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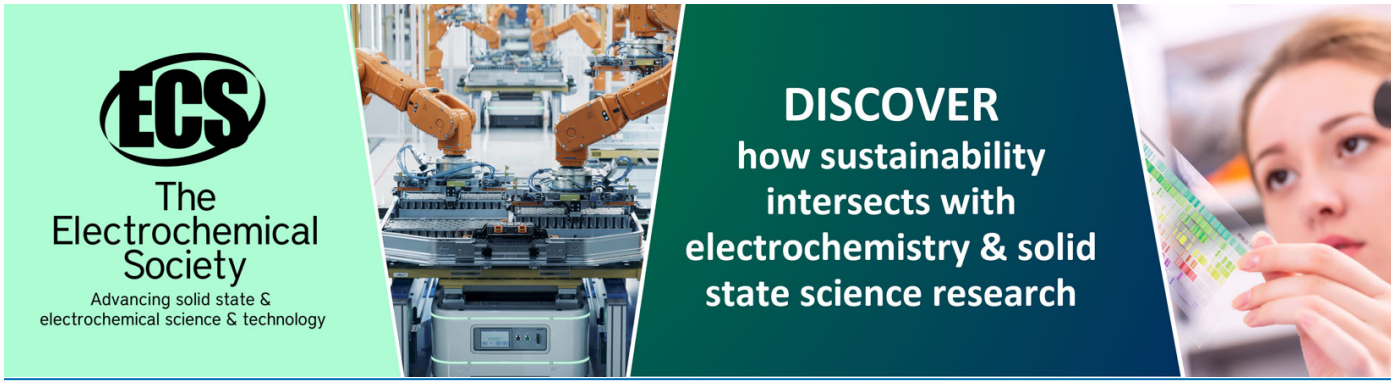

Numerical simulation of nanofluids based on power-law fluids with flow and heat transfer

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Abstract. In this paper, we investigate the heat transfer of nanofluids based on power-law fluids and movement of nanoparticles with the effect of thermophoresis in a rotating circular groove. The velocity of circular groove rotating is a constant and the temperature on the wall is kept to be zero all the time which is different from the temperature of nanofluids in the initial time. The effects of thermophoresis and Brownian diffusion are considered in temperature and concentration equations, and it is assumed that the thermal conductivity of nanofluids is a function of concentration of nanoparticles. Based on numerical results, it can be found that nanofluids improve the process of heat transfer than base fluids in a rotating circular groove. The enhancement of heat transfer increases as the power law index of base fluids decreases.

1. Introduction

Nanofluids which composed of nanoparticles and common fluids has been widely applied in many fields like physics, chemistry and biology, etc., in recent years. As nanofluids have been applied in solving many practical problems, many researchers become interested in studying nanofluids. For example, remedying the soil, removing oily soil and enhancing oil recovery can be done by nanofluids with surfactant micelles and it can improve the effect [1]. Also nanofluids has implication for cooling equipment and inkjet [2]. Actually the enhancement of heat transfer, one of the most important application of nanofluids, is catching more attention. “Nanofluids” is first proposed by Choi [3] who did research on the heat transfer about fluids. He found that putting nanoparticles in fluids could enhance thermal conductivities of base fluids. Since then lots of researches on the process of heat transfer of nanofluids come out and the theme is to find out reasons for the enhancement of thermal conductivity of nanofluids. At the beginning of these researches for the same kind of naoparticles and base fluids, nanoparticle size, concentration, dispersion, nanoparticle structure and microscopic motion are regraded as reasons [4-7]. The different kinds of nanoparticles and base fluids may also enhance heat transfer, e.g., nanofluid with Cu is better than the one with Al_2O_3 [5]. In those researches, it is found that experiment data are much higher than results of theoretical models like Maxwell’s equation [7]. Absence of microscopic motion and structure-dependent behavior in theoretical models are reasons for this error. In 2006, Buongiorno [16] chose some important reasons of enhancing thermal conductivity to discuss among inertia, Brownian diffusion, thermophoresis, magnus effect, diffusiophoresis, gravity and fluid drainage. He compared these seven physical quantities and concluded that Brownian diffusion and thermophoresis play the most important roles in enhancing the

thermal conductivity. Other five physical quantities also influenced the enhancement of thermal conductivity but the influence was too small and could be negligible. Then he proposed a model with the consideration of Brownian discussion and thermophoresis and this model includes the microscopic motion or Brownian diffusion.

