PAPER • OPEN ACCESS

Experimental Verification of Steel Pipe Collapse under Vacuum Pressure Conditions

To cite this article: R Autrique and E Rodal 2016 IOP Conf. Ser.: Earth Environ. Sci. 49 052008

View the article online for updates and enhancements.

You may also like

- Influence of steel pipe on mechanical properties of airport cement concrete <u>pavement</u> Y Lai, D X Ma, P Wang et al.

- The application of a reflected nonaxisymmetric torsional guided wave model for imaging crack-like defects in smalldiameter pipes Zhou Fang, Peter W Tse and Fan Xu
- Finite Element Analysis of Influencing Factors of Micro Round Steel Pipe Pile Based on Pure Bending Orthogonal Test Xianyuan Tang, Shihai Wang and Bin Guo





DISCOVER how sustainability intersects with electrochemistry & solid state science research



This content was downloaded from IP address 3.19.56.114 on 26/04/2024 at 19:44

Experimental Verification of Steel Pipe Collapse under Vacuum **Pressure Conditions**

R Autrique¹, E Rodal²

¹ Policonductos, Cracovia 54, México City, México, rautrique@prodigy.net.mx

² Instituto de Ingeniería, UNAM, México City, México, erc@pumas.ii.unam.mx

Abstract

Steel pipes are used widely in hydroelectric systems and in pumping systems. Both systems are subject to hydraulic transient effects caused by changes in boundary conditions, such as sudden valve closures, pump failures, or accidents. Water column separation, and its associated vaporization pressure inside the pipe, can cause the collapse of thin walled steel pipes subject to atmospheric pressure, as happened during the well known Oigawa Power Plant accident in Japan, in 1950.

The conditions under which thin walled pipes subject to external pressure can collapse have been studied mathematically since the second half of the XIX century, with classical authors Southwell and Von Mises obtaining definitive equations for long and short pipes in the second decade of the XX century, in which the fundamental variables are the diameter to thickness ratio D/t and the length to diameter ratio L/D.

In this paper, the predicted critical D/t ratio for steel pipe collapse is verified experimentally, in a physical model able to reproduce hydraulic transients, generating vacuum pressures through rapid upstream valve closures.

1. Theoretical developments

The determination of the maximum external pressure that an infinitely long thin cylindrical shell could resist without collapsing was studied for the first time by Jacques Bresse in 1859, although Franz Grashof reached the same result in the same year, continuing the studies made by William Fairbairn with fixed end tubes in 1858. Saint-Venant pupil Maurice Lévy reobtained the expression for the critical external pressure in 1884 [1].

The expression obtained was, for an elementary ring,

$$P_{crit} = 2 E \left(\frac{t}{D} \right)^3 \tag{1}$$

which, considering the stresses acting between neighboring rings, transforms into

$$P_{crit} = \frac{2E}{(1 - \nu^2)} \left(\frac{t}{D} \right)^3$$
(2)

In the above equations, P_{crit} is the critical collapse pressure, D is the pipe diameter, t is the pipe thickness, E is the Young's modulus of elasticity, and ν is the Poisson's ratio. Equation (2) is the formula that we find in the engineering literature [1, 2, 3], and which gives, for steel tubes exposed outside to the atmospheric pressure and inside to vapor pressure, a critical D/t ratio of 166 for sea level and 179 for a height of 1900 m above sea level, which is the elevation of the city where the experimental installation is located. It becomes clear that for an infinitely long pipe, the D/t ratio is the decisive parameter regarding resistance against collapse. Being a dimensionless parameter, the collapse problem can be conveniently studied under laboratory conditions.



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Published under licence by IOP Publishing Ltd

Further contributions to this problem were made by R.V. Southwell in 1913 [5], and by Richard Von Mises in 1914 [6], introducing in the calculation the L/D ratio, for "short" pipes, and n, the number of circumferential lobes that would form at collapse. The equations developed assume that the tubes were simply supported, with ends maintaining its circumference and being free to bend.

