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Bipolar-value fuzzy soft lie subalgebras

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Abstract. In this paper, the notions of bipolar-value fuzzy soft Lie subalgebras are given, some of their properties are studied. Furthermore, we shall define the definition of bipolar-value fuzzy soft homomorphism of bipolar-value fuzzy soft Lie subalgebras and show that the theorem of bipolar-value fuzzy soft homomorphic pre-image of bipolar-value fuzzy soft Lie subalgebra, we shall give a counterexample which the image of a bipolar-value fuzzy soft Lie subalgebra under bipolar-value fuzzy soft homomorphism needs not be bipolar-value fuzzy soft Lie subalgebra..

1 Introduction and preliminaries

The real world is too complex for our immediate and direct understanding. We create models of reality that are simplifications of aspects of the real word.In 1999 D.Molodtsov [18] introduced the concept of a soft set and started to develop basic of the theory as a new approach for modeling uncertainties. From then on, many works on soft set theory and its applications in various fields are progressing rapidly [1,3-7,9-11,14-25].

In this paper, the notions of bipolar-value fuzzy soft Lie subalgebras are given, some of their properties are studied. Furthermore, we shall define the definition of bipolar-value fuzzy soft homomorphism of bipolar-value fuzzy soft Lie subalgebras and show that the theorem of bipolar-value fuzzy soft homomorphic pre-image of bipolar-value fuzzy soft Lie subalgebra, we shall give a counterexample which the image of a bipolar-value fuzzy soft Lie subalgebra under bipolar-value fuzzy soft homomorphism is not need to be bipolar-value fuzzy soft Lie subalgebra.

Definition 1.1[16] Let X be a set, a pair (f, A) is called a fuzzy soft set on X, (where $A \subseteq E$ and E be a set of parameters), and $f: A \to I^X$ is a mapping from A into I^X, i.e. for each $e \in A,\ f(e) = f_e : X \to I$ is a fuzzy set on X.

Definition 1.2[13] Let X be a set, $\mu = (\mu^P, \mu^N)$ is called a bipolar-value fuzzy set on X iff both $\mu^P : X \to [0,1]$ and $\mu^P : X \to [-1,0]$ are mappings. The family of all bipolar-value fuzzy sets on X is denoted by $BF(X)$.

Definition 1.3[13] Let $\mu = (\mu^P, \mu^N)$ and $\lambda = (\lambda^P, \lambda^N)$ be two bipolar-value fuzzy set on X. Then

(1) $\mu \wedge \lambda = (\mu^P \wedge \lambda^P, \mu^N \wedge \lambda^N)$;

(2) $\mu \vee \lambda = (\mu^P \vee \lambda^P, \mu^N \vee \lambda^N)$;

(3) $\mu'(x) = (1 - \mu^P(x), -1 - \lambda^P(x))$.

Definition 1.4 Let X be a set, a pair (f, A) is called a bipolar-value fuzzy soft set on X iff $f: A \to BF(X)$ is a mapping from A into $BF(X)$, i.e. for each $e \in A$, $f(e) = f_e = (f_e^P, f_e^N)$ is a bipolar-value fuzzy set on X.

Definition 1.5 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, we say that (f, A) is a bipolar-value fuzzy soft subset of (g, B), and write $(f, A) \subseteq (g, B)$ if

(1) $A \subseteq B$, (2) $f_e \le g_e$ for each $e \in A$, i.e. f_e is a bipolar-value fuzzy subset of g_e, in other word, $f_e^P \le g_e^P$ and $f_e^N \ge g_e^N$.

Definition 1.6 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, (f, A) and (g, B) are said to be equal if $(f, A) \subseteq (g, B)$ and $(g, B) \subseteq (f, A)$.

Definition 1.7 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, union of two bipolar-value fuzzy soft sets (f, A) and (g, B) is the bipolar-value fuzzy soft set (g, B), where

$$C = A \cup B \text{ and } h(e) = \begin{cases} f_e, & if\ e \in A - B \\ g_e, & if\ e \in B - A, \forall e \in C \\ f_e \vee g_e, & if\ e \in A \cap B \end{cases}$$

We write $(f, A) \tilde{\cup} (g, B) = (h, C)$.

