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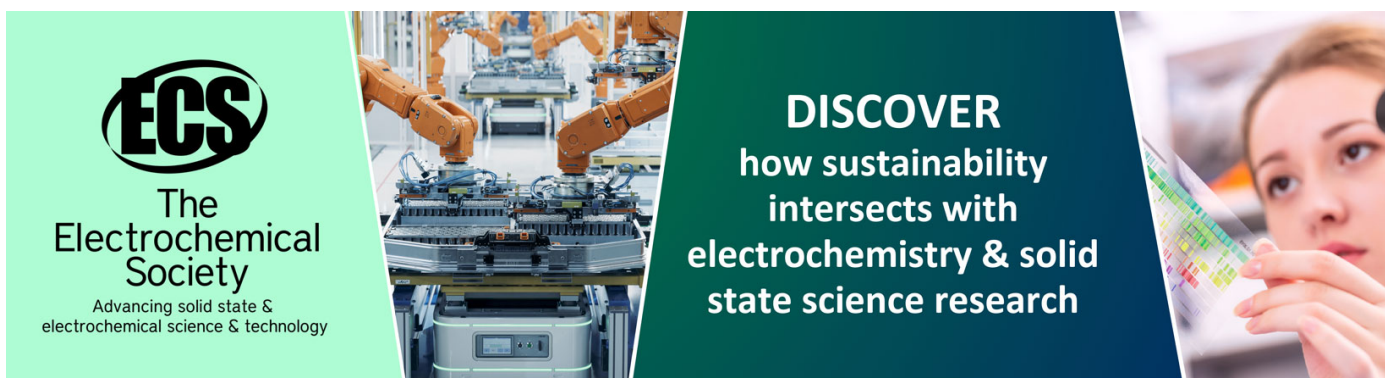
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Optimization and Investigation of a Free-Piston Stirling Engine based on Power and Frequency using Genetic Algorithm

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Abstract. This paper presents optimum design and performance investigation of a Free-Piston Stirling Engine (FPSE) by using genetic algorithm (GA). Mechanical and thermal analyses were considered in the optimization. Power and frequency were chosen primarily as the output performance. The mathematical model of the FPSE was provided from equation of motion in state space. The required engine operation frequency was set. The root of equation of motion is function of five unknowns including piston mass, displacer mass, stiffness of piston, stiffness of displacer and damping due to load. The engine operation with desired frequency occurs when two roots of equation of motion are imaginary part, which indicates operation frequency of the engine and other two roots are negative real part. The difference of operation frequency from equation of motion and desired frequency was set as the first objective function. The damping due to load which represents the work output of the engine was set as second objective function. Both objective functions were solved by GA.

1. Introduction

Nowadays, there is more demand for energy as the economy grows. The fossil fuel, the main source of energy is limited; fuel price and the cost of living are high. Furthermore, the use of fossil fuels is a major cause of global warming due to increased greenhouse gas emissions. This is the reason why there is great interest in alternative energy. Researchers are working to reduce the amount of greenhouse gases, and invent technology for use with clean energy sources.

Solar energy is an alternative energy that has the potential to generate electricity and it is environment friendly. It is a clean energy that reduces fossil fuel use. It is abundant in Thailand which is located in the latitudes 30°N and 30°S because the area is more exposed to the sun's rays than in other parts of the world. Throughout the year, exposure has an average of 5 kilowatt hours per square meter of sunlight per day [1]. Therefore, it is appropriate to build solar power plants as well as to promote the research and development of solar energy.

The Stirling engine is also called the hot gas engine. It works on the principle of compression and expansion of the gases with external heat source, so they can operate with a variety of fuels. They work with not only fossil fuel but also all types of heat source such as solar, geothermal, agriculture, incinerators, etc. In addition, the Stirling engine is an engine with only a few essential parts as shown in figure 1. It has high thermal efficiency; thus, this is powerful tool to use with clean energy sources.



