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PREFACE

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PREFACE

Lattice models and integrability: a special issue in honour of F Y Wu

Fa Yueh (Fred) Wu was born on 5 January 1932 in Nanking (now known as Nanjing), China, the capital of the Nationalist government. Wu began kindergarten in 1937 in a comfortable setting, as his father held a relatively high government position. But the Sino–Japanese war broke out in July 1937, and Nanking fell to Japanese hands in November. Fleeing the Japanese, his parents brought Wu to their hometown in Hunan, and then to the war capital Chungking (now Chongqing) in 1938, where they lived for eight years until the end of the war.

Around that time the Japanese began bombing Chungking, and Wu's childhood memories were dominated by air raids, bombings and burning not dissimilar to those experienced by Londoners during the war. At times the air raids lasted for days disrupting everyday lives in Chungking, including Wu's schooling. One day during a fierce bombing raid, a bomb fell in their garden reducing a pavilion and the surrounding pond to a huge crater; another bomb fell just a few metres from the tunnel where his family took shelter, almost sealing the only entrance. The family moved the very next day to the countryside.

As a result of the war, Wu attended seven schools before finishing his primary education. Fortunately, by the time he entered junior high school in 1943, the Japanese forces were on the wane and Wu entered the elite middle school, Nankai. His early academic performance in Nankai seemed to him mediocre, but he nevertheless impressed his geometry teacher by showing bursts of talent. With hindsight, this early interest in geometry may have led to his later insights in graphical analyses of statistical systems. The family returned to Nanking in 1946 after the Victory over Japan Day. By this time his father had become elected to the Legislative Yuan, the equivalent of the US Senate.

Wu entered high school in Nanking in 1946. Since he came from an elite school in Chungking, he excelled in most of his classes, especially mathematics. Notwithstanding his academic success, Wu probably spent more time playing Chinese chess, a board game similar to international chess. He ranked high in a city-wide tournament and often played blind-folded games. He also spent time playing bridge, a game he learned in Nankai and kept up throughout his years in the US. He also loved puzzles and riddles.

But the good days did not last long, as the civil war drew closer to Nanking with the Communists winning. The family fled Nanking once again, following a zigzag path and traveling by boat, train, car and then by boat again, eventually reaching Taiwan in June 1949. By this time, the Nationalists had lost most of China, and there was no hope of returning to the mainland. Wu entered the Naval College of Technology to study electrical engineering, giving up an opportunity to study mathematics in the National Taiwan University, although his real interest was in mathematics.

In 1954, Wu graduated from the Naval College with a BS degree and the commission of Ensign. Recognizing his outstanding academic record in the College, the Chinese Navy sent him to the US in 1955 to study radar and sonar and to receive training as an instructor. He stayed at the Naval School of Electronics in San Francisco and at the Instructor's School

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in San Diego. Wu felt that he benefited from the instructor's training much more than from the electronics school, as the training helped him to develop teaching and presentation skills that served him well throughout his career. The Navy assigned him to teach electronics in the Naval Academy when he returned to Taiwan in 1956.

Wu was interested in attending a graduate school. The only institution that offered a graduate degree in Taiwan at the time was Taiwan's newly re-established Tsing Hua University. In its hurried retreat to Taiwan, the Nationalist government left the original Tsing Hua University, one of China's best-known institutions of higher learning with a history dating back to the 19th century, behind in Beijing. In 1956, after gaining footing in Taiwan, the Nationalist government revived Tsing Hua, and began offering a two-year Master's degree in nuclear science. Wu decided to apply for admission but faced considerable obstacles since he was in the Navy. After one year's effort, mostly on his father's part, Wu finally entered Tsing Hua in 1957. He completed the two-year program with an experimental thesis in 1959. By this time, the US was pushing for a scientific renaissance after the launch of the Soviet satellite Sputnik. Wu received offers of teaching assistantships from several physics departments in the US, and chose to continue his graduate education at Washington University in St. Louis in 1959.

At Washington University he studied many-body theory under the late Eugene Feenberg and produced several influential papers [1, 2] on ground state properties of liquid helium-3 and liquid helium-4. In 1963, he published a paper on formulating cluster expansions in an N-body problem [3], extending the Mayer expansion to systems with indexed many-body interactions, which appeared to have escaped the attention of the community of statistical physics that it deserved. The expansion made extensive use of graphical terms, demonstrating his prowess in graphical analysis at an early stage of his career. Wu's interest in many-body theory continued over the years, with subsequent works on the electron gas, adsorbed systems, and the long-perplexing problem of density correlations in Fermi and Bose systems. After obtaining his PhD from Washington University in 1963, Wu went on to teach at Virginia Polytechnic Institute (VPI) as an assistant professor.

