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Kinematic Analysis of Four-Link Suspension of Steering Wheel by Means of Equation Sets of Geometrical Constraints with Various Structure

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Abstract. In research of the kinematic and dynamic properties of complex mechanical set-ups, results of numerical experiments are used. It is required to minimize the calculation time of various problems in the domain. For the multi-link suspension of the steered wheel, sets of the equations of the geometrical constraints were presented in two structurally different forms, scalar and vector. The vector set consists of the transcendental equations. Their solution was possible after previous expanding the trigonometric functions into power series. Because of the finite amount of the computer memory for the algorithm solving the vector form, it was possible to obtain solutions consisting of three terms. The number of terms in power series of equations' solutions determines the magnitudes of increments of input parameters (degrees of freedom). In this paper it is demonstrated, that fulfilling of this demand is possible by the change of the geometrical constraint's structure of the multi-link wheel suspension system.

1. Introduction

In theoretical problems in the domain of kinematics, guidance systems of steering wheel relative to the car body are represented by the spatial mechanisms with two degrees of freedom. The structure of these mechanisms is various. Their kinematics is solved by means of several different methods [2]. In the case of McPherson suspension and with two diagonal beams the matrix analysis is used [7].

Nonlinear sets of equations describing the geometrical constraints on the relative movement of the elements of multi-link suspension system are formulated in the scalar [3] or vector [6] form. These sets are usually solved by means of numerical methods. They can also be solved by use of the perturbation method [1]. The latter one makes possible to present the solutions of nonlinear sets of geometrical constraints as a series.

In papers [4, 5] from the domain of steered wheel's suspensions usually kinematic characteristics are analysed. These characteristics constitute the intersections of the spatial characteristics. These are usually the dependences between the relative change of wheel track and wheel camber and steering angles and levels of freedom, from which one has a fixed value. Such analysis is not precise, because in the wheel suspensions movement's space the mechanism's singularity can occurred.



In problems from the domain of the analysis of suspension synthesis and car dynamics modelling, calculation time minimization is desired. Because of this, a choosing the proper solution method is very important.

The wheel suspension system synthesis consists of several stages. At first, the structure of the suspension mechanism should be defined. Next, it's geometrical parameters are determined it means the coordinates of links joining beams with the body and with the stub axle or with the wheel support. Placement of these elements in the suspension movement's space follows the designed suspension's configuration. The synthesis of the steered wheel mechanisms is performed with the simultaneous change of two parameters. Usually these are the vertical translation of the wheel's centre and translating the rack of the steering gear [6].

Examination of the dynamic properties of designed cars is possible by means of the mathematical models with many degrees of freedom. In these models, suspension systems with different structure are considered.

2. The range and purpose of the paper

In the paper there will be presented two ways of solution of the multi-link steered wheels' suspension system kinematics with two degrees of freedom using the perturbation method [1]. In the first one, the set of 17 equations representing geometrical constraints of relative displacements of the suspension's elements will be formulated in the scalar form. In the second one, 5 equations in a vector form.

The main aim of the work will be a determination of the spatial kinematic characteristics of the four-link steered wheels' suspension mechanism. It will be presented the quantitative dependence between time of effective numerical computation and the structure of the geometrical constraints of the examined mechanism.

3. Structure of multi-link steered wheels' suspension system

In Fig. 1 it is presented the diagram of the structure of the four-link steered wheels' suspension system. Points B_1, B_2, B_4, B_5 are centres of the ball-joints linking the beam with the stub axle. Point B_3 is a centre of the ball-joint linking the extreme stub-axle mechanism's rod with the stub-axle beam.

