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Optimal Control of Malaria Transmission using Insecticide Treated Nets and Spraying

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Abstract. In this paper, we consider a model of the transmission of malaria which was developed by Silva and Torres equipped with two control variables, namely the use of insecticide treated nets (ITN) to reduce the number of human beings infected and spraying to reduce the number of mosquitoes. Pontryagin maximum principle was applied to derive the differential equation system as optimality conditions which must be satisfied by optimal control variables. The Mangasarian sufficiency theorem shows that Pontryagin maximum principle is necessary as well as sufficient conditions for optimization problem. The 4th-order Runge Kutta method was then performed to solve the differential equations system. The numerical results show that both controls given at once can reduce the number of infected individuals as well as the number of mosquitoes which reduce the impact of malaria transmission.

1. Introduction

Epidemiology of malaria is one of quite complicated biological system. The parasites and its vertebrate and the arthropods hosts interact in close dependence on environmental conditions. Malaria is caused by one of the four species of protozoa in the genus of Plasmodium, namely *P. falcifarum, P. vivax, P. malariae,* and *P. ovale* [1]. The parasites are transmitted to humans by female mosquitoes belonging to certain species of the genus Anopheles each time the infected insect takes human blood. Conversely, the female mosquito can pick up the infection when they bite infected humans. Mosquito density largely depends on the longevity of the adult mosquitoes and the availability of suitable breeding places.

Environmental condition in the tropics is the prime factor for it being endemic. The moderate-to-warm temperatures, high humidity and water bodies allow mosquito and parasites to reproduce. The epidemiological pattern of malaria usually vary with season because of the dependency of transmission on mosquitoes. Each year 1-3 million deaths are attributed worldwide to malaria, out of which one-third are children. However, malaria can be prevented and controlled if it is detected early. Therefore, everyone needs to understand the main parameters in the transmission of the disease and to develop strategies for effective solutions to the prevention and control [3].

Malaria control is carried out through the following recommended malaria treatment and prevention interventions. The choice of interventions depends on the malaria transmission level in the area. In most malaria-endemic countries, four interventions—case management (diagnosis and treatment), ITNs (insecticide-treated nets), IPTp (intermittent preventive treatment of malaria in pregnant women), and IRS (indoor residual spraying) make up the essential package of malaria interventions. Occasionally, other interventions are used: larval control and other vector control interventions and mass drug administration and mass fever treatment [2].

Sir Ronald Ross, while working in the Indian Medical Service in 1890's, demonstrated the-life cycle of the malaria parasite in mosquito. He developed a simple model, now known as 'Ross Model', which explains the relationship between the number of mosquitoes and malaria in humans. Ross introduced a deterministic model of malaria by the differential equations by dividing the susceptible human population (S_h) and infected (I_h), with classes being reinfected again to the vulnerable class and leads to the structure of SIS. The mosquito population also has only two compartments (S_m, I_m), but they do not recover from the infection because of their short lifespan, and leads to the structure of SI [4].

2. Controlled model

Silva and Torres [5] modify the Ross model with an additional possibility of reducing cases of malaria caused by the education. In this study, we consider a model of the transmission of malaria which is developed by Silva and Torres equipped with two control variables, namely the use of insecticide treated nets (ITN) to reduce the number of human beings infected and spraying to reduce the number of mosquitoes.

We consider a malaria transmission model which consists of human population as the host and mosquito population as the vector, each is divided into two classes namely the susceptible and infected compartments. We denote by S_h and S_v the classes of susceptible humans and mosquitos, respectively, by I_h and I_v the classes of infected humans and mosquitos, respectively. So that total population for human (N_h) is given by $N_h = S_h + I_h$, and total population for mosquito (N_v) is given by $N_v = S_v + I_v$. The model is constructed under the following assumptions: all newborns individuals are assumed to be susceptible and no infected individuals are assumed to come from outside the community. The human and mosquito recruitment rates are denoted by Λ_h and Λ_v, respectively. The disease is fast progressing, thus the exposed stage is minimal and is not considered. Infectious individuals can die from the disease or become susceptible after recovery while the mosquito population does not recover from infection. ITN contributes for the mortality of mosquitoes. The average number of bites per mosquito, per unit of time (mosquito-human contact rate), is given by

$$\beta = \beta_{\max}(1 - b)$$

where $\beta_{\max}$ denotes the maximum transmission rate and b the proportion of ITN usage. It is assumed that the minimum transmission rate is zero. The value of β is the same for human and mosquito population, so the average number of bites per human per unit of time is $\beta N_v / N_h$. Thus, the force of infection for susceptible humans (λ_h) and susceptible vectors (λ_v) are given by

$$\lambda_h = \frac{m_1 \beta I_v}{S_h + I_h} \qquad \text{And} \qquad \lambda_v = \frac{m_2 \beta I_h}{S_h + I_h},$$

where m_1 and m_2 are the transmission probability per bite from infectious mosquitoes to humans, and from infectious humans to mosquitoes, respectively. The death rate of the mosquitoes is modeled by $\mu_{vb} = \mu_v + \mu_{max}$, where μ_v is the natural death rate and μ_{max} is the death rate due to pesticide on ITNs.