Definition 1.8 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, intersection of two bipolar-value fuzzy soft sets (f, A) and (g, B) is the bipolar-value fuzzy soft set (h, C), where $C = A \cap B$ and $h_e = f_e \wedge g_e (\forall e \in C)$. We write $(f, A) \tilde{\cap} (g, B) = (h, C)$.

Definition 1.9 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, (f, A) and (g, B) is denoted as $(f, A) \wedge (g, B)$, is defined as follows: $(h, A \times B)$, where $h(a,b) = h_{a,b} = f_a \wedge g_b$ for each $(a,b) \in A \times B$.

Definition 1.10 [26] Let (f, A) and (g, B) are two bipolar-value fuzzy soft sets on X, (f, A) or (g, B) is denoted as $(f, A) \vee (g, B)$, is defined as follows: $(h, A \times B)$, where $h(a,b) = h_{a,b} = f_a \wedge g_b$ for each $(a,b) \in A \times B$.

Definition 1.11 [26] (1) A bipolar-value fuzzy soft set (f, A) is called a null bipolar-value fuzzy soft set iff $f_e^P(x) = 0$ and $f_e^N(x) = 0$ for each $e \in A$ and $x \in X$, we write (ϕ, A).

(2) A bipolar-value fuzzy soft set (g, B) is called an absolute bipolar-value fuzzy soft set iff $f_e^P = 0$ and $f_e^N = -1$ for each $e \in B$ and $x \in X$, we write (X, B).

Definition 1.12[8] A vector space L over a field F with an operation $[,]: L \times L \to L, (x, y) \to [x, y]$ is called a Lie algebra over F if the following axioms are satisfied:

(1) The bracket operation is bilinear, that is,

$$[ax + by, cz + dw] = ac[x, z] + cb[y, z] + ad[x, w] + bd[y, w].$$

(2) $[x, y] = -[y, x] (\forall x, y \in L)$.

(3) (Jacobi identity) $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0 (\forall x, y, z \in L)$.

A subspace K of L is called a Lie subalgebra of L if $[x,y]\in K,$, for all $x,y\in K$ and a subspace I of L is called a Lie ideal of L if for all $x\in I, y\in L$, implies $[x,y]\in I,$ .Obviously, a Lie ideal of L is a Lie subalgebra of L.

Remark 1.14 $[x,x]=0(\forall x\in L).[[x,y],z]=[x,[y,z]]$ does not established generally, that is, [,] does not satisfy the associative law, for example: Let L be all the 2×2 matrix of vector space over a field $\mathbb{F}$, A,B are 2×2 matrix, for bracket operation $[A,B]=AB-BA,$ but $[[A,B],C]\neq[A,[B,C]]$.

In fact, let $A=\begin{pmatrix}1 & -1\\ 0 & 0\end{pmatrix}, B=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}, C=\begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}$, we can show that

$$[[A,B,C]]=A=\begin{pmatrix}-1 & 2\\ 0 & 1\end{pmatrix}\neq\begin{pmatrix}0 & 2\\ 0 & 0\end{pmatrix}=[A,[B,C]]$$

2 Bipolar-value fuzzy soft Lie subalgebras

Definition 2.1 Let L is a Lie algebra over a field $\mathbb{F}$ and (f,A) is a bipolar-value fuzzy soft sets on L, (f,A) is called a bipolar-value fuzzy soft Lie subalgebra of L iff the following conditions are satisfied : for each $e\in A, x,y\in L$, and $\alpha\in\mathbb{F}$,

(1) $f_e^P(x+y)\geq f_e^P(x)\wedge f_e^P(y)$ and $f_e^N(x+y)\leq f_e^N(x)\vee f_e^N(y)$

(2) $f_e^P(\alpha x)\geq f_e^P(x)$ and $f_e^N(\alpha x)\leq f_e^N(x)$.

(3) $f_e^P([x,y])\geq f_e^P(x)\wedge f_e^P(y)$ and $f_e^N([x,y])\leq f_e^N(x)\vee f_e^N(y)$.

That is, for each $e\in A,$ f_e^P is a fuzzy Lie subalgebra o $L(see[12])$ and f_e^N is an anti fuzzy Lie subalgebra. If (f,A) is satisfied (1)–(2) and the condition (4),

(4) $f_e^P([x,y])\geq f_e^P(x)$ *and* $f_e^N([x,y])\leq f_e^N(x)$.