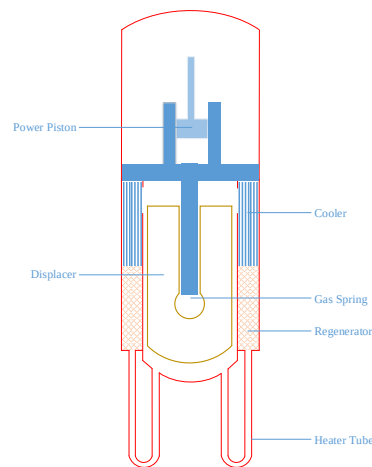


Figure 1. Schematic diagram of FPSE.

Free Piston Stirling Engine was invented by W. Beale [2]. FPSE's have no mechanism between moving parts. Instead, the elements move due to different pressures of working gas or other spring forces. The advantages of FPSE are quiet operation, no wear, no maintenance and long life. The disadvantage is the difficulty to stabilize the movements of piston and displacers [3]. Optimizations of the dynamic behavior of FPSEs are not a simple matter because of the lack of mechanical linkages that are able to fix strokes (displacer and piston) and phase angle for the moving elements (displacer and piston) [4].

Analysis of FPSE performance can be achieved by solving the equation of motion of the piston and the displacer. In 1985, R. W. Redlich and D. M. Berchowitz [5] presented the linear dynamics of FPSEs. They concluded that existence of stability criterion which relates to mechanical dynamics and thermodynamics and a minimum hot end temperature for which oscillation may be expected and frequency is generally load dependent. F. De Monte et al [6] presented the dynamic behavior prediction of FPSEs with hysteresis and viscous loss in heat exchanger. Rogdakis E.D. et al, [7] presented the thermodynamic investigation for the optimization of stable operation of FPSE. The equations of motion were solved analytically in terms of mass of the working gas and the displacer–piston phase angle of the machine. Using the criterion of stable engine cyclic steady operation, a mathematically rigorous form was obtained for the main parameters of the engine. Boucher J. et al [4] presented the dynamic behavior of a dual free-piston Stirling engine (DFPSE) coupled with an asynchronous linear alternator. They evaluated the thermo-mechanical conditions for stable operation of the engine. The DFPSE produces a mechanical power of 1 kW and it has a design operating point of 1.4 MPa corresponding to the frequency of about 22 Hz. Helium was used as the working fluid. New optimization technique called genetic algorithm (GA) was used to develop the Stirling engine system. Mohammad H. Ahmadi. et al [8] presented multi-objective thermodynamic-based optimization of output power of Solar Dish-Stirling engine by implementing an evolutionary algorithm. They used GA to optimize multi-objective function of the efficiency of solar dish-Stirling system. Sh. Zare, A.R. Tavakolpour-Saleh. et al [9] presented Frequency-based design of a free piston Stirling engine using genetic algorithm. They developed the equation of motion of FPSE which included five unknowns then solved the equation of motion by GA.

2. FPSE Analysis

2.1. Dynamic analysis

Based on the governing equation of motion of FPSE, Newton's law of motion was used to set the equation of motion and thermodynamics was used to consider the gas spring force

2.1.1. Equation of motion. From Newton's second law of motion and free-body diagram in figure 2, the dynamic equation of piston and displacer can be written as equation (1) and (2).

The equation of motion of the piston

$$M_p \ddot{x}_p = A_p(P_c - P_b) - (C_{load} - C_{Hp}) \dot{x}_p \quad (1)$$

The equation of motion of the displacer

$$M_d \ddot{x}_d = A_d(P_e - P_c) + A_R(P_c - P_d) - (C_{Hd}) \dot{x}_d \quad (2)$$

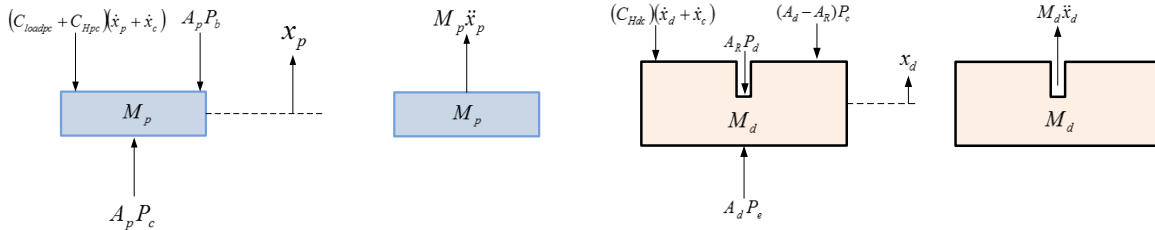


Figure 2. FBD of piston and displacer.

