R_I &= [1 - (1-R)^2]^{20}; \quad R_{II} = [1 - (1-R^2)(1-R^3)]^2 [1 - (1-R^2)^2]^6; \\ R_{III} &= [1 - (1-R^5)(1-R)(1-R^4)]^2 [1 - (1-R^4)^2(1-R)]^5; \\ R_{IV} &= [1 - (1-R)^2(1-R^5)(1-R^4)]^2 [1 - (1-R)^2(1-R^4)^2]^4; \\ R_V &= [1 - (1-R)^3(1-R^5)(1-R^4)]^2 [1 - (1-R)^3(1-R^4)^2]^3; \\ R_{VI} &= [1 - (1-R)^4(1-R^5)(1-R^4)]^2 [1 - (1-R)^4(1-R^4)^2]^2; \\ R_{VII} &= [1 - (1-R)^5(1-R^5)(1-R^4)]^2 [1 - (1-R)^5(1-R^4)^2]; \quad R_{VIII} = [1 - (1-R)^6(1-R^5)(1-R^4)]^2; \\ R_{IX} &= [1 - (1-R)^7(1-R^5)^2]. \end{aligned}$$

Table 1. Kinematically admissible failure mechanisms for $n=8$

KAFM	Number of MCSE	Number of causative elements	Number of mechanisms
I	20		
II	8	2	56
III	7	3	120
IV	6	4	104
V	5	5	88
VI	4	6	72
VII	3	7	56
VIII	2	8	40
IX	1	9	25

5. Conclusions

In the paper the reliability of plane trusses is considered. New strategy is presented to determine of the dominant failure paths and the structural system failure probability. The method allows to calculate the reliability for the statically indeterminate truss composed of n repeatable sections. Presented method consists of three steps: identification of reliability models and estimation of failure probabilities of individual modes and the overall system. To identification of reliability models kinematically admissible failure mechanisms which contain minimal critical sets of elements are determined. The spectral analysis of the linear stiffness matrix is used to identify mechanism. The problem is solved using the finite element method implemented in *Mathematica* environment. Basing on obtained results the universal relations on possible number of mechanism is determined for considered type of truss.

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