Leader-Follower Formation Control for Quadrotors

To cite this article: Falin Wu et al 2017 IOP Conf. Ser.: Mater. Sci. Eng. 187 012016

View the article online for updates and enhancements.

Related content
- Quadrotor trajectory tracking using PID cascade control
  M Idres, O Mustapha and M Okasha
- Intelligent passively stabilized quadrotor
  D Sayfeddine, A G Bulgakov and T N Kruglova
- Trajectory tracking in quadrotor platform by using PD controller and LQR control approach
  Maidul Islam, Mohamed Okasha and Moumen Mohammad Idres

Recent citations
- Backstepping Based Formation Control of Quadrotors with the State Transformation Technique
  Keun Lee et al
Leader-Follower Formation Control for Quadrotors

Falin Wu\textsuperscript{a}, Jiemin Chen\textsuperscript{b} and Yuan Liang\textsuperscript{c}

School of Instrumentation Science and Opto-electronics Engineering, Beihang University, No.37 XueYuan Road, HaiDian District, Beijing, China

\textsuperscript{a}falin.wu@buaa.edu.cn, \textsuperscript{b}chenjiemin0417@buaa.edu.cn, \textsuperscript{c}liangyuan@buaa.edu.cn

Abstract. Quadrotors are gaining an increasing interest in public and extensively explored in recent years. In many situations, a team of quadrotors is desired to operate in a certain shape, which is also called formation. In this paper, a linear PID controller is used to control each single quadrotor and a slide mode controller is adopted to solve the formation flying problem which employs the leader-follower structure. The formation simulations are run in the Matlab/Simulink environment to evaluate the performance of control laws.

1. Introduction
Quadrotor, or commonly known as drone and quadcopter, is a simple aerial vehicle which can operate in the narrow space with four propellers. With the pace of electronic device industry accelerating, quadrotors are being extensively explored and studied by an increasing number of scholars over the past few years. Carrying with sensors and other necessary devices, they make a difference to numerous civilian and military applications, including disaster mitigation, environment preservation, reconnaissance and academic research [1].

However, in many application scenarios, a team of unmanned aerial vehicles (UAVs) needs to follow the preset trajectory while maintaining a specific geometric shape. Compared with the conventional system, formation flight can increase the anti-interference performance and efficiency, improve the probability of success in search tasks, expand the region of surveillance and reduce the expense of military missions. As a result, the cooperation of multiple quadrotors is an active and popular research topic as formation has broad applications in accomplishing complicated tasks in real-world domains [2–4].

There are various methods to realize formation cooperation, such as leader-follower strategy [3; 5], virtual structure approach [6] and behavior-based method [7].

The leader-follower method is the most common hierarchical structure used in the formation flight. Each follower aircraft is controlled to maintain its position and velocity to a designated leader aircraft which flies along some predefined trajectory. This structure is characterized by its easy reconfiguration and simple expandability to new objects, for its function can be quickly taken over by other UAVs [8]. However, this strategy does not use direct feedback from the followers in general.

Virtual architecture approach enables quadrotors to fly as if they were embedded in a rigid structure. The formation has a so-called geometry center, which is a virtual point determined by shape, flying speed, orientation of the formation or some other factors. If one of the UAVs in the team loses its position, the overall flock can change the flight trajectory to help the lost one. The literature [9] describes this approach applied in formation of space crafts. However, this strategy is relatively difficult to be implemented in practical applications.
The behavior-based approach prescribes the desired behaviors and generates a vector based on a weighted average of sensory inputs. Possible behaviors include formation keeping [7], trajectory tracking and obstacle avoidance. This strategy is suitable for applications with multiple agents and has an advantage in communication among agents. However, it is difficult to assure some of the characteristics of the formation.

In this paper we adopt a slide mode controller [10] to manipulate formation flight of three UAVs, which takes the leader-follower structure. PID control laws are used and preformed for each of the three UAVs. To verify the control effect, simulations with six-degrees-of-freedom state space UAV model are presented, which are run in Matlab/Simulink software.

The outline for this paper is as follows: Section 2 introduces the dynamic model of the quadrotor. Section 3 lays out the controller strategy, including PID controller to manipulate a single quadrotor and a formation controller to guide the whole team of UAVs. The simulation results are summarized in Section 4 which evaluates the performance of the strategy. Section 5 is the conclusions.