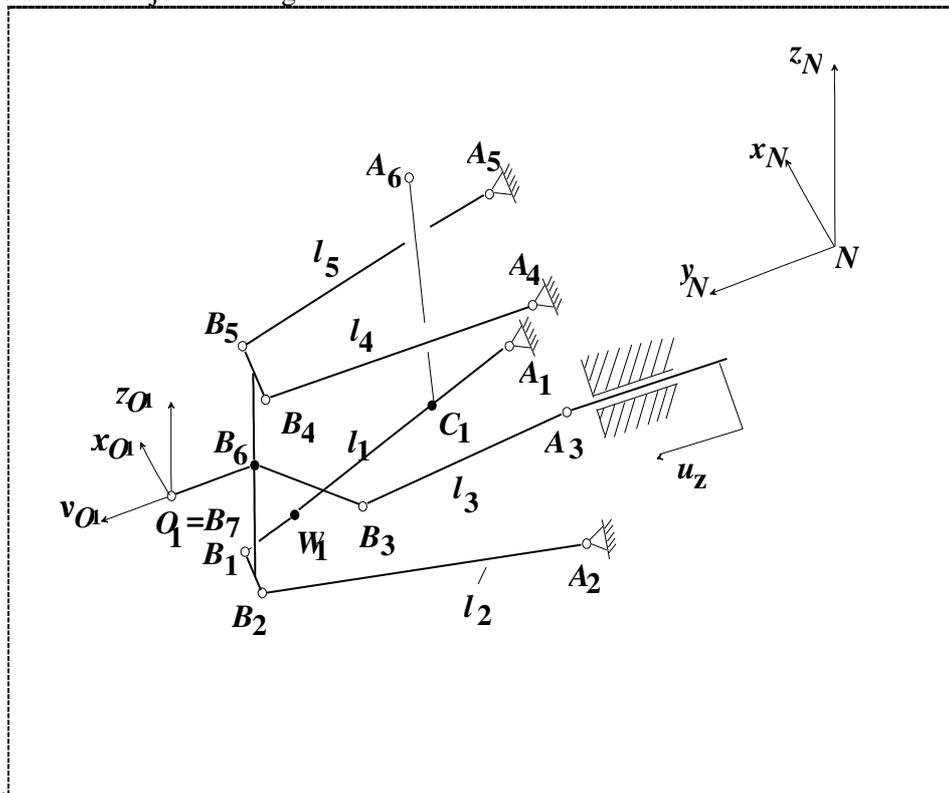


Figure 1. Scheme of the four-link steered wheel suspension mechanism .

Points A_1, A_2, A_4, A_5 are the centres of the ball-joints which replace the silent-block joints linking the beams with the body. Point A_3 is a centre of the ball-join joining the stub-axle mechanism's extreme rod with the steering gear's rack. The lower front beam, represented in the figure by the join A_1B_1 , is joined with the stabilizer in the point W_1 . In the point C_1 this beam is joined with the telescope column supporting the body in the point A_6 . Points B_6 and B_7 represent the wheel's rotation axis.

The sets of coordinates $\{N\}$, $\{O_1\}$ are connected with the body and the wheel's stub-axle respectively.

4. Equations of the mechanism's geometrical constraints

Geometrical constraints equations of the above presented suspension mechanism can be formulated as the set of 17 or 5 nonlinear algebraic equations. In the first one the equations express the squares of the distances between characteristic points of the mechanism

$$\begin{cases} \mathbf{r}_{AjBj}^T \mathbf{r}_{AjBj} = l_j^2, & \text{for } j = (1)5 \\ \mathbf{r}_{BjBk}^T \mathbf{r}_{BjBk} = l_{jk}^2, & \text{for } \begin{cases} j = 1 \text{ i } k = (2)5 \\ j = 2 \text{ i } k = (3)5 \\ j = (1)5 \text{ i } k = 7 \end{cases} \end{cases} \quad (1)$$

In the above set, the input parameters are coordinates q_3 of the point $O_1(q_1, q_2, q_3)$ and translation of the rack u_z , added to the coordinate y_{A3} of the point $A_3(x_{A3}, y_{A3}+u_z, z_{A3})$. At given parameters q_3 and u_z from the set (1) the coordinates of points $B_j(x_{Bj}, y_{Bj}, z_{Bj})$, for $j=(1)5$ and coordinates q_1 and q_2 of the point O_1 are determined.

In the second method, the equations express the squares of length of vectors with origins and ends in points A_j, B_j , for $j=(1)5$ respectively.

They are presented in the form:

$$(\mathbf{r}_{NO1.N} + \mathbf{A}_{NO1} \mathbf{r}_{O1Bj.O1} - \mathbf{r}_{NAj.N})^T (\mathbf{r}_{NO1.N} + \mathbf{A}_{NO1} \mathbf{r}_{O1Bj.O1} - \mathbf{r}_{NAj.N}) = l_j^2, \quad \text{for } j = (1)5. \quad (2)$$

where:

$$\mathbf{A}_{NO1} = \mathbf{A}_\gamma \mathbf{A}_\beta \mathbf{A}_\alpha = \begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \quad (3)$$

is a rotation matrix $\{O_1\}$ respectively to $\{N\}$.