The coefficient $(1 - u(t))$ on Silva and Torres, represents the effort of susceptible humans that become infected. The proportion of ITN usage (b) is static. In this study, u_1 as control of proportion of nets usage that is dependent on t or dynamic. The new control variable u_2 added to the model represents spraying to reduce the number of mosquitoes. If $u_1 = 1$ then the proportion of nets usage is maximum. The equation of motion of the disease transmission control is provided by the following first order nonlinear differential equation. Table 1 provides the symbols and descriptions for the differential equation system.

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - (1-u_1)\frac{m_1\beta_{\max}I_v}{S_h+I_h}\,S_h + \gamma_h I_h - \mu_h S_h,\\
\frac{dI_h}{dt} &= (1-u_1)\frac{m_1\beta_{\max}I_v}{S_h+I_h}\,S_h - (\mu_h+\gamma_h+\delta_h)I_h,\\
\frac{dS_v}{dt} &= \Lambda_v - (1-u_1)\frac{m_2\beta_{\max}I_h}{S_h+I_h}\,S_v - \mu_{vb}S_v - u_2 S_v,\\
\frac{dI_v}{dt} &= m_2(1-u_1)\frac{m_2\beta_{\max}I_h}{S_h+I_h}S_v - \mu_{vb}I_v - u_2 I_v.
\end{aligned} \tag{1}$$

Table 1. Symbol and description for differential equation system.

Symbol	Description	Value
Λ_h	Recruitment rate in humans	$10^3/(70\times 365)$
Λ_v	Recruitment rate in mosquitoes	$10^4/21$
μ_h	Natural mortality rate in humans	$1/(70\times 365)$
δ_h	Disease induced mortality rate in humans	10^{-3}
γ_h	Recovery rate of infectious	$1/4$
μ_v	Natural mortality rate of mosquitoes	$1/21$
$\mu_{\max}$	Mortality rate of mosquitoes due to treated net	$1/21$
$\beta_{\max}$	Maximum mosquito-human contact rate	0.1
m_1	Probability of disease transmission from mosquito to human	1
m_2	Probability of disease transmission from human to mosquito	1
$S_h(0)$	Susceptible individuals initial value	800
$I_h(0)$	Infectious individuals initial value	200
$S_v(0)$	Susceptible vectors initial value	4000
$I_v(0)$	Infectious vectors initial value	900

3. Optimal control problem

Our control effort aims to seek the optimal controls u_1 and u_2, that are carrying the system from initial state $(S_{h0}, I_{h0}, S_{v0}, I_{v0})$ to undetermined state $(S_h(t_f), I_h(t_f), S_v(t_f), I_v(t_f))$, such that the controls minimize the following control performance

$$J(u_1,u_2) = \int_0^{t_f} [A_1 I_h + A_2 S_v + A_3 I_v + (C_1 u_1^2 + C_2 u_2^2)]dt, \tag{2}$$

where A_1 , A_2, dan A_3 are weight constant on infectious human, suspected mosquitoes, and infectious mosquitoes, respectively. Weight coefficient, $C_1$and C_2 are measure of the relative cost of the interventions associated to the control u_1 and u_2. Minimizing the objective function is to minimize the number of I_h, S_v and I_v. Optimum control problem can be written as follows:

$$\min J,$$

with constraints:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - (1-u_1)\frac{m_1\beta_{\max}I_v}{S_h+I_h}\,S_h + \gamma_h I_h - \mu_h S_h,\\
\frac{dI_h}{dt} &= (1-u_1)\frac{m_1\beta_{\max}I_v}{S_h+I_h}\,S_h - (\mu_h+\gamma_h+\delta_h)I_h,\\
\frac{dS_v}{dt} &= \Lambda_v - (1-u_1)\frac{m_2\beta_{\max}I_h}{S_h+I_h}\,S_v - \mu_{vb}S_v - u_2 S_v,\\
\frac{dI_v}{dt} &= m_2(1-u_1)\frac{m_2\beta_{\max}I_h}{S_h+I_h}S_v - \mu_{vb}I_v - u_2 I_v,\\
&S_h(0) = S_{h0}, I_h(0) = I_{h0}, S_v(0) = S_{v0}, I_v(0) = I_{v0},\\
&S_h(t_f), I_h(t_f), S_v(t_f), I_v(t_f) \text{ free and } 0 \le u_i \le u_{i_{\max}}, i = 1,2.
\end{aligned} \tag{3}$$