The curves corresponding to four equations, two from Southwell, one from Von Mises, and one from Tokugawa, are shown in Figure 1, for L/D = 9.50 (see paragraph 5) and for n = 2, which is the typical number of lobes forming in pipes with L/D greater than 7.5 and D/t lower than 300, as can be calculated from equation (3), derived from [4]:

$$n = \frac{1.33}{(\alpha^{0.25}\beta^{0.5})} = \frac{1.67(D/t)^{0.25}}{(L/D)^{0.5}}$$
(3)

in which $\alpha = \left(\frac{t}{D}\right)$ and $\beta = \left(\frac{2L}{\pi D}\right)$

In Figure 1, the net external pressures, considering that the inside of the pipe is under vapor pressure, are shown for sea level (1.009 kg/cm^2) and for the elevation of 1900 m above sea level (0.804 kg/cm^2) , with the critical *D/t* ratios corresponding with the interceptions of those net external pressures and the equation curves. Pressures higher than the critical will cause collapse or buckling of the pipes, at stresses below the elastic limit.

The equations are the following:

Southwell (1913) [5]:

$$P_{crit} = 2 E \alpha \left(\frac{\alpha^2}{1 - \nu^2} + \frac{1}{48\beta^4} \right) \tag{4}$$

Von Mises (1914) [6]:

$$P_{crit} = 2 E \alpha \left\{ \frac{\alpha^2}{3(1-\nu^2)} \left(n^2 - 1 + \frac{2n^2 - 1 - \nu}{n^2 \beta^2 + 1} \right) \frac{1}{(n^2 - 1)(n^2 \beta^2 + 1)^2} \right\}$$
(5)

Tokugawa (1929) [6]:

$$P_{crit} = \frac{2 E \alpha}{n^2 - 1 + \frac{1}{2}\beta^{-2}} \left\{ \frac{\alpha^2}{3(1 - \nu^2)} \left[(n^2 + \beta^{-2})^2 - \frac{n^4(2n^2 - 1)}{(n^2 + \beta^{-2})^2} \right] + \frac{1}{(n^2\beta^2 + 1)^2} \right\}$$
(6)

Southwell (1915) [6] developed also a very simple expression:

$$P_{crit} = \frac{2.28 E}{(1-\nu^2)^{3/4}} \left(\frac{\alpha^{2.5}}{L/D}\right) = 2 E \alpha \left(\frac{0.726}{(1-\nu^2)^{3/4}}\right) \left(\frac{\alpha^{1.5}}{\beta}\right)$$
(7)

which gives values close to those of eqs (4), (5) and (6) for L/D between 9 and 12.

If eq (7) is made identical to eq (2), we obtain the L/D ratio for which the critical pressure will be equivalent to that of an infinitely long tube:

$$\left(\frac{L}{D}\right)_{crit} = 1.14 \ (1 - \nu^2)^{1/4} \left(\frac{D}{t}\right)^{1/2} \tag{8}$$



from which we can determine if a pipe will behave as an infinite pipe, or "long" pipe, or as a "short" pipe.

Figure 1. Critical pressure for a pipe of infinite length (eq 2), and for eqs 4, 5, 6 and 7, with L/D = 9.50. Ratio D/t of test pipes shown as vertical lines.

Initially investigated in relation with the design of boiler flues [1,7], the collapse of thin cylinders under external pressure is also studied by aeronautical, submarine, and hydroelectric and pumping systems designers. It belongs to the problems of "Elastic Stability"[1, 3], of which the typical example is the buckling of columns under axial compression [8], studied by Leonhard Euler in 1744 (*De Curvis Elasticis*, ("Elastic Curves")) [9], in which the maximum load, or critical load, that a structural element can resist before buckling or collapsing, is determined by its stiffness, that is, by its geometrical characteristics and its material properties, and not by its yield or allowable stresses. Euler's critical load, for a vertical column pinned at its ends, is

$$P_E = \frac{\pi^2 E I}{L^2} \tag{9}$$

Loads higher than the critical will cause collapse or buckling of the element, at stresses below the elastic limit.