Most people will regard the nanofluid as a mixture of nanoparticles and the base fluid. Not like a conventional mixture with solid and fluid, sometimes it behaves more like one kind of common fluids [8]. Based on this assumption, the concentration of nanoparticles is involved in all these new physics parameters such as the new viscosity which is also called the effective viscosity has a definition with $\mu_{eff} = \mu_f /(1-\phi)^{2.5}$, where μ_f and ϕ are the viscosity of the base fluid and concentration of nanoparticles, respectively. In many researches on nanofluids, this one-fluid model performs well and is widely applied to solve many problems especially on boundary layer flow and heat transfer [9-15]. In all these researches, it is assumed that the concentration of nanoparticles is a constant. But according to the model proposed in [16], the nanofluid was considered as a mixture and it has the relative motion between the base fluid and nanoparticles. Not a constant any more, the concentration of nanoparticles is an unknown variable coupled with flow equations and temperature equation. Results of the model are well agreed with some experiments and the model has been applied to solve various physical problems [17-22]. Two modeling ideas are sometimes combined and studied as well [23, 24, 25].

Following the idea proposed by Buongiorno [16], we choose to modify the model in [28] and study the flow and heat transfer of nanofluids based on power-law fluids in a rotating circular groove. In Section 2, the mathematical formulation of the physical problem is introduced. A temporally second order continuous finite element method is proposed in Section 3. Numerical results and discussion are presented in Section 4. Finally, Section 5 concludes the paper.

2. Mathematical formulation of the problem

Here the physical model we study is that nanofluids composed of nanofluids (e.g. Cu) and the power-law fluid flow in a rotating circular groove. The rotating angular velocity ω is $\pi/35$ (see also [28]). Meanwhile, the temperature change of nanofluids is also considered. Also the wall temperature of the circular groove is always zero during the whole process. The corresponding equations are as follows:

$$\nabla \cdot u = 0, \tag{1}$$

$$\rho_f(\partial u/\partial t + (u\cdot\nabla)u) = -\nabla p + \nabla\cdot\sigma_u, \tag{2}$$

$$\rho_f c_f(\partial T/\partial t + u\cdot\nabla T) = \nabla\cdot(k\nabla T) + \rho_p c_p[D_B \nabla C\cdot\nabla T + D_T/T_{in}\nabla T\cdot\nabla T], \tag{3}$$

$$\partial C/\partial t + u\cdot\nabla C = \nabla\cdot[D_B\sigma_C + D_T\nabla T/T_{in}]. \tag{4}$$

Where σ_u and σ_C represent the viscous part of stress tensor and Brownian diffusion, respectively, and their expressions are given as follows:

$$\sigma_u = \eta(2(\partial u_x/\partial x)^2 + (\partial u_x/\partial y + \partial u_y/\partial x)^2 + 2(\partial u_y/\partial y)^2)^{(m-1)/2}(\nabla u + \nabla u^T), \tag{5}$$

$$\sigma_C = ((\partial C/\partial x)^2 + (\partial C/\partial y)^2)^{(m-1)/2}\nabla C. \tag{6}$$