Pontryagin maximum principle is used to derive the differential equation system as optimality conditions which must be satisfied by optimal control variables. In general Hamilton function is defined by equation (3) and objective function (2) as follows:

$$
\begin{aligned}
H = {} & A_1 I_h + A_2 S_v + A_3 I_v + C_1 {u_1}^2 + C_2 {u_2}^2 \\
& + p_1 \left(\Lambda_h - (1-u_1) \frac{m_1 \beta_{\max} I_v}{S_h + I_h} S_h + \gamma_h I_h - \mu_h S_h \right) \\
& + p_2 \left((1-u_1) \frac{m_1 \beta_{\max} I_v}{S_h + I_h} S_h - (\mu_h + \gamma_h + \delta_h) I_h \right) \\
& + p_3 \left(\Lambda_v - (1-u_1) \frac{m_2 \beta_{\max} I_h}{S_h + I_h} S_v - \mu_{\mathrm{vb}} S_v - u_2 S_v \right) \\
& + p_4 \left(m_2 (1-u_1) \frac{m_2 \beta_{\max} I_h}{S_h + I_h} S_v - \mu_{\mathrm{vb}} I_v - u_2 I_v \right),
\end{aligned}
$$

with $p_1(t), p_2(t), p_3(t), p_4(t)$ are adjoin functions to be determined.

Control function u_1^* and u_2^* are determined based on the condition given by the Pontryagin maximum principle are:

$$
u_1^* = \frac{(p_2 - p_1)\beta_{\max} m_1 S_h I_v + (p_4 m_2 - p_3)\, \beta_{\max} m_2 S_v I_h}{2C_1 (I_h + S_h)},
$$

$$
u_2^* = \frac{p_3 S_v + p_4 I_v}{2C_2}.
$$

Use limited control function, we consider the following cases:

Case 1. $0 \le u_1 \le u_{1_{max}}$

$$
u_1^* = \begin{cases} 0 & ; \quad k < 0 \\ k & ; \quad 0 \le k \le u_{1_{max}} \\ u_{1_{max}} & ; \quad k > u_{1_{max}} \end{cases}.
$$

with

$$
k = \frac{(p_2 - p_1)\beta_{\max} m_1 S_h I_v + (p_4 m_2 - p_3)\, \beta_{\max} m_2 S_v I_h}{2C_1 (I_h + S_h)},
$$

or

$$
u_1^*(t) = \min\left\{ u_{1_{max}}, \max\left\{ 0, \frac{(p_2 - p_1)\beta_{\max} m_1 S_h I_v + (p_4 m_2 - p_3)\, \beta_{\max} m_2 S_v I_h}{2C_1 (I_h + S_h)} \right\} \right\}. \tag{4}
$$

Case 2. $0 \le u_2 \le u_{2_{max}}$

$$
u_2^* = \begin{cases} 0 & ; \quad l < 0 \\ l & ; \quad 0 \le l \le u_{2_{max}} \\ u_{2_{max}} & ; \quad l > u_{2_{max}} \end{cases}.
$$

with

$$
l = \frac{p_3 S_v + p_4 I_v}{2C_2},
$$

or

$$
u_2^*(t) = \min\left\{ u_{2_{max}}, \max\left\{ 0, \frac{p_3 S_v + p_4 I_v}{2C_2} \right\} \right\}. \tag{5}
$$