In the hydroelectric field, the problem has been particularly studied for the cases of linings in pressure and underwater tunnels, where the pipes are surrounded by concrete. Well known are the articles on the subject by Amstutz, Borot, and Vaughan [10]. In these cases, the pressures applied during grouting are much higher than atmospheric, and the stresses in the pipe can approach the yield stress.

The Oigawa accident, mentioned above, which occurred in 1950, was documented by C.C. Bonin [11] in 1960, and made widely known to a large number of generations of students of hydraulic transients by M. Hanif Chaudhry [12] in 1978. Oigawa, on the Oi river, was a 72 MW hydroelectric plant, finished in 1936 [13], with three Francis units and three 250 m long penstocks, 2.75 m in diameter, located downstream of a surge tank. The accident, caused by a series of wrong manoeuvers and equipment malfunctioning, resulted in the rupture by overpressure of an 8 m long section in one of the penstocks, originating a negative pressure wave and inside vacuum conditions that collapsed a 53 m long (L/D = 19.3) section of the penstock. The thickness range in the collapsed section was between 8.9 and 14.0 mm, and the D/t ratio between 196 and 308. The pipes with thicknesses between 8.9 and 11.9 mm, and D/t ratios between 230 and 308, had stiffeners, the details of which are not known, but that were apparently too far apart to be effective, as can be seen in the photographs in [11] and [12]. In spite of the different thicknesses and stiffeners, the penstock behaved as a "long" pipe.

Being the D/t ratio decisive, the aim of this paper is to reproduce in the laboratory the vacuum conditions of the Oigawa accident and the resulting pipe collapse, in order to verify the critical D/t ratios for atmospheric external pressure, compare them with the theoretical formulas, and make the required design and safety recommendations, for all types of pipeline projects. Tank or pipe drainage situations and its

28th IAHR symposium on Hydraulic Machinery and Systems (IAHR2016)

IOP Conf. Series: Earth and Environmental Science **49** (2016) 052008

associated risks can show up suddenly in many installations, as many events resulting in tank collapse can show. In the case of pipelines, although water column separation is generally avoided in the design phase, other conditions, normally not part of the current manoeuvres and operating procedures, can cause vacuum pressures and risk of collapse, concluding that for a safe design the D/t ratios must be equally safe, being able to resist any unexpected or unfavorable condition.

2. Previous experiments

A very large number of experiments have been made collapsing thin cylinders by axial pressure. Experiments have also been made for external pressure (also called lateral pressure), but for low L/D ratios and generally for low D/t ratios [7]. Measurements have been made in lined tunnels, leading to hydroelectric plants, during grouting operations [10]. Experiments reproducing the actual conditions of sudden vacuum pressures in pipelines with flowing water have not been made, except for polyethylene pipes [14]. In this last reference, it was shown that eq (2) was valid for long polyethylene pipes. The aim of this paper is to reproduce, as mentioned above, the actual collapse conditions in steel pipelines.

3. Experimental installation

The experiments were conducted in the city of San Luis Potosí, in central Mexico, 1900 m above sea level, in a model consisting of a 300 m long steel pipe, 114.3 mm (4.5 in) in diameter, D/t ratio of 67, with two hydropneumatic tanks in the extremes, acting as constant head and as pressure wave reflecting bodies (Figure 3). The details of the model have been described elsewhere [15]. Pressure wave celerity is 1120 m/s. Pressures are measured with strain gauge transducers, and its evolution in time is recorded. The upstream programmable butterfly valve allows for rapid closures, giving rise to, under adequate initial pressures and water velocities, water column separation, cavity formation, and vacuum pressures. Steel pipe test sections, 1219 mm long, and with different D/t ratios, are installed as shown in the figures, preceded by a transparent PVC pipe, used to verify the presence of vapor cavities. A check valve is located downstream of the test pipes, to keep the vacuum pressures inside the pipe for the desired time. The net external pressure (atmospheric pressure minus water vapor pressure) at the elevation of the city of San Luis Potosí (1900 masl), is 0.804 kg/cm^2 . The experiments were filmed, to visualize the collapse.