The variation in compression and expansion space volume are functions of position of piston and displacer as in equation (3) and (4), respectively.

$$V_c = A_p x_p + (A_d - A_r)(S_d - x_d) \quad (3)$$

$$V_e = A_d x_d \quad (4)$$

$$p = \frac{MR}{S} \left[1 + \frac{A_p x_p - (A_d - A_{dr}) x_d}{T_k S} + \frac{A_d x_d}{T_h S} \right]^{-1} \quad (5)$$

Where

$$S = \left(\frac{(A_d - A_{dr}) S_d}{T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h / T_k)}{T_h - T_k} + \frac{V_h}{T_h} \right)^{-1} \quad (6)$$

For active gas spring in displacer and bounce space, the variation of pressure in these spaces can be written as follows:

$$P_b = P_{mean} \left[\frac{V_B}{V_B - A_p x_p} \right]^\gamma \quad (7)$$

$$P_d = P_{mean} \left[\frac{V_D}{V_d - A_R x_d} \right]^\gamma \quad (8)$$

The pressure in equation (5), (7) and (8) are non-linear. In order to linearize these pressures, binomial expansion was used. We get:

$$p = P_{mean} \left(1 - \frac{A_p x_p - (A_d - A_{dr}) x_d}{T_k S} - \frac{A_d x_d}{T_h S} \right) \quad (9)$$

$$P_b = P_{mean} \left(1 + \gamma \frac{A_p}{V_B} x_p \right) \quad (10)$$

$$P_d = P_{mean} \left(1 + \gamma \frac{A_R}{V_D} x_d \right) \quad (11)$$

In order to consider Term $P_c - P_b$, $P_e - P_d$, binomial expansion was used,

$$P_c - P_b = -P_{mean} \left[\frac{A_p x_p - (A_d - A_{dr}) x_d}{T_k S} + \frac{A_d (x_d - x_c)}{T_h S} + \gamma \frac{A_p}{V_b} (x_p + x_c) \right] \quad (12)$$

$$P_c - P_d = -P_{mean} \left[\frac{A_p x_p - (A_d - A_{dr}) x_d}{T_k S} + \frac{A_d (x_d - x_c)}{T_h S} + \gamma \frac{A_r}{V_d} (x_d + x_c) \right] \quad (13)$$

2.1.2. Pressure drop in heat exchanger. The pressure drop in heat exchanger strongly influenced the dynamic behaviour of the FPSE which consists of two reciprocating parts. The pressure drop represents the dissipation of energy (damping) in the engine. The pressure can be written as follows [10]:

$$A_D \Delta p = C_p \dot{x}_p + C_d \dot{x}_d \quad (14)$$

Where

$$C_p = A_p A_d P_c \quad (15)$$

$$C_d = -(2A_d - A_{dr}) A_d P_c \quad (16)$$

$$P_c = \frac{4}{3\pi} \left[\frac{\rho_k U_k (f_k + k_k)}{A_k} + \frac{\rho_h U_h (f_h + k_h)}{A_h} \right] + \frac{f_r}{A_r} \quad (17)$$