In addition, here the vector $\vec{x} = (x, y)$ denotes the coordinate system (x and y indicate the streamwise direction and the normal direction, respectively.). Also $u = (u_x, u_y)$ represents the velocity of the nanofluid, where u_x and u_y denote the velocity components along $x-$ and $y-$axes. P, T and C represent the hydrostatic pressure, the temperature and the concentration of nanofluids, respectively. m is the power law index of power law fluids. When $m-1$, the fluid becomes Newtonian fluid. The parameters ρ_f, ρ_p, c_f, c_p, η are the fluid density, nanoparticles density, fluid specific heat, nanoparticles specific heat and relative viscosity coefficient for power law fluids, respectively. As pointed out in [9], here the thermal conductivity k can be written as

$$k = \frac{k_f((k_p + 2k_f) - 2C(k_f - k_p))}{(k_p + 2k_f) + C(k_f - k_p)}, \tag{7}$$

Where k_f and k_p are the thermal conductivity of base fluids and thermal conductivity of nanoparticles, respectively. D_B and D_T denote the Brownian diffusion coefficient and the thermophoresis diffusion coefficient, respectively. The corresponding boundary condition at the edge of the circular groove are

$$u_x = \omega y, \quad u_y = -\omega x, \quad T = T_\omega, \quad \partial C / \partial n = 0. \tag{8}$$

The following dimensionless quantities are introduced:

$$U = u/V, \quad t^* = tV/L, \quad \theta = (T - T_\omega)/(T_{in} - T_\omega), \tag{9}$$

$$p^* = pL^m/(\eta V^m), \quad X = x/L, \quad \phi = C/C_\infty, \tag{10}$$

where L is the characteristic length and V is the characteristic speed. T_{in} is the initial temperature of nanofluids and C_∞ is the characteristic concentration. When substituting these dimensionless quantities into Eqs.(1)-(4) and denoting $u = U$, $x = X$, $p = p^*$, $t = t^*$, we can obtain

$$\nabla \cdot u = 0, \tag{11}$$

$$\mathrm{Re}(\partial u/\partial t + (u \cdot \nabla)u) = -\nabla p + \nabla \cdot \sigma_u', \tag{12}$$

$$\mathrm{Re} \cdot \mathrm{Pr}(\partial \theta/\partial t + u \cdot \nabla \theta) = \nabla \cdot (k/k_f \nabla \theta) + \mathrm{Re} \cdot \mathrm{Pr}[N_B \nabla \phi \cdot \nabla \theta + N_T \nabla \theta \cdot \nabla \theta], \tag{13}$$

$$\mathrm{Re} \cdot Sc(\partial \phi/\partial t + u \cdot \nabla \phi) = \nabla \cdot \sigma_\phi + N_T/N_B \Delta \theta. \tag{14}$$

Some dimensionless parameters in Eqs.(11)-(14) become

$$N_B = \rho_p c_p D_B/(\rho_f c_f VL), \quad N_T = \rho_p c_p D_T (T_{in} - T_\omega)/\rho_f c_f T_{in} VL, \tag{15}$$

$$\mathrm{Re} = \rho_f VL(V/L)^{1-m}/\eta, \quad \mathrm{Pr} = c_f \eta (L/V)^{1-m}/k_f, \tag{16}$$

$$Sc = \eta (L/V)^{1-m}/\rho_f D_B, \quad \sigma_\phi = ((\partial \phi/\partial x)^2 + (\partial \phi/\partial y)^2)^{(m-1)/2} \nabla \phi, \tag{17}$$

$$\sigma_u' = (2(\partial u_x/\partial x)^2 + (\partial u_y/\partial x + \partial u_x/\partial y)^2 + 2(\partial u_y/\partial y)^2)^{(m-1)/2} (\nabla u + \nabla u^T). \tag{18}$$

The boundary conditions given in (8) become

$$u_x = \omega y, \quad u_y = -\omega x, \quad \theta = 0, \quad \partial \phi/\partial n = 0. \tag{19}$$