Then (f,A) is called a bipolar-value fuzzy soft Lie ideal of L , that is , for each $e\in A,$ f_e^P is a fuzzy Lie ideal of $L(see[12])$ and f_e^N is an anti fuzzy Lie ideal (see [2], if a bipolar-value fuzzy soft set (f,A) on L is only satisfied (2) and (4), then (f,A) is a bipolar-value pre-fuzzy soft Lie ideal on L.

Remark 2.2 Obviously, a bipolar-value fuzzy soft Lie ideal is a bipolar-value fuzzy soft Lie subalgebra. A bipolar-value fuzzy soft Lie ideal is a bipolar-value pre-fuzzy soft Lie ideal .

Theorem 2.3 Let L is a Lie algebra over a field$\mathbb{F}$, (f,A) is a bipolar-value fuzzy soft set on L, if (f,A) is a bipolar-value fuzzy soft Lie subalgebra on L, then for each $e\in A, x,y\in L$ and $\alpha\in\mathbb{F}$ $-\{0\}$, we have

(i) $f_e^P(0)\geq f_e^P(x)$ and) $f_e^N(0)\leq f_e^N(x)$;

(ii) $f_e^P(\alpha x)=f_e^P(x)$ and $f_e^N(\alpha x)=f_e^N(x)$;

(iii) If $f_e^P(x)<f_e^P(y)$,then $f_e^P(x-y)=f_e^P(x)=f_e^P(y-x)$ and if $f_e^N(x)<f_e^N(y)$, then

$$f_e^N(x-y)=f_e^N(y)=f_e^N(y-x);$$

(iv) $f_e^P([y,x])=f_e^P(-[y,x])=f_e^P([x,y])$.

Proof The proof is fair straightforward and we omit it.

Example 2.4 Let $L=\mathbb{R}^3$ and $[x,y]=x\times y(\forall x,y\in L)$,where $\times$ is cross product. Then L is a Lie algebra over a field $\mathbb{F}$. let $\mathbb{Z}$ is the set of all integer set.

(1) Define $f:\mathbb{Z}\to BF(\mathbb{R}^3)$ as follows:

$$\forall n\in\mathbb{Z},\ f_n^P(x)=\begin{cases}1, & if\ \ x=(0,0,0),\\ 0.5 & if\ \ x=(a,0,0), a\neq 0, \forall n\in\mathbb{Z}-\mathbb{N},\\ 0 & otherwise.\end{cases}\quad f_n^P(x)=\begin{cases}0.9, & if\ \ x=(0,0,0),\\ 0.8, & if\ \ x=(0,0,b), b\neq 0\\ 0, & otherwise.\end{cases}$$

and $$\forall n\in\mathbb{Z},\ f_n^N(x)=\begin{cases}-0.8, & if\ \ x=(0,0,0),\\ 0 & otherwise.\end{cases}$$

Then the pair $(f,\mathbb{Z})$ is a bipolar-value fuzzy soft set on $\mathbb{R}$,and we can show that $(f,\mathbb{Z})$ is a bipolar-value fuzzy soft Lie subalgebra.

(2)Define $f:\mathbb{Z}\to BF(\mathbb{R}^3)$ as follows:

$$\forall n\in\mathbb{Z},\ f_n^P(x)=\begin{cases}0.9, & if\ \ x=(0,0,0),\\ 0.3 & otherwise.\end{cases}\quad and\quad f_n^N(x)=\begin{cases}-0.7, & if\ \ x=(0,0,0),\\ -0.2 & otherwise.\end{cases}$$

Then the pair $(f,\mathbb{Z})$ is a bipolar-value fuzzy soft set on $\mathbb{R}^3$, and we can show that $(f,\mathbb{Z})$ is a bipolarvalue fuzzy soft Lie ideal.

In the following content, we shall study bipolar-value fuzzy soft Lie subalgebras and bipolar-value pre-fuzzy soft Lie ideals .

Theorem 2.5 Let (f,A) and (g,B) are bipolar-value fuzzy soft Lie subalgebras on L (resp.bipolarvalue fuzzy soft Lie ideal, bipolar-value pre-fuzzy soft Lie ideal), then $(f,A)\tilde{\cap}(g,B)$ is a bipolar-value fuzzy soft Lie subalgebra on L (resp.bipolar-value fuzzy soft Lie ideal, bipolar-value pre-fuzzy soft Lie ideal).