2. Dynamic model of single quadrotor

The kinematic of quadrotor is formalized in earth frame $E$ and body-fixed frame $B$. The position and Euler angles (roll, pitch and yaw) of the vehicle are defined as $r = [x\ y\ z]^T$ and $\eta = [\phi\ \theta\ \psi]^T$ respectively, with mass $m$ and inertia matrix $I$. The angular velocity $\omega = [p\ q\ r]$ is defined in the body-fixed frame $B$.

Some reasonable assumptions need to be considered to simplify the design which can be referred in literature [11]. Firstly, the quadrotor is a symmetric rigid body with the center of mass and body-fixed frame origin coinciding. So the inertia matrix $I$ can be simplified as $I = \text{diag} [I_{xx}\ I_{yy}\ I_{zz}]$. Secondly, the quadrotor operates in the hover state where roll and pitch are small ($c\phi \approx 1$, $c\theta \approx 1$, $s\phi \approx \phi$, $s\theta \approx \theta$), so the angular velocity $\omega$ can be seen equal to $\dot{\eta}$ and the gyroscopic torque can be neglected.

The translational and rotational dynamics equations are as follows:

$$m\ddot{r} = -mg e_z + T_B R_B^e e_z$$

$$I\ddot{\omega} = -\omega \times I\omega + \tau$$

where $g$ is the gravity acceleration; $e_z = [0\ 0\ 1]^T$ is the unit vector w.r.t. earth frame $E$; $T_B$ is the total force generated by the four rotors w.r.t. body-fixed frame $B$; $R_B^e \in SO(3)$ is the rotation matrix from body-fixed frame to earth frame; $\tau$ is the control torque in three dimensions obtained by changing the rotor speeds. The speeds of four rotors are manipulated by adjusting the total force $T_B$ and torque $\tau$, both of which actually are the components of inputs $u$ of the system. The relationship between inputs and the speed of rotors is as follows:

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} T_B \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} k & k & k & \Omega_1^x \\ 0 & -lk & 0 & lk \\ lk & 0 & -lk & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1^x \\ \Omega_2^x \\ \Omega_2^y \\ \Omega_2^z \end{bmatrix}$$

where $k$ is the thrust factor; $l$ is the distance from the center of mass to any one of the rotors; $d$ is the drag coefficient; $\Omega_i$ is the speed of the rotor $i$.

3. Control strategy

In this section, a PID law is utilized to make the single UAV move along the predefined trajectory. Then we focus on leader-follower formation control according to the geometry of formation and the relative dynamics.
3.1. **PID control**

The controller is designed to calculate the attitude and velocity needed for UAVs to follow the desired trajectory. Each quadrotor is controlled independently by the nested feedback loops shown in figure 1. The outer and inner loops control position and attitude respectively. The reference signals for velocity control are the sums of the reference values given by reference trajectory generator and correction signals calculated by rigid body dynamics. The superscript $d$ represents the corresponding value is desired.

![Figure 1. The nested control strategy block diagram](image)

### 3.1.1. **Attitude control.**

The attitude controller is designed at a point where roll and pitch are small. According to the assumptions, the products of inertia are zero and $I_{xx} \approx I_{yy}$ because of the symmetry then:

$$
I_{xx} \dot{\phi} = u_2 - qr(I_{zz} - I_{yy})
$$

(4)

$$
I_{yy} \dot{\theta} = u_3 - pr(I_{xx} - I_{zz})
$$

(5)

$$
I_{zz} \dot{\psi} = u_4
$$

(6)

The component of the angular velocity $r$ is very small. So the rightmost terms in (4) and (5) can be ignored compared with other terms. The system is presented in the hover state, so $\omega \approx \dot{\eta}$. For those reasons, simple proportional derivative control laws can be employed as follows:

$$
u_2^d = k_{p,\phi}(\dot{\phi}^d - \phi) + k_{d,\phi}(\ddot{\phi}^d - \dot{\phi})
$$

(7)

$$
u_3^d = k_{p,\theta}(\dot{\theta}^d - \theta) + k_{d,\theta}(\ddot{\theta}^d - \dot{\theta})
$$