However

$$l_j^2 = \mathbf{r}_{AjBj.N}^T \mathbf{r}_{AjBj.N}.$$

At given parameters q_3 and u_z from the set (1) are calculated the coordinates q_1 and q_2 of the wheel centre $O_1(q_1, q_2, q_3)$ and rotation angles $\{O_1\}$ respectively to $\{N\}$: α, β, γ .

For ensuring the equivalence of the calculation range of the algorithms derived on the basis of sets (1) and (2) it is necessary to determine the rotation angles $\{O_1\}$: α, β and γ respectively to $\{N\}$. So, the set (1) must be supplemented with the calculations of the mentioned angles.

After calculation of coordinates of points O_1 , and $B_j(j=(1)5)$ it is possible to create three vectors \mathbf{r}_{O1Bj} for $j \in \{1,2,3,4,5\}$. For every of these vector a matrix equation is fulfilled

$$\mathbf{r}_{O1Bj.O1} = \mathbf{A}_{O1N} \mathbf{r}_{O1Bj.N} \quad (4)$$

where:

$\mathbf{r}_{O1Bj.O1}$ – vector in $\{O_1\}$,
 $\mathbf{r}_{O1Bj.N}$ – vector in $\{N\}$,

$$\mathbf{A}_{O1N} = \mathbf{A}_{NO1}^T = \begin{bmatrix} c\beta c\gamma & c\beta s\gamma & -s\beta \\ sas\beta c\gamma - cas\gamma & sas\beta s\gamma + sac\gamma & sac\beta \\ cas\beta c\gamma + sas\gamma & cas\beta s\gamma - sac\gamma & cac\beta \end{bmatrix}$$

Denoting the coordinates of vectors: $\mathbf{r}_{O1Bj.O1} = [x_{bj} \ y_{bj} \ z_{bj}]^T$.

$\mathbf{r}_{O1Bj.N} = [x_{jb} \ y_{jb} \ z_{jb}]^T$ and putting $j = n, m, v$; on the basis (4) we obtain:

$$\left. \begin{aligned} x_{bn} &= (x_{nb}c\gamma + y_{nb}s\gamma)c\beta - z_{nb}s\beta \\ x_{bm} &= (x_{mb}c\gamma + y_{mb}s\gamma)c\beta - z_{mb}s\beta \\ x_{bv} &= (x_{vb}c\gamma + y_{vb}s\gamma)c\beta - z_{vb}s\beta \end{aligned} \right\} \quad (5)$$

From the set (5) the rotation angles β and γ are calculated. For calculation of the rotation angle α we use equation:

$$y_{bv} = (x_{vb}s\beta c\gamma)sa - (x_{vb}s\gamma)ca + (y_{vb}s\beta s\gamma)sa + (y_{vb}c\gamma)ca + (z_{vb}c\beta)sa \quad (6)$$

Next, having the rotation angles $\{N\}$ respectively to $\{O_1\}$, the $\mathbf{r}_{B6B7.N}$ vector's coordinates were calculated.

$$\mathbf{r}_{B6B7.N} = \mathbf{A}_{NO1} \mathbf{r}_{B6B7.O1}, \quad (7)$$

The unit vector of the wheel rotation axis $\mathbf{e}_k = [e_{kx} \ e_{ky} \ e_{kz}]^T$ and steering and camber angles of the wheel:

$$\delta_k = -\arctg\left(\frac{e_{kx}}{e_{ky}}\right), \quad (8)$$

$$\gamma_k = -\arcsin(e_{kz}). \quad (9)$$

Calculation of the angles δ_k and γ_k in both algorithms is analogous.

5. Solution of the equations' sets of the suspension mechanism's geometrical constraints

The solution of sets (1) and (2) were obtained by means of the perturbation method [1].