Adjoin functions are determined from $\dot{p}^*(t) = -\frac{\partial H}{\partial x}$ with $x \in \{S_h, I_h, S_v, I_v\}$, then we will get differential equations for $p_i(t)$:

$$\begin{aligned}
\dot{p_1} &= (p_4 m_2 - p_3)\left(\frac{m_2(1-u_1)\beta_{\max} S_v I_h}{(I_h+S_h)^2}\right) \\
&\quad + (p_1 - p_2)\left(\frac{m_1(1-u_1)\beta_{\max} S_h I_v}{I_h+S_h}\right)\left(1-\frac{S_h}{I_h+S_h}\right) + p_1\mu_h, \\
\dot{p_2} &= -A_1 + (p_4 m_2 - p_3)\left(\frac{m_2(1-u_1)\beta_{\max} S_v I_h}{I_h+S_h}\right)\left(\frac{I_h}{I_h+S_h} - 1\right) \\
&\quad + (p_2 - p_1)\left(\frac{m_1(1-u_1)\beta_{\max} S_h I_v}{(I_h+S_h)^2}\right) - p_1\gamma_h \\
&\quad + p_2(\gamma_h + \delta_h + \mu_h)\,, \\
\dot{p_3} &= -A_2 + (p_3 - p_4 m_2)\left(\frac{m_2(1-u_1)\beta_{\max} I_h}{I_h+S_h}\right) + p_3(u_2 + \mu_{\text{vb}}), \\
\dot{p_4} &= -A_3 + (p_1 - p_2)\left(\frac{m_1(1-u_1)\beta_{\max} S_h}{I_h+S_h}\right) + p_4(u_2 + \mu_{\text{vb}}).
\end{aligned} \tag{6}$$

By assuming $S_h(t_f), I_h(t_f), S_v(t_f)$ and $I_v(t_f)$ are free, then the following transversality requirement must hold:

$$p_1(t_f) = 0, p_2(t_f) = 0, p_3(t_f) = 0 \text{ and } p_4(t_f) = 0.$$

Finally, because f_0 and $f_i, \forall i$ are nonlinear and convex functions on (x, u) and $p_i(t)$ are assumed nonnegative $\forall t$, show that Pontryagin maximum principle is necessary as well as sufficient conditions for minimize $J(x)$.

4. Numerical result and discussion

This section will show the effectiveness of control. Scenario made is

- Without control ($u_1 = 0, u_2 = 0$),
- With control usage insecticed treated net ($u_1 \neq 0, u_2 = 0$),
- With control spraying ($u_1 = 0, u_2 \neq 0$), and
- With control usage insecticed treated net and spraying ($u_1 \neq 0, u_2 \neq 0$).

Parameter values used are given in table 2 as follows:

Table 2. Parameter values.

Symbol	Value
A_1	25
A_2	0
A_3	25
t_f	100
u_{1max}	1
u_{2max}	1
C_1	25
C_2	40

In figure 1, we can see number of susceptible individuals without control, with u_1 control (nets usage), with u_2 control (spraying), and both. Using the mosquito nets S_h increased almost the same as using nets and spraying at once. Spraying also can raise S_h, but the results are not optimal. Giving only u_1 control will result almost the same by giving control u_1 and u_2 at once. Giving spraying control can also reduce the number of susceptible individuals, but the results are not optimal.

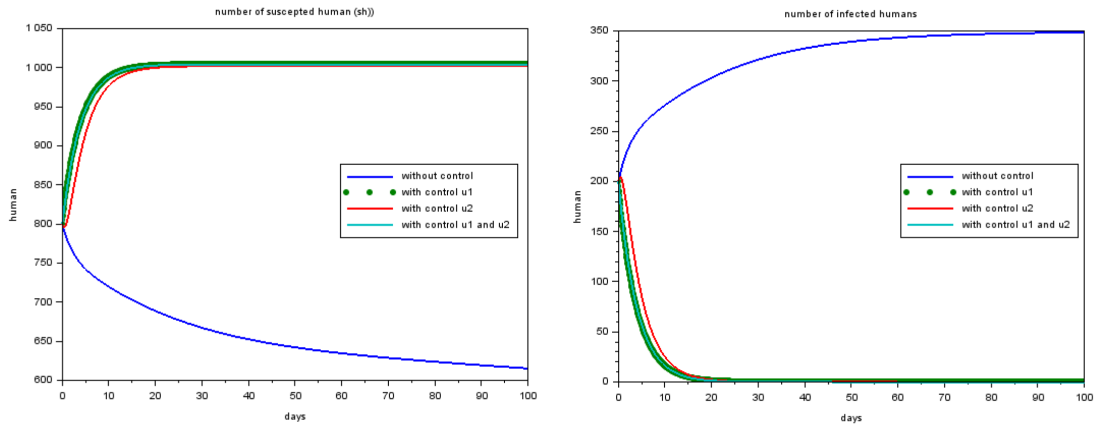

Figure 1. The number of susceptible and infected humans, with and without control.