3. Finite element scheme

The main aim of this section is to present finite element scheme on Eqs.(11)-(14). Before we do this, some notations used in the subsequent computation are introduced. Ω denotes the domain of the circular groove in R^2. $\Delta t(>0)$ denotes the time step size and $t^{n+1} = (n+1)\Delta t$. So u_h^n, p_h^n, θ_h^n, and ϕ_h^n represent an approximation of u, p, θ and ϕ at the time t^n. On the one hand, to have a more stable formulation for the pressure p [27], the continuity equation (1) is rewritten as

$$\nabla \cdot u + \delta p = 0, \quad \delta = 10^{-6}. \tag{20}$$

On the other hand, based on the work in [27, 29, 30], the midpoint scheme on the time is used to Eqs.(11)-(14). As a result, the discretized formulas on them become as follows:

$$\int_\Omega ((\nabla \cdot u_h^{n+1/2})q + \delta p_h^{n+1/2} q) dX = 0, \tag{21}$$

$$\int_\Omega (\mathrm{Re}(u_{\bar{t}}^{n+1/2}\cdot v+(u_h^{n+1/2}\cdot\nabla u_h^{n+1/2})\cdot v)+p_h^{n+1/2}(\nabla\cdot v)+\sigma_{uh}^{'n+1/2}:\nabla v)dX=0, \quad (22)$$

$$\int_\Omega (\mathrm{Re}\cdot\mathrm{Pr}(\theta_{\bar{t}}^{n+1}\omega+(u_h^{n+1/2}\cdot\nabla\theta_h^{n+1/2})\omega)+(k_h^{n+1/2}/k_f)\nabla\theta_h^{n+1/2}\cdot\nabla\omega)dX$$

$$-\int_\Omega \mathrm{Re}\cdot\mathrm{Pr}(N_B(\nabla\phi_h^{n+1/2}\cdot\nabla\theta_h^{n+1/2})\omega+N_T(\nabla\theta_h^{n+1/2}\cdot\nabla\theta_h^{n+1/2}\omega))dX=0, \quad (23)$$

$$\int_\Omega (\mathrm{Re}\cdot Sc(\phi_{\bar{t}}^{n+1}\varphi+(u_h^{n+1/2}\cdot\nabla\phi_h^{n+1/2})\varphi)+\sigma_{\phi,h}^{n+1/2}\cdot\nabla\varphi+N_T/N_B\cdot\nabla\theta_h^{n+1/2}\cdot\nabla\varphi)dX$$

$$-\int_\Gamma (N_T/N_B\cdot(\nabla\theta_h^{n+1/2}\cdot n)\varphi)ds=0, \quad (24)$$

where $u_h^{n+1/2}=(u_{x,h}^{n+1/2},u_{y,h}^{n+1/2})$, $p_h^{n+1/2}=(p_h^n+p_h^{n+1})/2$, $u_{\bar{t}}^{n+1/2}=(u_h^{n+1}-u_h^n)/\Delta t$, $u_h^{n+1/2}=(u_h^{n+1}+u_h^n)/2$,

$\theta_{\bar{t}}^{n+1/2}=(\theta_h^{n+1}-\theta_h^n)/\Delta t$, $\theta_h^{n+1/2}=(\theta_h^{n+1}+\theta_h^n)/2$, $\phi_{\bar{t}}^{n+1/2}=(\phi_h^{n+1}-\phi_h^n)/\Delta t$, $\phi_h^{n+1/2}=(\phi_h^{n+1}+\phi_h^n)/2$. In addition, we can obtain

$$k_h^{n+1/2}/k_f=(1-2\phi_h^{n+1/2})k_p+2(1-\phi_h^{n+1/2})k_f/((2+\phi_h^{n+1/2})k_f+(1-\phi_h^{n+1/2})k_p), \quad (25)$$

$$\sigma_{uh}^{'n+1/2}=[(2(\partial u_{x,h}^{n+1}/\partial x)^2+(\partial u_{x,h}^{n+1}/\partial y+\partial u_{y,h}^{n+1}/\partial x)^2+2(\partial u_{y,h}^{n+1}/\partial y)^2)^{(m-1)/2}$$

$$+(2(\partial u_{x,h}^{n+1}/\partial x)^2+(\partial u_{x,h}^{n+1}/\partial y+\partial u_{y,h}^{n+1}/\partial x)^2+2(\partial u_{y,h}^{n+1}/\partial y)^2)^{(m-1)/2}](\nabla(u_h^{n+1}+u_h^n)$$
$$+\nabla(u_h^{n+1}+u_h^n)^T)/4, \quad (26)$$