(8)

$$
u_4^d = k_{p,\psi}(\dot{\psi}^d - \psi) + k_{d,\psi}(\ddot{\psi}^d - \dot{\psi})
$$

(9)

### 3.1.2. **Position control.**

We use pitch and roll to control position in XY plane, $u_4$ to control yaw angle, and $u_1$ to control height. The desired position is denoted as $r_i^d$, which can be calculated from PID controller

$$(\vec{r}_{i,T} - \vec{r}_i^d) + k_{d,\phi}(\vec{r}_{i,T} - \vec{r}_i) + k_{p,\phi}(r_{i,T} - r_i) + k_{d,\theta}(\vec{r}_{i,T} - \vec{r}_i) + k_{p,\theta}(r_{i,T} - r_i) = 0
$$

(10)

where $\dot{r}_{i,T} = \vec{r}_{i,T} = 0$ for hover.

By linearizing equation (1), the relationship between the desired acceleration and angles (roll and pitch) is obtained according to following assumptions:

a. The roll and pitch are small, so $c\phi \approx 1$, $c\theta \approx 1$, $s\phi \approx \phi$, $s\theta \approx \theta$.

b. The yaw keeps unchanged, which means $\psi = \psi_i = \psi_o$, where $\psi_o$ is the initial yaw and $\psi_i$ is the desired yaw.

c. In hover state $\sum T_B \approx mg$.

After linearization, we can get

$$
\phi^d = \frac{1}{g}(\vec{r}_i^d \sin \psi_i - \vec{r}_z^d \cos \psi_i)
$$

(11)

$$
\theta^d = \frac{1}{g}(\vec{r}_1^d \cos \psi_i + \vec{r}_2^d \sin \psi_i)
$$

(12)
\[ u^d_l = m^d_i \]  

(13)

3.2. Leader-follower formation control

The process of formation flight is as follows: the leader flies along a predefined trajectory which is stipulated by the trajectory tracking controller. Then the formation controller is designed to keep a specific shape in XY plane (e.g. the shape shown in figure 3), according to the relative kinematics between the leader and the follower. After the formation controller generating the desired velocity for the follower, the follower can track this velocity so as to keep the relative distance \( \lambda \) and orientation angle \( \phi \) constant at the same height. The leader tracks the predefined trajectory of the formation while the follower tracks the desired velocity to keep the shape, but the same tracking control strategy can be used for both UAVs. The formation flying strategy is shown in figure 2.

\[ \text{Figure 2. Control strategy block diagram} \]

The leader-follower structure for three UAVs is shown in figure 3, with a geometry called ‘V’ shape. The position of UAV can be determined by velocity \( v_i \) and angle velocity \( \omega_i \) for the yaw angle, which are the reference inputs to the trajectory tracking controller of the follower. More specifically, \( v_{ix} \) and \( v_{iy} \) are the velocity components in X and Y direction respectively w.r.t. body-fixed frame coordinate \( B \), the subscript \( i \) denotes the leader (\( i = L \)) or the follower (\( i = F \)). \( \psi_i \) is the angle between the heading direction and the \( x \) axis of earth frame \( E \).

Considering the dynamic characteristics of the quadrotor, it can be found that its vertical subsystem and horizontal subsystem are completely decoupling. Therefore, it is possible to design these two subsystems separately. For vertical subsystem,

\[ \dot{z}_i = v_{iz}, i = \{F, L\} \]  

(14)
where \( v_z \) is the velocity component in \( z \) axis w.r.t. body-fixed frame \( B \) and \( \dot{z}_i \) is the velocity component w.r.t. earth frame \( E \).

Then we mainly focus on the relative translational kinematics in XY plane at the same altitude, which is demonstrated in figure 4. \( \dot{x}_i \) and \( \dot{y}_i \) are the velocity components w.r.t. earth frame \( E \), while \( v_{ix} \) and \( v_{iy} \) are the velocity components w.r.t. body-fixed frame \( B \). The translational dynamics in the XY plane can be described as

\[
\begin{align*}
\dot{x}_i &= v_{ix} \cos \psi_L - v_{iy} \sin \psi_L \\
\dot{y}_i &= v_{ix} \sin \psi_L + v_{iy} \cos \psi_L \\
\dot{\psi}_L &= \omega_L
\end{align*}
\]