Figure 2. Schematic representation of physical model, showing Configurations No. 1 and No. 2.

The general arrangement of the installation is shown in Figure 2, for the two test configurations. In Configuration No. 1, the test pipes are placed in line with the model pipes; the two ends of the pipe are fixed and simply supported. In Configuration No. 2, the test pipes are placed perpendicular to the model pipes, the test section forming an inverted "T"; in this case, one of the ends of the pipe is fixed and the other end is free: we can also call it a flagpole position. Maximum water hammer effects are obtained through the rapid closure of the upstream butterfly valve, and through the long pipeline of the model, which produces long vapor cavities and long vacuum periods. Two valve cycles were selected. In both cycles, the valve was closed during 10 seconds and opened during 12.5 seconds, to allow the recovery of the steady state flow conditions (initial pressure, 1.6 kg/cm²; flow, 7 l/s; water velocity, 0.62 m/s). In Cycle No. 1, the check valve downstream of the test pipe was allowed to close normally upon return of the flow, maintaining the vapor pressure inside the pipe while the valve was closed, that is, during 10 seconds. In Cycle No. 2, the check valve was blocked, and the duration of the vacuum periods was determined by the normal water hammer pressure variations. Cycles 1 and 2 are shown in Figures 3 and 4.

28th IAHR symposium on Hydraulic Machinery and Systems (IAHR2016)

IOP Conf. Series: Earth and Environmental Science 49 (2016) 052008





Figure 3. Valve cycle No. 1

Figure 4. Valve cycle No. 2

4. Experiments

The experiments were made with 114.3 mm (4.5 in) diameter pipes, made from thin gauge steel sheets (grade SAE 1008), that were roll formed, seam jointed (double edge folded), TIG welded, and hydrostatically tested, to ensure air tightness. Pipe thicknesses were 0.61, 0.45, and 0.38 mm (commercial U.S. calibers 24, 26 and 28), resulting in D/t ratios of 187, 254 and 301, respectively. Length of the test pipes was 1219 mm (48 in), and, given the difficulty of joining two test pipes together, due to their reduced thicknesses, single test pipes were TIG welded in their extremes to stub ends, and held tight to the model through bolted idle flanges, obtaining test pipes with built-in edges, with an "free" length of 1086 mm (42.75 in) and an L/D ratio of 9.50. The theoretical D/t critical ratios obtained from equation (2), for an infinitely long pipe, and from equations (4), (5), (6) and (7), are shown in Table 1.

Table 1.

	Atm	Net ext	D/t	D/t	D/t	D/t	D/t
(masl)	Pressure	Pressure	Eq (2)	Eq (4)	Eq (5)	Eq (6)	Eq (7)
	Kg/cm ²	Kg/cm ²	$L/D = \infty$	L/D = 9.5	L/D = 9.5	L/D = 9.5	L/D = 9.5
0	1.033	1.009	166	191	192	192	196
1900	0.827	0.804	179	211	211	212	215

From Table 1, an infinitely long pipe with D/t greater than 179 should collapse. Table 2 shows the critical L/D ratios, calculated with eq (8), showing that, for the three D/t ratios tested, the pipes are "short", as their L/D is 9.50. The critical D/t ratios for collapse are therefore greater than the theoretical ratio of 179, being now 212, if we consider the average of eqs (4) to (7). This means that pipes with D/t = 187 will not collapse, and that pipes with D/t = 254 and 301 should collapse.

Table 3 shows the results of the experiments. We can observe the following:

I. General observations

- 1. According to the assumptions made in the derivation of eqs (4) to (7), the pipe ends must be simply supported, free to bend, and should keep their circular shape. In Configuration 1, with test pipes in line, the pipes are bolted to the model and unable to bend.
- 2. In Configuration 2, with test pipes in inverted T, or flagpole position, the lower end is fixed and unable to bend, but the upper end is totally free. In this position, the test pipes bend at the top during collapse, as shown in Photos 5 and 8.
- 3. The pipes that collapse recover their shape when pressurized, as their deformations remain in the elastic range. They can stand several cycles of collapse and recovery, until the wrinkles, that develop in the weak zones, generally in the middle of the pipes, fail and produce small holes through which small water jets leak out (Photo 10).