In respect to the transcendental set (2), trigonometric functions were resolved into trigonometric series:

$$\sin(x_0 + x) = \sin x_0 + x \cos x_0 - \frac{x^2 \sin x_0}{2}, \quad (10)$$

$$\cos(x_0 + x) = \cos x_0 - x \sin x_0 - \frac{x^2 \cos x_0}{2} \quad (11)$$

The resulting set of equations can be presented in the general form:

$$f_j(q_1, q_2, \alpha, \beta, \gamma) = 0, \quad j = (1)5 \quad (12)$$

Solutions of the set (12) were split into nonlinear and linear parts:

$$f_{jN}(q_1, q_2, \alpha, \beta, \gamma) + f_{jL}(q_1, q_2, \alpha, \beta, \gamma) = 0, \quad j = (1)5 \quad (13)$$

Nonlinear parts of these equations were multiplied by the perturbation parameter ε and an auxiliary set of equations was obtained

$$g_j(\varepsilon, q_1, q_2, \alpha, \beta, \gamma) = \varepsilon f_{jN} + f_{jL}, \quad j = (1)5, \quad (14)$$

For $\varepsilon=1$ the sets (13) and (14) are identical, but for $\varepsilon=0$ the set (13) it is formed only from the linear part.

It was assumed, that the solutions of set (14) is presented in the form of the series:

$$\left. \begin{aligned} q_1 &= \sum_{i=0}^m \varepsilon^i q_{1i}, & q_2 &= \sum_{i=0}^m \varepsilon^i q_{2i}, \\ \alpha &= \sum_{i=0}^m \varepsilon^i \alpha_i, & \beta &= \sum_{i=0}^m \varepsilon^i \beta_i, & \gamma &= \sum_{i=0}^m \varepsilon^i \gamma_i, \end{aligned} \right\} \quad (15)$$

After substitution of (15) to (14) we obtain

$$g_j(\varepsilon, q_1(\varepsilon), q_2(\varepsilon), \alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon)) = 0, \quad j = (1)5, \quad (16)$$

The set (16) was expanded into series respectively to the powers of ε :

$$\sum_{i=0}^2 \varepsilon^i g_{ji} = 0, \quad j = (1)5 \quad (17)$$

Next, the linear sets of equations $g_{ji} = 0$ were solved, at first for $i = 0$, next for $i=1,2$. The obtained solutions can be presented in the general form:

$$\left. \begin{aligned} q_1 &= \sum_{i=0}^2 q_{1i}, & q_2 &= \sum_{i=0}^2 q_{2i}, \\ \alpha &= \sum_{i=0}^2 \alpha_i, & \beta &= \sum_{i=0}^2 \beta_i, & \gamma &= \sum_{i=0}^2 \gamma_i, \end{aligned} \right\} \quad (18)$$

The set (1) was solved analogously.

6. Numerical example

The characteristic points were assigned to the designing placement of the suspension mechanism from Fig.1. Their values of coordinates (in mm) are as follows:

$A_1(132.1, 347.4, -93.8),$	$A_2(-245.3, 379.0, -114.3)$
$A_3(-88.05, 385.61, 280.92),$	$A_4(138.74, 431.18, 389.65)$
$A_5(6.47, 400.47, 381.42),$	
$B_1(29.0, 697.0, -98.2),$	$B_2(-23.7, 693.3, -132.5)$
$B_3(-140.3, 619.7, 284.0),$	$B_4(9.3, 675.3, 389.0)$
$B_5(-24.0, 642.0, 387.0),$	
$B_6(-0.2, 650.0, 0.5),$	$B_7(0.0, 750.0, 0.0)$

The solutions of sets (1), (2) were obtained for given input parameters q_3 and u_z with the same step equal 0.5 mm. Comparable solutions of the mentioned sets at this step were almost the same.

In Figures 2 and 3 kinematic characteristics of steering and camber angles together with the computation time are presented.