In February 1967, Wu met Elliott Lieb who was visiting VPI to give a talk on the Bethe ansatz evaluation of the entropy of two-dimensional ice, a 6-vertex model. Wu soon realized the underlying graphical aspects of two-dimensional vertex models and solved the thermodynamics of a related 5-vertex model using the Pfaffian approach. The result was published in the April issue of *Physical Review Letters* (PRL) of the same year [4], and in September 1967, Wu moved to Northeastern University to join Lieb's group.

Wu taught at Northeastern for 39 years until his retirement in 2006 as the Matthews Distinguished University Professor of Physics. Over the years, Wu has published more than 230 papers and monographs, and he continues to publish after retirement. Most of his research since 1967 is in exact and rigorous analyses of lattice models and integrable systems, which is the theme of this special issue.

In 1968, after Wu's arrival at Northeastern, Lieb and Wu obtained the exact solution of the ground state of the one-dimensional Hubbard model and published the result in PRL [5], a work which has since become highly important after the advent of high-temperature superconductivity. This Lieb–Wu paper and Wu's 1982 review of the Potts model in *Reviews of Modern Physics* [37] are among the most cited papers in condensed matter physics. Later in 1968 Lieb departed Northeastern for MIT. As a result, the full version of the solution was not published until 34 years later [38] when Lieb and Wu collaborated to work on the manuscript on the occasion of Wu's 70th birthday.

Wu spent the summer of 1968 at Stony Brook as the guest of C N Yang. Working with Yang's student, C Fan, he extended the Pfaffian solution of the Ising model to general lattices and termed such models 'free-fermion', a term now in common use [6].

In 1972, Wu visited R J Baxter, whom he had met earlier in 1968 at MIT, in Canberra, Australia, with the support of a Fulbright grant. They solved the triangular-lattice Ising model with 3-spin interactions [7], a model now known as the Baxter–Wu model. It was an ideal collaboration. While Baxter derived the solution algebraically, Wu used graphical methods to reduce the problem to an Ashkin–Teller model, which greatly simplifies the presentation. While in Canberra, Wu also studied the 8-vertex model on the honeycomb lattice [8], a model which proved to be useful in his later research.

In 1973, Wu returned to Tsing Hua as a visiting professor and worked with colleague K Y Lin. They published two important papers introducing staggered vertex models for the first time [10, 11]. In other important work they clarified the nature of the phase diagram of the Ashkin–Teller model, and found it to have two phase transitions [9].

In the 1970s Wu traveled to Taiwan, Australia, Europe and to China when it re-opened. He met H N V Temperley in Aberdeen, Scotland in 1976, and collaborated with H J Brascamp and H Kunz in Lausanne to establish a number of rigorous results on vertex models, including a proof of the equivalence of boundary conditions for the 6-vertex model [13, 14]. From 1979 to 1980, Wu resided in the Netherlands and Germany, where he was the guest of Piet Kasteleyn at Leiden, Hans van Leeuwen at Delft and Kurt Binder in Juelich.

It was in Juelich that Wu completed the 1982 review paper on the Potts model [37], a paper that has been cited 70 or more times every year since its publication. Another important work in that period is a 1976 graphical analysis of the Potts model on the triangular lattice in collaboration with Baxter and Kelland [15]. This paper provided an elegant and conceptually easy description of the duality relation of the model, complementary to the algebraic analysis of Temperley and Lieb [16]. Four years later, Wu and Lin further refined the graphical aspects to reduce the model to a 5-vertex model, under which the duality relation appears as a simple spatial symmetry [18]. The Wu–Lin formulation of the Potts model is used by Jacobsen and Sculland in an analysis of the kagome-lattice Potts model in their first paper in this issue [39]. In other pioneering work in 1976, Wu and Y K Wang introduced a spin model with chiral interactions and its duality relation in Fourier space [19]. Prior to that time, studies of spin models had invariably been confined to models with symmetric interactions.

In 1977 Wu published an influential paper on spanning trees [20]. In it, he derived the spanning tree constants of the regular two-dimensional lattices. Since then, he has been the co-author of several papers extending this work to a variety of other two-dimensional Archimedean lattices [21–23]. In this issue Guttmann[1] and Rogers solve the three-dimensional version of this problem, which has resisted attack for more than 30 years [40]. The connection between spanning trees and dimers was previously highlighted by Neville Temperley in 1974 [17].