IOP Publishing

28th IAHR symposium on Hydraulic Machinery and Systems (IAHR2016)



- 4. The longitudinal welds did not prevent or stop the collapse, as some specimens collapsed with the weld along the concave surface, contrary to what could be expected, that is, that the "strong line" of the weld would remain in the convex surfaces (Photo 9).
- 5. The "out of roundness" effect, reducing the critical pressure, was verified in Tests 5 and 9, although with a high initial deformation. This fact agrees with previous theory and observations related with the reduction of critical pressures caused by "out of roundness".
- 6. In the cases in which the pipes resist vacuum and do not collapse, no fatigue effects are to be expected considering the actual low stresses during the vacuum periods. Being well below the elastic limit, we can assume that the number of cycles they resist is in excess of $(10)^6$ [14, 16].

28th IAHR symposium on Hydraulic Machinery and Systems (IAHR2016)

 IOP Conf. Series: Earth and Environmental Science 49 (2016) 052008
 doi:10.1088/1755-1315/49/5/052008

7. All experiments were repeated at least twice, with good repeatability and no scatter effects.

5. Results

Table 3. Data and results of experiments.

No.	Test pipe	t mm	D/t	Config.	Cycle	Initial deformation	Result	Observations
1	24-1	0.61	187	1: in line	1			200 cycles
2	24-1	0.61	187	2: flag pole	1			25 cycles
3	24-1	0.61	187	2: flag pole	1	10%		25 cycles
4	24-1	0.61	187	2: flag pole	1	20%		50 cycles
5	24-1	0.61	187	2: flag pole	1	30%	Collapse	
6	24-2	0.61	187	2: flag pole	1			20 cycles
7	24-2	0.61	187	2: flag pole	1	10%		20 cycles
8	24-2	0.61	187	2: flag pole	1	20%		20 cycles
9	24-2	0.61	187	2: flag pole	1	30%	Collapse	
10	26-2	0.45	254	1: in line	1			100 cycles 0° 100 cycles 90°
11	26-4	0.45	254	2: flag pole	1		Collapse	
12a	26-1	0.45	254	2: flag pole	1		Collapse	
12b	26-1	0.45	254	2: flag pole	2		Collapse	
13	26-3	0.45	254	1: in line	2			2000 cycles
14	28-2	0.38	301	1: in line	2		Collapse	
15	28-1	0.38	301	1: in line	2		Collapse	

II. Pipes with t = 0.61 mm and D/t = 187

- 1. These pipes did not collapse, when placed in line, with the two ends fixed (Test 1). The first test tube resisted 200 cycles of vacuum.
- 2. They did not collapse either when placed in the inverted T position, with one free end (Tests 2 and 6).
- 3. The collapse was achieved, however, in two different experiments, in the inverted T or flagpole position, when the pipes were subject to an initial deformation of 30 % of their initial diameter, deformation which was made and measured at the middle of the pipe (Tests 5 and 9). Smaller deformations of 10 % (Tests 3 and 7) and 20 % (Tests 4 and 8), resisted several cycles of inside vacuum pressures, without collapsing. In one of the specimens (Test 9), the collapsed shape was "freezed" during a vacuum cycle (Photo 4), as the pipes, being in their elastic range, recovered their initial shape when pressurized again.