Similarly, \( \dot{x}_i, \dot{y}_i \) w.r.t. \( E \) can be transformed to \( v_{ix}, v_{iy} \) w.r.t. \( B \).

\[
\begin{align*}
\dot{x}_i &= \dot{x}_i \cos \psi_L + \dot{y}_i \sin \psi_L \\
\dot{y}_i &= -\dot{x}_i \sin \psi_L + \dot{y}_i \cos \psi_L
\end{align*}
\]

As shown in figure 4, \( \lambda \) is the distance from the mass center of the leader to one of the followers, \( \lambda_x, \lambda_y \) are the \( x, y \) coordinates in the leader’s body-fixed frame( \( B_L \)). Then,

\[
\lambda_x = (x_L - x_L) \cos \psi_L - (y_L - y_L) \sin \psi_L
\]

\[
\lambda_y = (x_L - x_L) \sin \psi_L - (y_L - y_L) \cos \psi_L
\]

Differentiating (20) w.r.t. time and using (15), (16), (17), (18) and (19), we can obtain

\[
\dot{\lambda}_x = \lambda_x \omega_L + \dot{x}_i \cos \psi_L + \dot{y}_i \sin \psi_L - \psi_L
\]

By defining \( e_{\psi} = \psi_F - \psi_L \), using the equation (15), (16) with the trigonometric identities, one gets

\[
\dot{\lambda}_x = \lambda_x \omega_L + v_{ix} \cos e_{\psi} - v_{iy} \sin e_{\psi}
\]

Similarly, \( \dot{\lambda}_y \) can be written as

\[
\dot{\lambda}_y = -\lambda_y \omega_L + v_{ix} \sin e_{\psi} + v_{iy} \cos e_{\psi} - \psi_L
\]

The desired distance \( \lambda^d \) and angle \( \phi^d \) are constant, so \( \dot{\lambda}_x = \dot{\lambda}_y = 0 \). Define the formation error

\[
\begin{align*}
e_x &= \lambda_x^d - \lambda_x \\
e_y &= \lambda_y^d - \lambda_y \\
e_{\psi} &= \psi_F - \psi_L
\end{align*}
\]

Differentiating (25), (26), (27) w.r.t. time, one gets

\[
\begin{align*}
\dot{e}_x &= -\left(\lambda_x^d - e_x\right) \omega_L - v_{ix} \cos e_{\psi} + v_{iy} \sin e_{\psi} + \psi_L
\end{align*}
\]

\[
\begin{align*}
\dot{e}_y &= \left(\lambda_y^d - e_y\right) \omega_L - v_{ix} \sin e_{\psi} - v_{iy} \cos e_{\psi} + \psi_L
\end{align*}
\]

\[
\dot{e}_{\psi} = \omega_F - \omega_L
\]

From the formation dynamics (28), (29) and (30), a control law is designed to make the errors converge to zero. For doing this, \( v_{ix}, v_{iy} \) and \( \omega_F \) are considered as the formation control inputs and the dynamics of the formation error is written as

\[
\dot{\chi} = F(\chi) + G(\chi)\psi
\]

where

\[
\chi = \begin{bmatrix} e_x & e_y & e_{\psi} \end{bmatrix}^T
\]

\[
\psi = \begin{bmatrix} v_{ix} & v_{iy} & \omega_F \end{bmatrix}^T
\]
\[ F(\chi) = \begin{bmatrix} e_1 \omega_L + \gamma_1 \\ -e_1 \omega_L + \gamma_2 \\ -\omega_L \end{bmatrix} \] (34)

\[ G(\chi) = \begin{bmatrix} -ce_\nu & se_\nu & 0 \\ -se_\nu & -ce_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (35)

with

\[ \gamma_1 = v_{L_x} - \alpha L \lambda_\nu^d \] (36)

\[ \gamma_2 = v_{L_y} + \alpha L \lambda_\nu^d \] (37)

The slide mode technique is used to design a robust controller. The switching function is

\[ \sigma_z = \chi + k_j \int_0^t \chi \, dt \] (38)

where \( k_j \) is a constant matrix. The dynamics of the formation error are expected to keep on a surface defined by \( \sigma_z = 0 \) through choosing \( k_j \) appropriately, on which the errors converge to zero. On this surface \( \dot{\sigma}_z = 0 \), so we can get

\[ \dot{\sigma}_z = \dot{\chi} + k_j \chi = F(\chi) + G(\chi) \nu_{eq} + k_j \chi = 0 \] (39)