$$\sigma_{\phi,h}^{n+1/2}=[((\partial\phi_h^{n+1}/\partial x)^2+(\partial\phi_h^{n+1}/\partial y)^2)^{(m-1)/2}+((\partial\phi_h^n/\partial x)^2+(\partial\phi_h^n/\partial y)^2)^{(m-1)/2}]\nabla(\phi_h^{n+1}+\phi_h^n)/4. \quad (27)$$

Since Eqs.(21)-(24) are nonlinear implicit equations, a linearization for them should be considered. Here Newtonian method and the fixed point method shown in [27, 31] are used to solve the linear system at each time step. Here $\bar{u}_s=(\bar{u}_{x,s},\bar{u}_{y,s})$, $\bar{p}_s$, $\bar{\theta}_s$ and $\bar{\phi}_s$ represent an approximation of u_h^{n+1}, p_h^{n+1}, θ_h^{n+1} and ϕ_h^{n+1}, respectively. where the initial guess is chosen as $\bar{u}_0=u_h^n=0$, $\bar{p}_0=p_h^n=0$, $\bar{\theta}_0=\theta_h^n=1$ and $\bar{\phi}_0=\phi_h^n=0.01$. Also the parameter α, in (30) becomes

$$\alpha=((1-(\bar{\phi}_{s-1}+\phi_h^n))k_p=2(1-(\bar{\phi}_{s-1}+\phi_h^n)/2)k_f)/((2+(\bar{\phi}_{s-1}+\phi_h^n)/2)k_f+(1-(\bar{\phi}_{s-1}+\phi_h^n)/2)k_p), \quad (28)$$

In addition, the above computation are based on the Freefem++ platform [26].

4. Numerical results and discussion

The main aim of this section is to present numerical results and discussion. We mainly consider the case: for $m=0.91$, the values of some dimensionless parameters are used as follows:

$$\mathrm{Re}=70.73,\quad \mathrm{Pr}=41.08,\quad Sc=15.80. \quad (29)$$

In addition, $N_B = N_T = 3.29\times10^{-5}$, $k_p = 400W/mK$, $k_f = 0.613W/mK$, $\Delta t = 10^{-5}$ are used for this case.

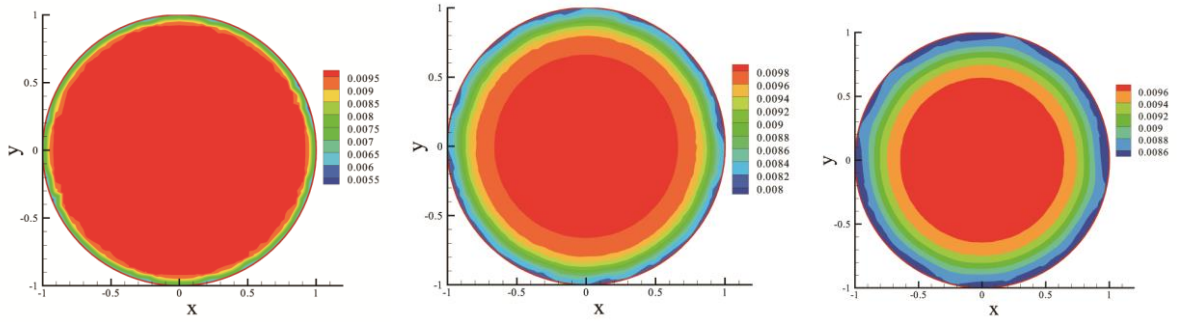

FIGURE 1. The concentration of nanoparticles changes for the nanofluid based on the pseudo-plastic fluid ($m = 0.91$) when $t = 0.04$, 0.3 and 0.8.