Photo 3



Photo 4



Photo 5

- III. Pipes with t = 0.45 mm and D/t = 254
- 1. These pipes did not collapse when placed in line, that is, with their ends fixed. The first specimen supported 200 cycles of vacuum (Test 10), with 100 cycles with the longitudinal weld looking down and 100 cycles with the weld rotated 90 degrees clockwise. The rotation was made to verify any possible influence of the position of the longitudinal weld.
- 2. The pipes of this thickness did collapse, when placed in inverted T or flagpole position, being totally free at the upper end. A first specimen collapsed under type 1 vacuum cycle (Test 11), and a second specimen collapsed under cycle 1, supporting 8 cycles, and then again under cycle 2, supporting 3 cycles before starting to leak (Tests 12a and 12b). In this case, and in general, the experiments were stopped when the pipes started to leak, that is, when the wrinkles created by the successive cycles of collapse and circular section recovery had fatigued and perforated the pipes (Photos 8 and 10).
- 3. A third specimen in this thickness (0.45 mm) was installed in line in order to verify how many cycles of vacuum it could resist. It supported 2000 cycles of vacuum of type 2 (Test 13).
- *IV.* Pipes with t = 0.38 mm and D/t = 301
- 1. These pipes collapsed when placed in line. Two specimens collapsed (Tests 14 and 15), as shown in Table 2 and in Photos 6 and 7.





We can try to explain what was observed as follows:

- 1. Test pipes with D/t = 187 did not collapse, as expected, and pipes with D/t = 301 collapsed in line, as expected.
- 2. However, test pipes with D/t = 254 did not collapse in line, but collapsed in the inverted T or flagpole position. From Figures 1 and 2, they should have easily collapsed, being the calculated collapse pressure 0.55 kg/cm² and the external pressure 0.80 kg/cm².
- 3. To explain the above, we can reason as follows: the test pipes in line have fixed ends and are unable to bend. Their ends are really built-in ends, as they are welded to stub ends, and embedded 67 mm in a strong circular surface. On the other hand, the test pipes in inverted T, or flagpole position, have a built-in lower end but an upper end totally free.
- 4. We can make now an analogy with the Euler column under compression. The Euler critical load of eq (9) assumes pin-ended columns, free to bend. If the If the end conditions change, a new length, called the effective length *Le*, is used for the calculation of the critical buckling load. When both ends are

IOP Conf. Series: Earth and Environmental Science 49 (2016) 052008

doi:10.1088/1755-1315/49/5/052008

built-in, or embedded, the effective column length is 0.5 L. If the lower end is embedded, unable to bend, and the upper end is free, the effective column length is 2L [8].

- 5. From the above analogy, the test pipes in line should have an effective length smaller than their free length, and the test pipes in flagpole position should have an effective length larger than their free length. We must therefore look for effective lengths and L/D values and curves that can explain the experiments.
- 6. The proposed solution and curves are shown in Figure 6.
 - 6.1. For L/D = 7.75, for the test pipes in line, we have a critical pressure above the atmospheric for D/t = 254 (no collapse), and below the atmospheric for D/t = 301 (collapse).
 - 6.2. For L/D = 11.50, for the pipes in flagpole position, we have a critical pressure above the atmospheric for D/t = 187 (no collapse), and below the atmospheric for D/t = 254 (collapse).
 - 6.3. The resulting effective lengths proposed for the two configurations are: Configuration 1, test pipes in line, Le/D = 7.75, and Le = (7.75/9.50) L = 0.82 L. Configuration 2, test pipes in flagpole position, Le/D = 11.50, and Le = (11.50/9.50) L = 1.21 L.
 - 6.4. As in the case of the columns under compression, the effective length coefficients for the cases analyzed (0.5 and 2 for the columns, and 0.82 and 1.21 for the collapsing pipes) are each the inverse of the other.

In Figure 6, the red dots correspond to test pipes that collapsed, and the green dots correspond to test pipes that did not collapse.