From figure 2, it can be seen that the number of susceptible mosquitoes is most optimal with u_2 control than others. Conversely, giving u_1 control will increase the number of mosquitoes. The decline in the number of susceptible mosquitoes is most optimal when giving u_2 control at $t \in [0,10]$ because maximum spraying occurs in that time. The nets usage and spraying can reduce the number of infected mosquitoes (figure 2). The result will be more optimal when using nets and spraying at once rather than just with one of the two controls.

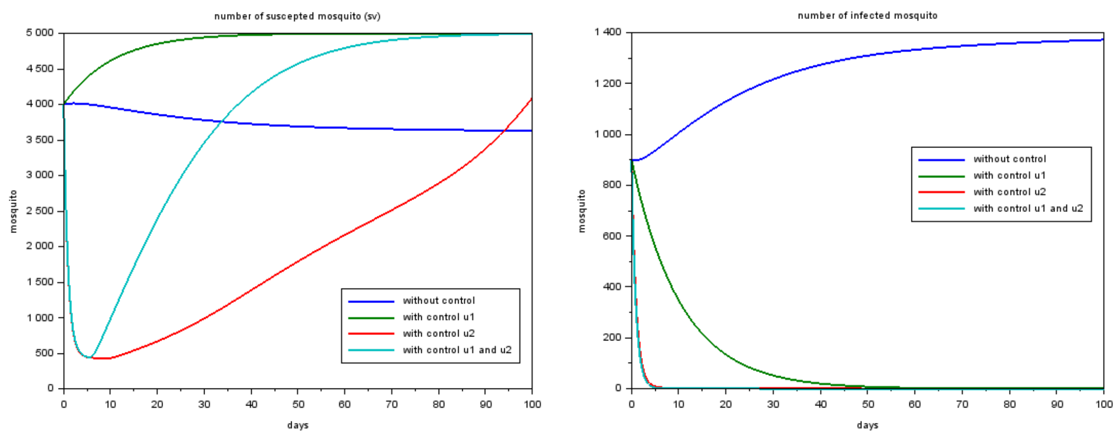

Figure 2. The number of susceptible and infected mosquitoes, with and without control.

Giving u_1 and u_2 control is illustrated in figure 3. For $t \in [0,100]$, $u_1 = 1$, means that nets are used every day. Maximum spraying is only at interval [0,7]. Furthermore, spraying can be reduced every day.

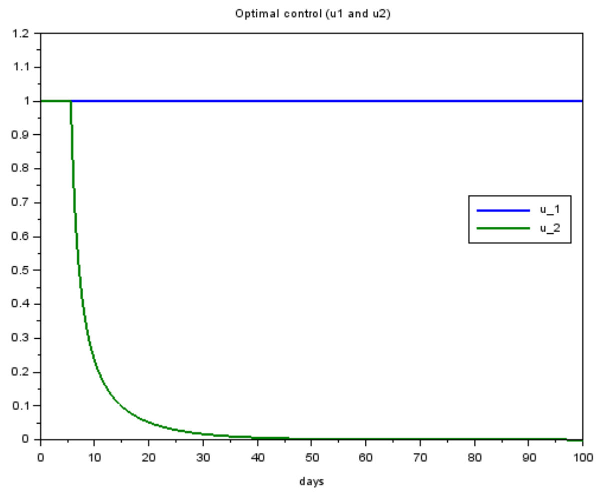

Figure 3. Control variables.

5. Conclusion

Giving u_1 control (nets usage) is very important in order to reduce the number of human beings infected. Giving u_2 control (spraying) to reduce the number of mosquitoes is also important. When giving the two controls at once, after 20 days, nobody is infected and the same will occur when only giving control 1 after 23 days. This means that only by giving control 1 is sufficient to reduce the number of infected individuals. Meanwhile the population of infected mosquitoes after being given controls at once is close to 0 after 8 days, if only giving control 2 it will take after 14 days, but if only giving control 1 it will take 74 days. It means giving control 2 is more optimal than control 1 to reduce the number of infected mosquitoes. In this case, using nets every day and spraying at first time is most optimal to reduce malaria cases.

References

[1] Arsin A A 2012 *Malaria di Indonesia – Tinjauan Aspek Epidemiologi* (Makasar: Masagena Press) p 37
[2] Centers for Disease Control and Prevention 2012 *How Can Malaria Cases and Deaths Be Reduced?*
[3] Hazarika G C and Bhattacharjee A 2011 *Proc. Indian Acad. Sci* **121(1)** 93–109
[4] Mandal S, Sarkar R R, and Sinha S 2011 *Malaria Journal* **10(202)** 1-19
[5] Silva C J and Torres D F M 2013 *Conference Papers in Mathematics* **2013** 1-8