Fig.1 shows planar figures about the change of concentration of nanoparticles in the nanofluid based on the pseudo-plastic fluid ($m = 0.91$) for $t = 0.04$, 0.3 and 0.8. Similarly, the change of temperature of nanofluids is shown in Fig.2. As observed in Figs.1-2, we can find that the concentration is not homogeneous close to the boundary of the circular groove. However, the temperature close to the boundary of the groove is still homogeneous when the thermal conductivity k of nanofluids is a function of the concentration of nanoparticles (see (7)). All these facts illustrate that the effect of the concentration on the thermal conductivity of nanofluids is very small for the case.

The changes of the concentration and temperature of the nanofluid based on $m = 0.91$ at $y = 0$ and $x \in [-1,0]$ are shown in Fig.3. When the time increases, nanoparticles go to the boundary from the center of the circular groove and the temperature of the nanofluid decreases. As seen in the left figure of Fig.4, the temperature of nanofluid with three different base fluids ($m = 0.85, m = 0.91, m = 1.0$) at $y = 0$ and $x \in [-1,0]$ when $t = 1$ is shown. Obviously, the dimensionless temperature θ increases as m increases, and the viscosity of the base fluid still is the main reason to enhance the heat transfer.

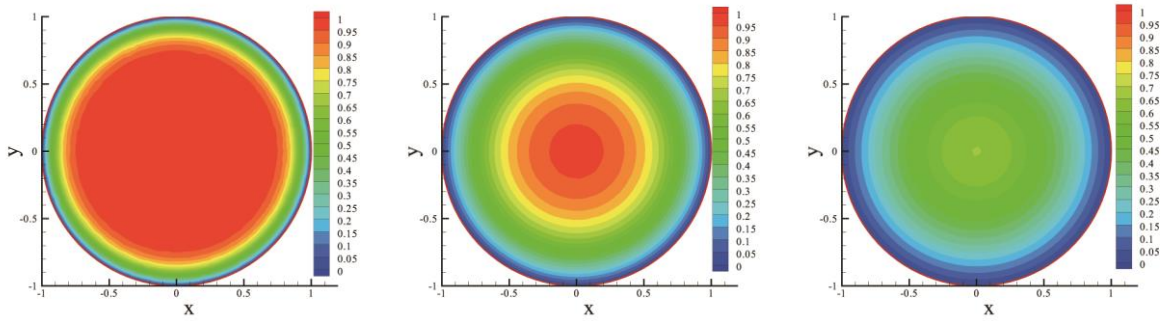

FIGURE 2. The temperature changes for the nanofluid based on the pseudo-plastic fluid ($m = 0.91$) when $t = 0.04$, 0.3 and 0.8.

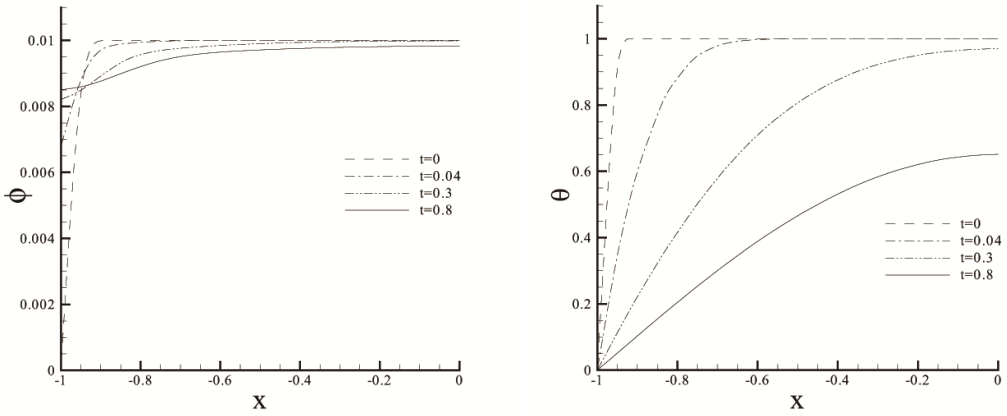

FIGURE 3. The change of physical quantities of nanofluid for pseudo-plastic fluid ($m = 0.91$) with t = 0, 0.04, 0.3 and 0.8. Left: Concentration of nanoparticles; Right: Temperature of nanofluid.