6. Conclusions

- 1. Experimental tests reproducing the actual conditions of vacuum pressure in steel pipelines exposed to atmospheric pressure were developed, for D/t ratios between 187 and 301.
- 2. In particular, the experiments could reproduce the conditions of the Oigawa plant accident of 1950 in Japan, an event very well known by the hydraulic transients community.
- 3. The experimental results were compared with the existing theoretical equations, showing good agreement, with adjustments made to consider the particular end conditions of the test pipes.
- 4. The theoretical equations, very close to the experimental results, do not contain any safety factor, which should be considered for a safe design. In particular, "out of roundness" effects, which by the nature of the fabrication and erection procedures of pipes and pipelines will be always present, should be considered.
- 5. In steel pipelines conveying water, although column separation is generally avoided in the design phase, other conditions, normally not part of the current manoeuvres and operating procedures, can cause vacuum pressures and risk of collapse, concluding that for a safe design the D/t ratios must be equally safe, being able to resist any unexpected or unfavorable condition. Typical velocities in steel pipelines, ranging from medium in aqueducts to high in penstocks, can produce high to very high Joukowsky heads and severe accidental transients which must be considered as a possibility at the design phase.
- 6. An analogy was proposed between the collapse of thin pipes and the buckling of columns under compression, which was useful to explain the results of the experiments.
- 7. New experiments are necessary, with longer tubes and a wider range in the L/D ratio, to confirm or reconsider the results of the present experiments.
- 8. Although most unstiffened pipes in hydroelectric and pumping installations will be typically "long", reducing the importance of its edges or boundaries, this condition must be verified.
- 9. The paper is expected to contribute to the general knowledge of the collapse of exposed steel pipes under internal vaporization pressures, and to provide useful information and guidelines for design. Complete records of the experiments are available.

7. References

- [1] Timoshenko S 1983. *History of Strength of Materials*. Dover, New York.
- [2] Young W and Budynas R 2002. *Roark's formulas for stress and strain*. 7th Ed, McGraw Hill, New York.
- [3] Timoshenko S and Gere J 1961. *Theory of Elastic Stability*. 2nd Ed, McGraw Hill, New York.

IOP Conf. Series: Earth and Environmental Science **49** (2016) 052008 doi:10.1088/1755-1315/49/5/052008

- [4] Wang J H 2009 *Buckling behavior and design of steel liner in deep water tunnel under external hydrostatic pressure.* PhD dissertation. Waseda University, Japan.
- [5] Southwell R V 1913. "On the general theory of elastic stability". *Philosophical Transactions, Royal Society,* London, 213A, Aug, pp 187-244.
- [6] Windenburg D and Trilling C 1934. "Collapse by Instability of Thin Cylindrical Shells Under External Pressure". *Transactions ASME*, 56, pp 819-825.
- [7] Singer J, Arbocz J and Weller T 2002. Buckling experiments. *Experimental methods in buckling of thin walled structures. Vol 2. Shells, built-up structures, composites.* Wiley, New York.
- [8] Shanley F R 1967. *Mechanics of Materials*. McGraw Hill, New York.
- [9] Oldfather W A, Ellis C A and Brown D M 1933. Translation of Leonhard Euler *De Curvis Elasticis* ("Elastic Curves"), Princeton University, *Isis* Vol 20 No 1 pp 72-160, published in line by The University of Chicago Press
- [10] Mc Caig, I W and Folberth, P 1962. "The buckling resistance of steel liners for circular pressure tunnels". *Water Power*, July, pp 272-278, and "Correspondence", Jan 1963, pp 5-7.
- [11] Bonin C C 1960. "Water-Hammer Damage to Oigawa Power Station". Journal of Engineering for Power. April, pp 111-119
- [12] Chaudhry M Hanif 1979. Applied Hydraulic Transients. Van Nostrand Reinhold, New York.
- [13] Japan Commission on Large Dams 2009. *Dams in Japan. Past, present and future.* CRC Press/Balkema, Netherlands; Taylor and Francis, London, p 51 and 215.
- [14] Autrique R and Rodal E 2012. "Fatiga en tuberías de polietileno sometidas a presiones extremas producidas por golpe de ariete", *XXV Congreso Latinoamericano de Hidráulica, IAHR*, San José, Costa Rica (In Spanish).
- [15] Autrique R, Rodal E, Carmona L and Sánchez H A 2012. "Physical model studies of water column separation" XXVI IAHR Symposium on Hydraulic Machinery and Systems, Beijing, China.
- [16] Autrique R and Rodal E 2014. "Estudio en laboratorio de fatiga en tuberías de polietileno sometidas a presiones transitorias extremas". *XXVI Congreso Latinoamericano de Hidráulica, IAHR*, Santiago, Chile (In Spanish).