Proof $(f,A)\tilde{\cap}(g,B)=(h,C)$. Where $C=A\cap B$ and $h_e=f_e\wedge g_e(\forall e\in C)$, if (f,A) and (g,B) are two bipolar-value fuzzy soft Lie subalgebras on L , then we have

(1) $h_e^P(x+y)=f_e^P(x+y)\wedge g_e^P(x+y)\geq f_e^P(x)\wedge f_e^P(y)\wedge g_e^P(x)\wedge g_e^P(y)=h_e^P(x)\wedge h_e^P(y),\ and$
$h_e^N(x+y)=f_e^N(x+y)\vee g_e^N(x+y)\leq f_e^N(x)\vee f_e^N(y)\vee g_e^N(x)\vee g_e^N(y)=h_e^N(x)\vee h_e^N(y);$

(2) $h_e^P(\alpha x)=f_e^P(\alpha x)\wedge g_e^P(\alpha x)\geq f_e^P(x)\wedge g_e^P(x)=h_e^P(x),\ and\ h_e^N(\alpha x)=f_e^N(\alpha x)\vee g_e^N(\alpha x)\leq$
$f_e^N(x)\vee g_e^N(x)=h_e^N(x);$

(3) $h_e^P([x,y])=f_e^P([x,y])\wedge g_e^P([x,y])\geq f_e^P(x)\wedge f_e^P(y)\wedge g_e^P(x)\wedge g_e^P(y)=h_e^P(x)\wedge h_e^P(y),\ and$
$h_e^N([x,y])=f_e^N([x,y])\vee g_e^N([x,y])\leq f_e^N(x)\vee f_e^N(y)\vee g_e^N(x)\vee g_e^N(y)=h_e^N(x)\vee h_e^N(y);$

If (f,A) and (g,B) are bipolar-value fuzzy soft Lie ideals on L ,we have (1,(2), and (4):

(4) $h_e^P([x,y]) = f_e^P([x,y]) \wedge g_e^P([x,y]) \geq f_e^P(x) \wedge g_e^P(x) = h_e^P(x),\ and\ h_e^N([x,y]) = f_e^N([x,y]) \vee g_e^N([x,y]) \leq f_e^N(x) \vee g_e^N(x) = h_e^N(x)$

If (f,A) and (g,B) are bipolar-value pre-fuzzy soft Lie ideals on L, we have (2) and (4).

Hence $(f,A)\tilde{\cap}(g,B)$ is a bipolar-value fuzzy soft Lie subalgebra on L (resp. bipolar-value fuzzy soft Lie ideal, bipolar-value pre-fuzzy soft Lie ideal).

Similar to the proof of Theorem 2.5 , we can show that the following Theorem 2.6:

Theorem 2.6 Let (f,A) and (f,B) are bipolar-value fuzzy soft Lie subalgebras on L (resp. bipolar-value fuzzy soft Lie ideal, bipolar-value pre-fuzzy soft Lie ideal), then (f,A) AND (g,B) is a bipolar-value fuzzy soft Lie subalgebra on L (resp. bipolar-value fuzzy soft Lie ideal, bipolar-value pre-fuzzy soft Lie ideal).

Proof Trivial.

Theorem 2.7 Let (f,A) and (f,B) are bipolar-value pre-fuzzy soft Lie ideals, then $(f,A)\tilde{\cup}(g,B)$ is a bipolar-value pre-fuzzy soft Lie ideal on L.

Proof Let $(f,A)\tilde{\cup}(g,B)=(h,C)$, if $e\in A-B$, ,then $h_e = f_e$, is a bipolar-value pre-fuzzy soft Lie ideal on L; if $e\in B-A$, ,then $h_e = g_e$, is a bipolar-value pre-fuzzy soft Lie ideal on L; if $e\in A\cap B$, ,then $h_e = f_e \vee g_e$, ,we have

(2) $h_e^P(\alpha x) = f_e^P(\alpha x) \wedge g_e^P(\alpha x) \geq f_e^P(x) \wedge g_e^P(x) = h_e^P(x),\ and\ h_e^N(\alpha x) = f_e^N(\alpha x) \vee g_e^N(\alpha x) \leq f_e^N(x) \vee g_e^N(x) = h_e^N(x);$

(4) $h_e^P([x,y]) = f_e^P([x,y]) \wedge g_e^P([x,y]) \geq f_e^P(x) \wedge g_e^P(x) = h_e^P(x),\ and\ h_e^N([x,y]) = f_e^N([x,y]) \vee g_e^N([x,y]) \leq f_e^N(x) \vee g_e^N(x) = h_e^N(x)$

Hence $(f,A)\tilde{\cup}(g,B)$ is a bipolar-value pre-fuzzy soft Lie ideal on L.