2.2. State space

The state space of equation motion was used to represent the equation of motion, given by

$$\begin{bmatrix} \ddot{x}_p \\ \ddot{x}_d \end{bmatrix} = \begin{bmatrix} -\frac{P_{mean} A_p^2}{M_p} \left(\frac{1}{T_k S} + \frac{\gamma}{V_b} \right) & -\frac{P_{mean} A_p}{M_p S} \left(\frac{A_d}{T_h} - \frac{A_d - A_{dr}}{T_k} \right) \\ -\frac{P_{mean} A_r A_p}{T_k S M_d} & -\frac{P_{mean} A_{dr}}{M_d} \left(\frac{A_d}{T_h S} - \frac{A_d - A_{dr}}{T_k S} + \frac{\gamma A_r}{V_d} \right) \end{bmatrix} \begin{bmatrix} x_p \\ x_d \end{bmatrix} + \begin{bmatrix} -\left(\frac{C_{load} + C_{Hp}}{M_p} \right) & 0 \\ \frac{C_p}{M_d} & \frac{C_d - C_{Hd}}{M_d} \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{x}_d \end{bmatrix} \quad (18)$$

Compared with the form below, where S_{ij} and D_{ij} are stiffness and damping per unit mass, respectively. The suffixes i and j indicate the influence on i -component due to the movement of j -component.

$$\begin{bmatrix} \ddot{x}_p \\ \ddot{x}_d \end{bmatrix} = \begin{bmatrix} S_{pp} & S_{pd} \\ S_{dp} & S_{dd} \end{bmatrix} \begin{bmatrix} x_p \\ x_d \end{bmatrix} + \begin{bmatrix} D_{pp} & D_{pd} \\ D_{dp} & D_{dd} \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{x}_d \end{bmatrix} \quad (19)$$

We have

$$S_{pp} = -\frac{P_{mean} A_p^2}{M_p} \left(\frac{1}{T_k S} + \frac{\gamma}{V_b} \right) \quad (20)$$

$$S_{pd} = -\frac{P_{mean} A_p}{M_p S} P_{mean} \left(\frac{A_d}{T_h} - \frac{A_d - A_{dr}}{T_k} \right) \quad (21)$$

$$S_{dp} = -\frac{P_{mean} A_p A_{dr}}{T_k S M_d} \quad (22)$$

$$S_{dd} = -\frac{P_{mean} A_r}{M_d} \left(\frac{A_d}{T_h S} - \frac{A_d - A_{dr}}{T_k S} + \frac{\gamma A_{dr}}{V_b} \right) \quad (23)$$

$$D_{pp} = -\frac{C_{load} + C_{Hp}}{M_p} \quad (24)$$

$$D_{pd} = 0 \quad (25)$$

$$D_{dp} = \frac{C_p}{M_d} \quad (26)$$

$$D_{dd} = \frac{C_d + C_{Hd}}{M_d} \quad (27)$$

2.3. Vibration analysis

The state space in equation (19) can be rearranged as follows:

$$\begin{bmatrix} \ddot{x}_p \\ \ddot{x}_d \\ \dot{x}_p \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} D_{pp} & D_{pd} & S_{pp} & S_{pd} \\ D_{dp} & D_{dd} & S_{dp} & S_{dd} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{x}_d \\ x_p \\ x_d \end{bmatrix} \quad (28)$$

Which is in form of

$$\dot{X} = AX \quad (29)$$

The solution of equation (29) is based on the root of the following polynomial:

$$r^4 + a_1 r^3 + a_2 r^2 + a_3 r + a_4 = 0 \quad (30)$$

Where a_1 , a_2 , a_3 and a_4 are eigenvalue of A as follows:

$$a_1 = D_{pp} + D_{dd} \quad (31)$$

$$a_2 = D_{pp}D_{dd} - D_{pd}D_{dp} - S_{pp} - S_{dd} \quad (32)$$

$$a_3 = D_{pd}S_{dp} + D_{dp}S_{pd} - D_{pp}S_{dd} - D_{dd}S_{pp} \quad (33)$$

$$a_4 = S_{pp}S_{dd} - S_{pd}S_{dp} \quad (34)$$

The general solution of equation (31) is:

$$X = c_1 e^{(r_1 t)} n_1 + c_2 e^{(r_2 t)} n_2 + c_3 e^{(r_3 t)} n_3 + c_4 e^{(r_4 t)} n_4 \quad (35)$$

Where c_1 , c_2 , c_3 and c_4 are initial conditions, n_1 , n_2 , n_3 and n_4 are eigenvector of A. Furthermore, we can find the engine operation frequency from the following equations [7]:

$$\omega_1 = \sqrt{\frac{(S_{pd}D_{dp} + S_{dp}D_{pd} - S_{pp}D_{dd} - S_{dd}D_{pp})}{(D_{pp} + D_{dd})}} \quad (36)$$

And

$$\omega_2 = \sqrt{\frac{a_2 - \sqrt{a_2^2 - 4a_4}}{2}} \quad (37)$$

The spring and damping coefficients are functions of known and unknown variables. In order to stabilize the existing engine, the geometry and working condition of the engine are presented in table 1.