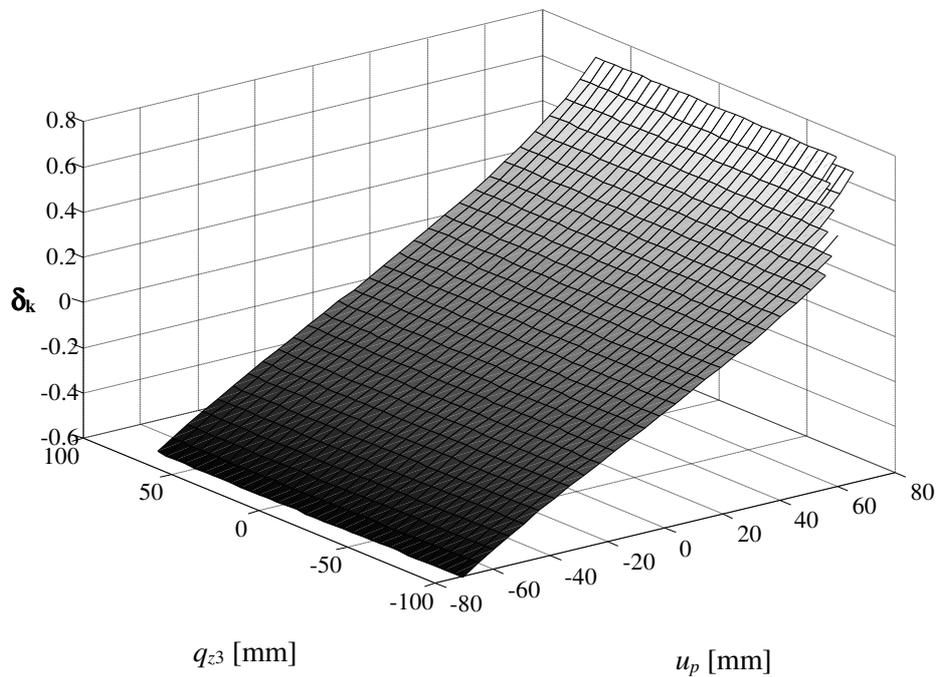


Figure 2. Dependence of steering angle – δ_k on the vertical displacement of the wheel centre – q_3 and on the displacement of the rack – u_p . Computation time for set of equation (1) = 516.02 s; Computation time for set of equation (2) = 15132.94 s.

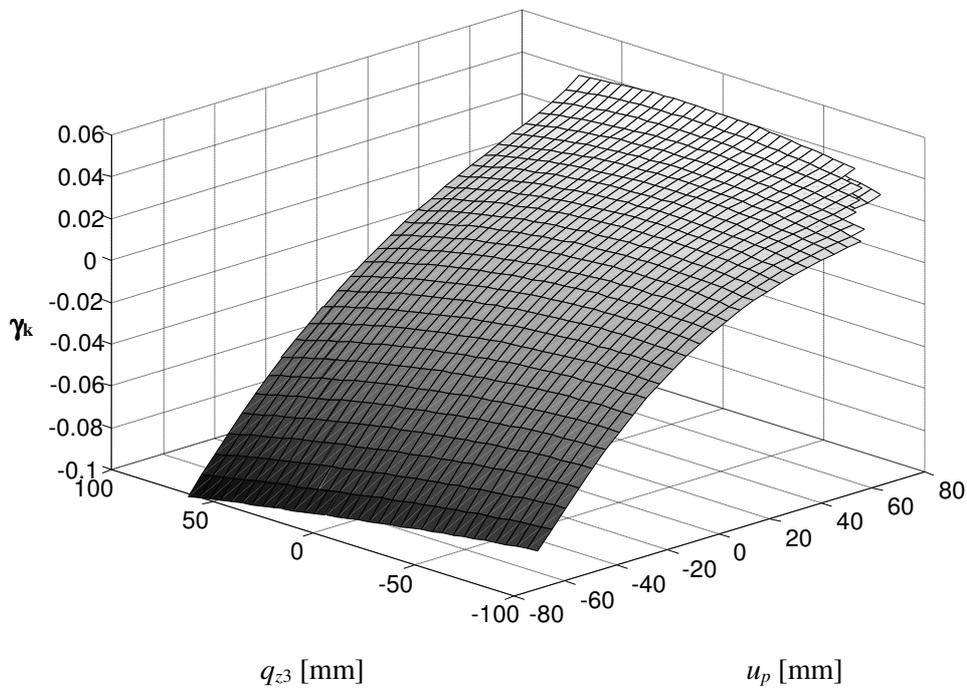


Figure 3. Dependence of camber angle – γ_k on the vertical displacement of the wheel centre – q_3 and on the displacement of the rack – u_p . Computation time for set of equation (1) = 516.02 s; Computation time for set of equation (2) = 15132.94 s.