Similar to the proof of Theorem 2.7 , we can show that the following Theorem 2.8:

Theorem 2.8 Let (f,A) and (f,B) are bipolar-value pre-fuzzy soft Lie ideals, then (f,A) OR (f,B) is a bipolar-value pre-fuzzy soft Lie ideal on L.

Proof Trivial.

Remark 2.9 If (f,A) and (f,B) are bipolar-value fuzzy soft Lie subalgebras on L, then $(f,A)\tilde{\cup}(g,B)$ need not be a bipolar-value fuzzy soft Lie subalgebra on L. For example let $L=\mathbb{R}^3$ and $[x,y]=x\times y(\forall x,y\in L)$, where $\times$is cross product, then L is a Lie algebra over a field $\mathbb{R}$. Let$\mathbb{Z}$ is the set of all natural number.

Define . $f:\mathbb{N}\to BF(\mathbb{R}^3)$ as follows: for each $n\in\mathbb{N}$,

$$f_n^p(x)=\begin{cases}0.9, & if\ x=(0,0,0),\\ 0.8, & if\ x=(0,0,a), a\neq 0,\\ 0, & otherwise.\end{cases} \quad f_n^N(x)=\begin{cases}-0.5, & if\ x=(0,0,0),\\ -0.3, & otherwise.\end{cases}$$

And define $g:\mathbb{N}\to BF(\mathbb{R}^3)$ as follows:

$$g_n^P(x)=\begin{cases}0.9, & if\ x=(0,0,0),\\ 0.4, & if\ x=(b,0,0), b\neq 0,\\ 0.2, & otherwise.\end{cases}\quad g_n^N(x)=\begin{cases}-1, & if\ x=(0,0,0),\\ -0.2, & otherwise.\end{cases}$$

Then both $(f,\mathbb{N})$ and $(g,\mathbb{N})$ are bipolar-value fuzzy soft sets of $\mathbb{R}^3$, we can show that both $(f,\mathbb{N})$ and $(g,\mathbb{N})$ are bipolar-value fuzzy soft Lie subalgebras of L, since

$(f_n\vee g_n)^P((0,0,a)+(b,0,0))=(f_n\vee g_n)^P((b,0,a))=0.2, but\ (f_n\vee g_n)^P(0,0,a)\wedge(f_n\vee g_n)^P(b,0,0)$
$=(0.8\vee 0.2)\ \wedge(0\vee 0.4)=0.2=0.8\wedge 0.4=0.4.$

. Therefore $(f_n\vee g_n)^P((0,0,a)+(b,0,0))<(f_n\vee g_n)^P((0,0,a))\wedge(f_n\vee g_n)^P((b,0,0))$ Hence $(f,A)\tilde{\cup}(g,B)$ need not be a bipolar-value fuzzy soft Lie subalgebra of L.

3 Homomorphism of bipolar-value fuzzy soft Lie subalgebras

Definition 3.1 Let (f,A) and (g,B) be two bipolar-value fuzzy soft sets on X and Y, respectively. $\Xi: X\to Y$ and $\Theta: A\to B$ be two functions, where A and B are parameter sets. Define (Ξ,Θ) as follows $(\Xi,\Theta)(f,A)=(\Xi(f),\Theta(A))$ where $\Xi,(f)_a$ is satisfied the following condition: for each $a\in\Theta(A),\forall y\in Y$

$$\Xi(f)_a^P(y)=\begin{cases}\bigvee\limits_{\Xi(x)=y}\bigvee\limits_{\Theta(e)=a} f_e(x), & if\ x\in\Xi^{-1}(y)\\ 0, & otherwise\end{cases}$$

and

$$\Xi(f)_a^P(y)=\begin{cases}\bigwedge\limits_{\Xi(x)=y}\bigwedge\limits_{\Theta(e)=a} f_e(x), & if\ x\in\Xi^{-1}(y)\\ -1, & otherwise\end{cases}$$