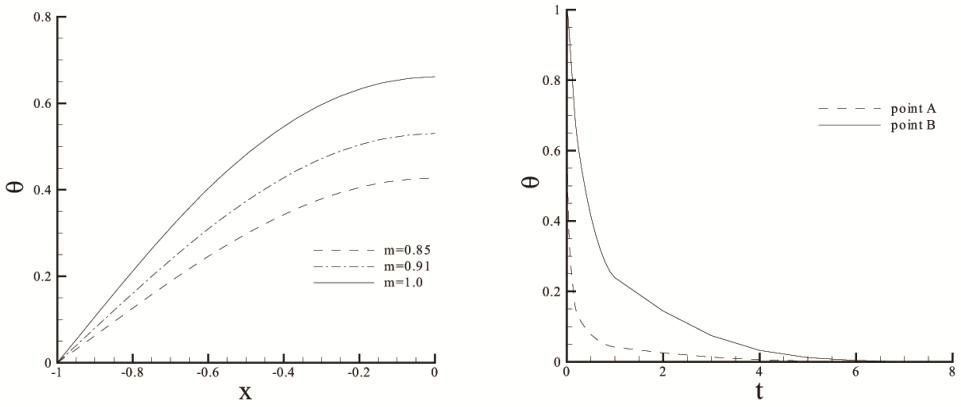

FIGURE 4. The change in temperature for three nanofluids with different base fluids; Right: The change of temperature of nanofluids for the base fluid (i.e. $m = 0.91$).

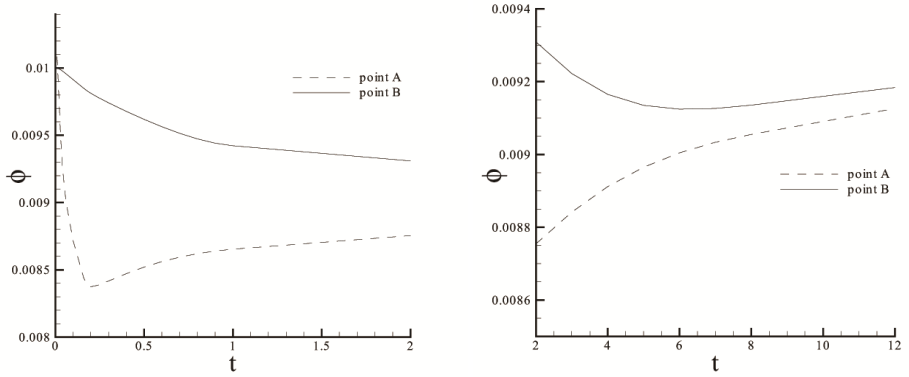

FIGURE 5. The compare on dimensionless concentration ϕ between point A and point B. Left: t from 0 to 2; Right: t from 2 to 12.

To show the difference of heat transfer on the concentration, $x = -0.774178$, $y = 0.556429$ and $x = -0.486749$, $y = 0.544328$ are chosen as point A and point B, respectively. According to the domain Ω, point A is much closer to the boundary than point B. The change of the concentration of

nanoparticles for point A and point B is shown in Figs.4-5. Obviously, for point A, the concentration decreases first when t goes from 0 to 0.2 and increases later, and its change is greater than the concentration for point B.

5. Conclusions

In this paper, the heat transfer of nanofluids based on power-law fluids in a rotating circular groove is investigated numerically with the finite element method. The concentration equation of nanoparticles is coupled with the temperature equation with considering Brownian diffusion and thermophoresis. We have assumed that the thermal conductivity of nanofluids is a function of the concentration of nanoparticles. Some phenomena of nanofluids in this dynamic heat transfer process are discussed (see Section 4), and these results may be helpful to better understand the movement of nanoparticles.

Acknowledgments

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