Table 1. Known variables.

Variable	Value	Variable	Value
A_p	$5.8 \times 10^{-4} \text{ m}^2$	T_h	550 K
A_d	$1.7 \times 10^{-3} \text{ m}^2$	T_c	323 K
A_{dr}	$1.8 \times 10^{-5} \text{ m}^2$	V_{h0}	$2.33 \times 10^{-5} \text{ m}^3$
P_0	101325 Pa	V_{c0}	$4.58 \times 10^{-5} \text{ m}^3$
R	287 N-m/kg-K	ω	10 Hz

Note that D_{pp} are load coefficients. Since they appear in the frequency equation, it can be seen that free-piston Stirling engines have load-dependent frequency [5].

2.4. Stability conditions

For stability operation of FPSE, the operation frequency from equation (36) and (37) must be equal. In addition, the root of equation (30) needs to follow this condition: two imaginary roots and two roots with negative real part. The two roots with negative real part should be far enough from imaginary axis to quickly meet the stability operation.

3. Genetic Algorithm(GA)

The GA is a new optimization method based on natural selection. The GA repeatedly modifies a population of individual solutions. At each step, the GA selects current population randomly to be parents and uses them to produce the next generation. In each generation, the population get toward to optimal solution. The GA uses three main types of rules at each step to generate the next population from current population. These are selection, crossover, and mutation rules.

3.1. Objective function

From the stability analysis in Section 2.3, we need to find the unknown variable which satisfies $\omega_1 = \omega_2 = \omega$. In this research, the desire operation frequency was set at 10 Hz so that the first objective function was set as equation (38). Another condition for stable operation is the roots of equation (30) are two imaginary roots and two roots with negative real part. Thus, the objective function was set as equation (39).

$$\text{Objective 1} = |\omega - \omega_1| + |\omega - \omega_2| \quad (38)$$

$$\text{Objective 2} = |\text{real}(r)| \quad (39)$$

In order to get the maximum power from the engine, the damping due to load (Cload) in equation (24) need to be maximized, so the third objective function is defined as:

$$\text{Objective 3} = -C_{load} \quad (40)$$

3.2. GA operation

The GA toolbox in MatLab was used to minimize the multi-objective function as in equation (38), (39) and (40). Genetic algorithm options for optimization problem are listed in table 2.

Table 2. GA options for optimization.

Option in GA	Value
Population type	Double vector
Population size	500
Selection process	Tournament
Tournament size	2
Mutation	Constraint dependent
Maximum number of generations	500

4. Results

4.1. Optimize unknown variable

The mathematical model of FPSE was constructed using thermodynamic and dynamic motion theories. The equation of motion consists of spring and damping coefficients which are the functions of five unknowns. The GA toolbox in MatLab was used to optimize those unknowns to get stability and maximum power of the FPSE. The optimum unknown variables are shown in Table 3.

Table 3. Optimum unknown variables.

Variable	Optimum value
M_p	0.368 kg
M_d	0.206 kg
k_p	1328 N/m
k_d	687 N/m
C_{load}	20.06 N-s/m

4.2. Stability verification

The stable operation result from GA was verified by equation (35). The displacement and velocity of piston and displacer were represented in the form of a phase diagram in figures 3 and 4, respectively. Both phase diagrams show that the displacement and velocity of piston and displacer are repeated in each cycle, which indicates the stable operation of the engine. In addition, the displacement of piston and displacer with time at stable operation frequency of 10 Hz is shown in figure 5.