Define $(\Xi,\Theta)^{-1}(g,B)=(\Xi^{-1}(g),\Theta^{-1}(B)),$ where $\Xi^{-1}(g)_e(x)=g_{\Theta(e)}(\Xi(x))(\forall e\in\Theta^{-1}(B),\forall x\in X)$. The pair (Ξ,Θ) is called a bipolar-value fuzzy soft function from X to Y, and $(\Xi,\Theta)(f,A)$ is called the image of (f,A) under the bipolar-value fuzzy soft function (Ξ,Θ), and $(\Xi,\Theta)^{-1}(g,B)=(\Xi^{-1}(g),\Theta^{-1}(B)),$ is called the pre-image of (g,B) under the bipolar-value fuzzy soft function (Ξ,Θ).

Definition 3.2 Let X , Y are Lie subalgebras on field $\mathbb{F}$, (Ξ,Θ) is a bipolar-value fuzzy soft function from X to Y, if Ξ is a isomorphism from the Lie subalgebra X to the Lie subalgebra Y, then (Ξ,Θ) is said to be a homomorphism of bipolar-value fuzzy soft Lie subalgebras over X and Y.

Theorem 3.3 Let (g,B) is a bipolar-value fuzzy soft Lie subalgebra (resp. bipolar-value fuzzy soft Lie ideal) on Lie subalgebra Y, (Ξ,Θ) is a homomorphism of bipolar-value fuzzy soft Lie subalgebras over X and Y, then $(\Xi,\Theta)^{-1}(g,B)$ is a bipolar-value fuzzy soft Lie subalgebras(resp.bipolar-value fuzzy soft Lie ideal) on X.

Proof We only show that $(\Xi,\Theta)^{-1}(g,B)$ is a bipolar-value fuzzy soft Lie subalgebras on X. Since $(\Xi,\Theta)^{-1}(g,B)=(\Xi^{-1}(g),\Theta^{-1}(B))$, for each $e\in\Theta^{-1}(B)$, we have

$$(1)\ \Xi^{-1}(g)_e^P(x+y)=g_{\Theta(e)}^P(\Xi(x+y))=g_{\Theta(e)}^P(\Xi(x)+\Xi(y))\geq g_{\Theta(e)}^P(\Xi(x))\wedge g_{\Theta(e)}^P(\Xi(y))$$
$$=\Xi^{-1}(g)_e^P(x)\wedge\Xi^{-1}(g)_e^P(y),\ and\ \Xi^{-1}(g)_e^N(x+y)=g_{\Theta(e)}^N(\Xi(x+y))=$$
$$g_{\Theta(e)}^N(\Xi(x)+\Xi(y))\leq g_{\Theta(e)}^N(\Xi(x))\vee g_{\Theta(e)}^N(\Xi(y))=\Xi^{-1}(g)_e^N(x)\vee\Xi^{-1}(g)_e^N(y).$$
$$(2)\ \Xi^{-1}(g)_e^P(\alpha x)=g_{\Theta(e)}^P(\Xi(\alpha x))=g_{\Theta(e)}^P(\alpha\Xi(x))\geq g_{\Theta(e)}^P(\Xi(x))=\Xi^{-1}(g)_e^P(x),$$
$$and\ \Xi^{-1}(g)_e^N(x+y)=g_{\Theta(e)}^N(\Xi(\alpha x))=g_{\Theta(e)}^N(\alpha\Xi(x))\leq g_{\Theta(e)}^N(\Xi(x))=\Xi^{-1}(g)_e^N(x).$$
$$(3)\ \Xi^{-1}(g)_e^P([x,y])=g_{\Theta(e)}^P(\Xi[x,y])=g_{\Theta(e)}^P([\Xi(x),\Xi(y)])\geq g_{\Theta(e)}^P(\Xi(x))\wedge g_{\Theta(e)}^P(\Xi(y))$$
$$=\Xi^{-1}(g)_e^P(x)\wedge\Xi^{-1}(g)_e^P(y),\ and\ \Xi^{-1}(g)_e^N([x,y])=g_{\Theta(e)}^N(\Xi[x,y])=$$
$$g_{\Theta(e)}^N([\Xi(x),\Xi(y)])\leq g_{\Theta(e)}^N(\Xi(x))\vee g_{\Theta(e)}^N(\Xi(y))=\Xi^{-1}(g)_e^N(x)\vee\Xi^{-1}(g)_e^N(y).$$

Hence $(\Xi,\Theta)^{-1}(g,B)$ is a fuzzy soft Lie subalgebra of X.