The ideas from number theory needed to obtain the spanning tree constant of three-dimensional lattices, notably logarithmic Mahler measures, are further discussed in the article by Glasser[2] in this issue [41].

Wu has had a long and productive collaboration with Maillard, particularly on aspects of the Ising model. Maillard also wrote the definitive description of Wu's many scientific contributions at the time of Wu's 70th birthday [24]. The paper was later included among the

[1] Wu met Tony Guttmann at the University of Newcastle, Australia, back in 1973 when Guttmann invited him to visit. Over the years their paths have crossed countless times at conferences and workshops, and during Wu's visits to Australia and Guttmann's to America; their families became close friends in the process, with Wu's wife Jane assisting Guttmann's wife Susette in her professional duties when they both visited Taiwan.

[2] Wu met Larry Glasser in 1968 at MIT and also visited him later at Clarkson. They collaborated in 2003 on a paper later published in the *Ramanujan Journal* in 2005, in which they evaluated an integral for the entropy of spanning trees on the triangular lattice.

biographies of great names such as Newton and Feynman in the *History of Physics: Individual Biographies* section in the *MIT Net Advance of Physics* website [59]. Further developments in the Ising model are highlighted in the article by Boukraa, Hassani and Maillard[3] in this issue [42]. Maillard's article also appears as the introduction to a wonderful collection of Wu's works that appeared in 2009 [25], entitled *Exactly Solved Models: A Journey in Statistical Mechanics.*

The relation between bond percolation and the random-cluster formulation of the Potts model was pioneered by Kasteleyn and Fortuin in 1969 [26]. Later, in a 1977 paper, Wu showed how to rederive this relation in a different setting and used it to obtain various quantities of interest in the bond percolation problem, including critical exponents, from the exact solution of the Potts model [27]. A few months later, in collaboration with Kunz, he showed that site percolation can also be related to the Potts model [28]. Problems in bond percolation are treated in this issue by several works. The paper by Hu, Blöte[4] and Deng[5] investigates how the imposition of a 'canonical' constraint, that there be an equal number of open and closed edges, affects the universal properties [43]. The paper by Ziff[6], Scullard, Wierman and Sedlock exactly solves inhomogeneous percolation on lattices of the bow-tie and checkerboard types [44].

In a 1979 paper on Potts model critical points, Wu proposed a conjecture, now known as the homogeneity hypothesis, on the location of the critical point of the kagome lattice [29]. Since then, numerous studies have been carried out to test the validity of that conjecture [12]. However, many of these tests proved to be inconclusive since they produced results extremely close to the conjectured value. The puzzle is finally solved by Jacobsen and Scullard in their two papers in this issue [39, 45]. Using a graphical analysis based on the Wu–Lin 5-vertex formulation, they recover the Wu conjecture of the kagome-lattice critical point as the first-order approximation in a well-defined graphical analysis. This establishes once and for all the approximate nature of the Wu conjecture.

These investigations, and the exact solutions found by Wu, raised the question as to the conditions under which a lattice model is exactly solvable. Quite recently, such questions have been addressed through the technique of discrete holomorphicity (DH). This direction is represented in this issue by the contributions of Alam and Batchelor[7], where the connection between DH and Yang–Baxter integrability is investigated [46]. DH is also a key ingredient in recent rigorous proofs that certain lattice models converge, in the continuum limit, to conformally invariant probabilistic processes known as Schramm–Loewner evolution (SLE). The theme of SLE appears within this issue in the article by Alberts, Kozdron and Lawler [47]. Finally, DH observables are used in this issue by Duminil-Copin to prove the divergence

[3] Wu and Jean-Marie Maillard enjoyed joint research grants, organised between the NSF and the CNRS. They also got together frequently in Taiwan and at conferences including one in Paris on the Yang–Baxter equation in 1992. They have many joint papers, including one of Wu's favorites, a 1992 *J. Phys. A: Math. Gen.* paper on thermal transmissivity. In that paper they put the loosely defined term *transmissivity* onto a rigorous footing.

[4] Henk Blöte and Wu first met in 1973 in Delft. Since then they have visited each other frequently, as Blöte made regular visits to the University of Rhode Island (near Boston) and Beijing Normal University, intersecting those of Wu. They first collaborated in a 1989 paper in which they obtained a closed-form expression for the critical curve of the honeycomb antiferromagnetic Ising model and checked the formula against finite-size analysis. This combination of checking an *a priori* derivation against high-precision numerical analysis set the tone of Wu's later collaborations with Blöte and his students.