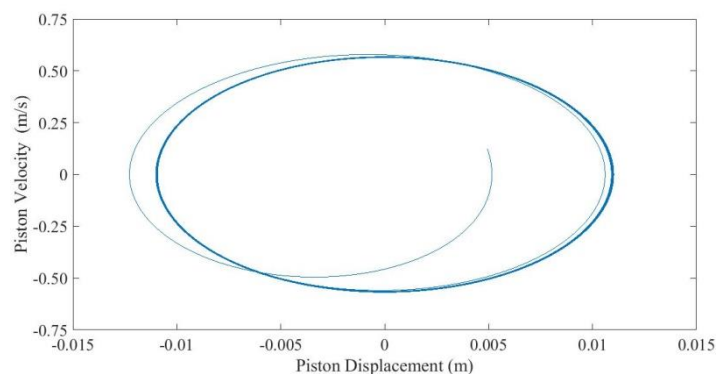


Figure 3. Phase diagram of the power piston.

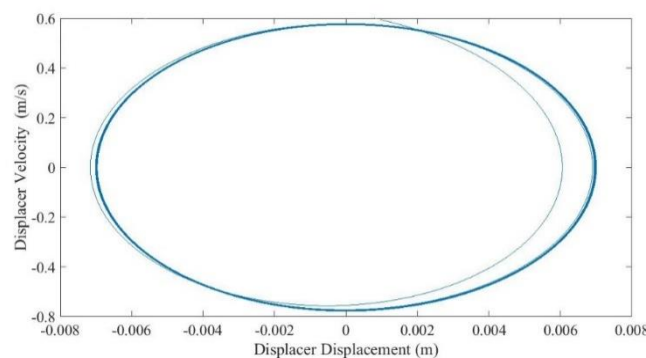


Figure 4. Phase diagram of the power displacer.

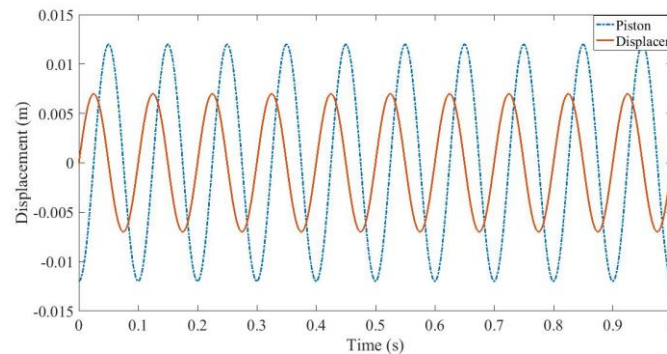


Figure 5. Displacement of piston and displacer.

5. Conclusion

The mathematical model of the FPSE was provided from equation of motion in state space with desired operation frequency of 10 Hz. The genetic algorithm was used to optimize the five unknowns to achieve stable operation and highest maximum output from the engine. The result from GA shows the optimum piston and displacer mass of 0.368 kg and 0.206 kg, respectively. The optimum piston and displacer stiffness are 1328 N/m and 687 N/m, respectively. Lastly, the maximum damping due to load is 20.06 N-s/m. These results were verified by the phase diagram, which indicates that the result from GA can achieve the stable and desired frequency operation of FPSE.

Acknowledgements

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Nomenclature

A	The cross-section area (m ²)	Geek symbol	
C	The damping coefficient (N-m/s)	γ	Polytropic index
f	The friction factor	ρ	Gas density (kg/m ³)
k	Head loss coefficient	ω	Frequency (rad/s)
M	Mass (kg)		
P	Pressure (Pa)	Suffix	
R	Gas constant (N-m/kg-K)	B, b	Bounce space
T	Temperature (K)	c	Compression space
U	Gas velocity (m/s)	D, d	Displacer
V	Volume (m ³)	dr	Displacer rod
$\ddot{x}$	Acceleration (m/s ²)	e	Expansion space
$\dot{x}$	Velocity (m/s)	k	Cooler
x	Displacement (m)	h	Heater
		p	Piston
		r	Regenerator

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