\( \nu_{eq} \) can be obtained by (39) as

\[ \nu_{eq} = G^{-1}(\chi)(-F(\chi) - k_j \chi) \] (40)

To make sure the system keep on the surface \( \sigma_z = 0 \) considering perturbations, it can be obtained

\[ \dot{\sigma}_z = \dot{\chi} + k_j \chi = F(\chi) + G(\chi) \nu_{eq} + k_j \chi = -L \text{sgn}(\sigma_z) \] (41)

where \( L \) is a positive constant. Then the control input can be obtained by

\[ \nu_{eq} = G^{-1}(\chi)(-F(\chi) - k_j \chi - L \text{sgn}(\sigma_z)) \] (42)

The \( \text{sgn}(x) \) is defined as

\[ \text{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases} \] (43)

and the vector

\[ \text{sgn}(\sigma_z) = [\text{sgn}(\sigma_{z1}) \text{sgn}(\sigma_{z2}) \text{sgn}(\sigma_{z3})]^T \] (44)

4. Simulation and test results

The main aim of the paper is a numerical simulation which is run in Matlab/Simulink software. The controller system is highly sensitive to parameters. Improper parameters result in large overshoot, as a consequence, the expected distance among UAVs cannot be maintained. So the gains for PID controller are tuned experimentally to provide as short as possible convergence time and non-oscillatory character of the system response.

The quadrotors used in the simulation are all the same, with mass \( m = 0.58 \, \text{kg} \), and moments of inertia \( I_{xx} = I_{yy} = 0.1147 \, \text{kg} \cdot \text{m}^2 \) and \( I_{zz} = 0.0522 \, \text{kg} \cdot \text{m}^2 \). The products of inertia are assumed to be zero. These are typical values for small UAVs. In the simulation, the leader and the followers are set to follow the similar trajectory which is an s-shape curve actually. The velocity of the leader is provided to the followers. The initial position of quadrotor formation is \([0 \ 0 \ 0]^T\) and the selected distance \( \lambda_x = \lambda_y = 5 \, \text{m} \). The simulation time is 250 seconds. The red line is the leader’s trajectory while the black and blue line describe the trajectory of Follower 1 and Follower 2 respectively.
Simulation results of the quadrotor formation flight trajectory tracking in three dimensions and XY plane are presented in figure 5 and figure 6 respectively, indicating that the agents try to follow the leader’s trajectory fairly well.

The position errors of Follower 1 in X and Y direction in the first 10 seconds are shown in figure 7 and figure 8. Due to the fact that the three quadrotors start from the same position, the initial position errors in X and Y direction are both \(5\, m\) according to the predefined \(\lambda_x\) and \(\lambda_y\). Then the Follower 1 moves to (5, 5) while the Follower 2 moves to (-5, -5) in the XY plane w.r.t. body-fixed frame \(B\) of the leader. So \(e_x\) and \(e_y\) converge to zero during the first 5 seconds. As can be seen from the figures, the errors converge to zero at a fast rate and fluctuate around zero after 5 seconds. The position errors of Follower 2 in X and Y direction are similar to Follower 1. In addition, the error of yaw also oscillates between -0.04 rad and 0.08 rad in the first 5 seconds and then fluctuates around zero after 5 seconds, which is not illustrated in the figure. The error is relatively small without considering the communication delay and disturbance. The analysis of error demonstrates that the performance of the formation control is quite satisfactory under the circumstance where the roll and pitch are small.

5. Conclusions
The paper focuses on formation control with a leader-follower structure designed for three quadrotors. PID control law is presented to manipulate a single vehicle and a slide mode controller is adopted for formation control. The experiments illustrate the effective performance of control laws. However, this research is restricted to simulations without considering the practical noise and communication delay. Future research will focus on the hardware platform for formation flight experiments.

6. References
[9] Ren W and Beard R 2004 Decentralized scheme for spacecraft formation flying via the virtual structure approach J. Guid., Control, Dyna. 27 73-82

Acknowledgements
This work was supported by Postgraduate Innovation Research Fund of Beihang University under Grant YCSJ-01-2016-10.