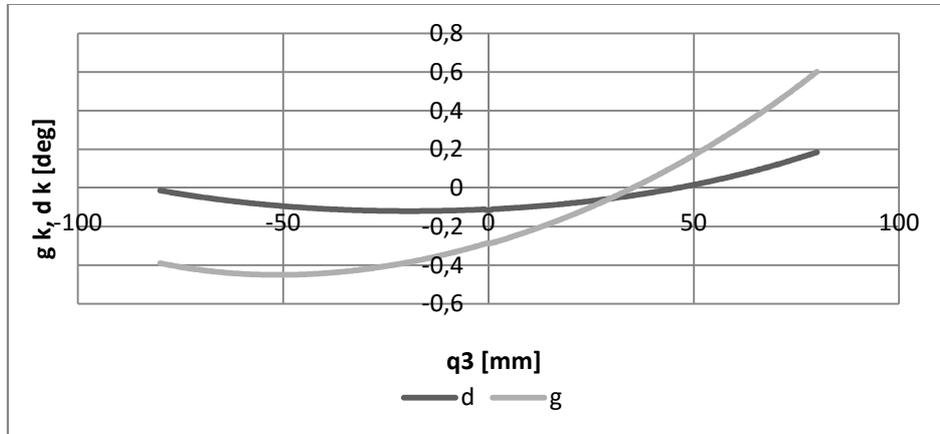


Figure 4. Steering angle and wheel camber characteristics, $u_z = 0$. Computation time for set of equation (1) = 2.276 s; Computation time for set of equation (2) = 45.216 s.

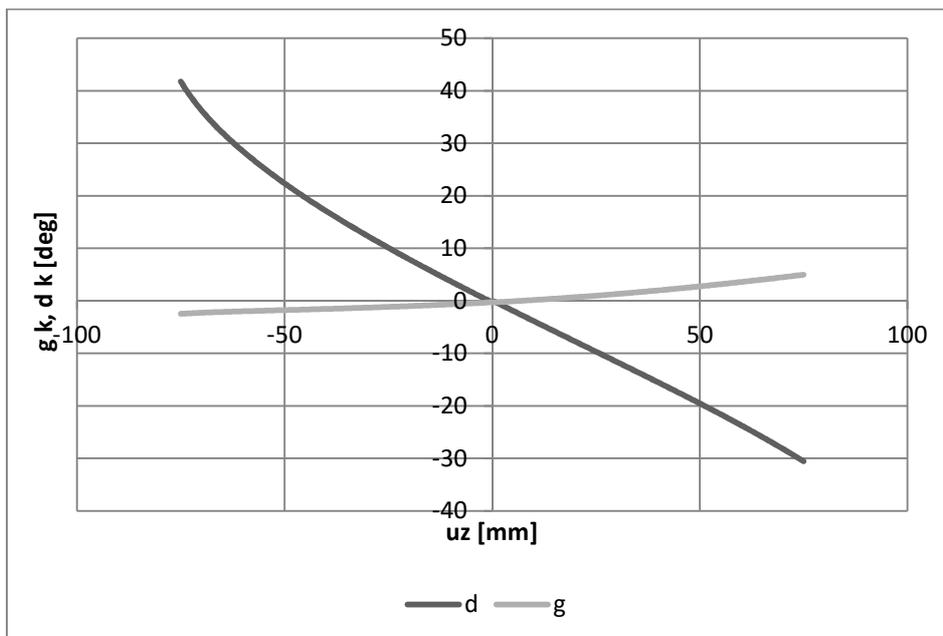


Figure 5. Steering angle and wheel camber characteristics, $q_3 = 0$. Computation time for set of equation (1) = 2.184 s; Computation time for set of equation (2) = 46.334 s.

7. Summary

For the multi-link suspension of the steered wheel, sets of the equations of the geometrical constraints were presented in two structurally different forms, scalar and vector. The vector set consists of the transcendental equations. Their solution was possible after previous expanding the trigonometric functions into power series (10) and (11).

Because of the finite amount of the computer memory for the algorithm solving the vector form, it was possible to obtain solutions consisting of three terms (18). The number of terms in power series of equations' solutions determines the magnitudes of increments of input parameters (degrees of freedom). Solutions of the set (1) are numerical series containing 10 terms each.

The calculations were performed on a computer with Intel Core i5 M430 CPU (2.27 GHz) and 4 GB RAM. Computation times of our own programs run in the MATLAB environment, containing 5 and 17 equations for the same problems in the suspension kinematics domain considerably differ.

Computation time for the set of 5 equations is many times greater (that is about 20-30 times) shown in all figures.

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