Remark 3.4 Let (f,A) is a bipolar-value fuzzy soft Lie subalgebra on X, (Ξ,Θ) is a homomorphism of bipolar-value fuzzy soft Lie subalgebras over X and Y. Then $(\Xi,\Theta)(f,A)$ need not be bipolar-value fuzzy soft Lie subalgebra on Y, thus $(\Xi,\Theta)(f,A)$ need not be bipolar-value fuzzy soft Lie ideal on Y. For example: let $L=\mathbb{R}^3$ and $[x,y]=x\times y(\forall x,y\in L)$, where $\times$ is cross product. Then L is a fuzzy Lie subalgebra of $\mathbb{R}$. Let $\mathbb{Z}$ is the set of all integer. Define $f:\mathbb{Z}\to BF(\mathbb{R}^3)$ as follows:

$$\forall n\in\mathbb{Z},\ f_n^P(x)=\begin{cases}0.9, & if\ x=(0,0,0),\\ 0.8, & if\ x=(0,0,a),a\neq 0,\\ 0, & otherwise.\end{cases}\quad and\ f_n^N(x)=-1(\forall x\in\mathbb{R}^3)$$

$$\forall n\in\mathbb{Z}-\mathbb{N},\ f_n^P(x)=\begin{cases}0.9, & if\ x=(0,0,0),\\ 0.4, & if\ x=(b,0,0),b\neq 0,\\ 0.2, & otherwise.\end{cases}\quad and\ f_n^N(x)=-1(\forall x\in\mathbb{R}^3)$$

Then the pair $(f,\mathbb{Z})$ is a bipolar-value fuzzy soft Lie subalgebra on L.

Let $L'=\mathbb{R}^{3\,3}$ and $[x,y]=x\times y(\forall x,y\in L')$, then L' is a fuzzy Lie subalgebra of $\mathbb{R}$. Let $\mathbb{N}$ is the set of all natural number. Define $f:L\to L'$ as follows: $\Xi(x)=x(\forall x\in L)$. Define $\Theta:\mathbb{Z}\to\mathbb{N}$, as follows:

$$\Theta(x)=\begin{cases}n, & if\ \ x=n,-n(\forall n\in\mathbb{N}-\{0\},\\ 0, & otherwise.\end{cases}$$

Then (Ξ,Θ) is a homomorphism of bipolar-value fuzzy soft Lie subalgebras over L and L'. Therefore $(\Xi,\Theta)(f,\mathbb{Z})=(\Xi(f),\mathbb{N})$, Hence, for each $n\in\mathbb{N}-\{0\}$ and $y\in L',\Xi(f)_n^P(y)=f_n^P(y)\vee f_{-n}^P(y)$, take $y_1=(0,0,a),y_2=(b,0,0),$ *we have* $y_1=(0,0,a),y_2=(b,0,0)$

$$but\ \ \Xi(f)_n^P(y_1)\wedge\Xi(f)_n^P(y_2)=\Xi(f)_n^P(0,0,a)\wedge\Xi(f)_n^P(b,0,0)=(f_n^P(0,0,a)\vee f_{-n}^P(0,0,a))\wedge$$
$$(f_n^P(b,0,0)\vee f_{-n}^P(b,0,0))=(0.8\vee 0.2)\wedge(0\vee 0.4)=0.8\wedge 0.4=0.4.$$

Therefore $\Xi(f)_n^P(y_1+y_2)<\Xi(f)_n^P(y_1)\wedge\Xi(f)_n^P(y_2)$. Hence $(\Xi,\Theta)(f,\mathbb{Z})=(\Xi(f),\mathbb{N})$ need not be a bipolar-value fuzzy soft Lie subalgebra of L'.

4 Conclusion

In this paper, the notions of bipolar-value fuzzy soft Lie subalgebras were given, some of their properties were studied. A counterexample which the image of a bipolar-value fuzzy soft Lie subalgebra under bipolar-value fuzzy soft homomorphism needs not be bipolar-value fuzzy soft Lie subalgebra was given. However, the bipolar-value fuzzy soft homomorphic theorem was not given.

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