[5] Youjin Deng obtained his PhD in 2004 under the direction of Blöte at Delft. Wu served on Deng's Dissertation Committee and participated in his graduation ceremony.

[6] Through Wu's recent works on the Potts model he got to know Bob Ziff well. They exchanged preprints and e-mails, and often had lengthy discussions on minute points, including the use and origin of the term 'hemp-leaf lattice'.

[7] Wu and Murray Batchelor met at the Australia National University in 1990 and again in 1995, and their paths have crossed at many conferences and workshops.

of the correlation length for the Potts model (in its formulation in terms of Fortuin–Kasteleyn clusters) when $1 \leqslant q \leqslant 4$ [48].

Establishing the phase diagrams of lattice models is a recurrent theme in Wu's works. In an interesting but little-known work from 2000 with Guo and Blöte [30], he has shown that, contrary to common belief, the O(n) model on the honeycomb lattice has a second-order phase transition for $n > 2$. The question of phase diagrams for O(n)-type models is taken up in this issue by Blöte, Wang and Guo[8] [49].

In 1983–84, Wu joined the National Science Foundation as the Director of the Condensed Matter Theory Program for 18 months. His duty was managing funding to individual researchers in condensed matter theory in the US. The 18-month tour in Washington offered Wu a bird's-eye view of condensed matter physics research in US universities, an understanding that proved useful to his later researches.

Throughout his career, Wu has insisted on the general applicability of graphical analysis to a variety of lattices. This aspect was highlighted in his 1988 paper on the Potts model and graph theory [31], in which he derived a number of equivalences with (di)chromatic and flow polynomials on arbitrary planar graphs, both for the partition function and correlation functions. An earlier result in the same vein is the equivalence of the Potts model on a planar graph with a loop model on the corresponding medial graph, found in 1976 in collaboration with Baxter and Kelland [15]. Building on these results, and on recent progress in the combinatorial approach to planar maps, Borot, Bouttier and Guitter systematically investigate properties of percolation and Potts models on random planar maps in their contribution to this issue [50].

Wu has published extensively on dimer enumerations. His work includes exact enumerations on non-orientable surfaces and surfaces with a single boundary defect. In this issue, Lu and Zhang consider dimer enumerations on the Klein bottle, which is an example of a non-orientable surface [51]. Another contribution is the paper by Ciucu and Fischer, considering dimer coverings of a domain with a defect (hole) in the interior [52].

Wu has also worked extensively in knot theory. He has constructed new knot invariants based on statistical mechanical models [61, 62], and published a well-received review of knot invariants for physicists [32], which elucidates the connection of knot invariants with statistical mechanics.

In 2004, Wu presented a new formulation of resistance networks [33], which permits the derivation of the exact expression of the resistance between two arbitrary nodes in any network. He later extended the formulation to impedance networks [34], a work which has since attracted interest in applications in petroleum research. These works can perhaps be seen as a distant echo of Wu's Navy training in electronics, more than 50 years earlier. In recent years Wu has developed this topic in joint work with Essam[9], who together with Brak has related work on lattice paths in this issue [53]. A cognate paper by Arrowsmith, Bhatti and Essam also appears [54].

Wu has made other contributions to asymptotic analysis, for example in relation to dimers in his recent papers, where he also uses results from conformal invariance [60]. This thread is taken up by the article of Izmailian[10] in this issue [55].

[8] Wenan Guo likewise obtained his PhD under the supervision of Blöte in Delft. Wu and Guo know each other well from Wu's visits to the Beijing Normal University where he is an honorary professor. He has collaborated with Guo, on the subject of finite-size analysis using the transfer matrix approach, in several of his recent papers.

[9] Wu first met John Essam at King's College, London in 1978. Followoing Wu's 2006 closed-form expression of the corner-to-corner resistance of an $M \times N$ resistor network in the form of a double summation, they combined forces in 2008 at a workshop in Cambridge, and derived the asymptotic expansion of that expression.

[10] Nickolay Izmailian holds positions in Armenia and Taiwan. Wu and Izmailian collaborated in a paper in 2000 on the exact solution of a 6-vertex model with bond defects. Most recently they collaborated on the exact enumeration of dimers on a cylinder with a single boundary defect.

In 1997, Wu reported, in a short paper, a new formulation of duality relations of Potts correlation functions for n spins residing on the boundary of a lattice [35]. He gave the examples of $n = 2$ and 3, and remarked that the formulation can be extended to higher values of n 'in a straightforward fashion'. But the extension is by no means straightforward[11] and its solution was eventually found by Wu and his student H Y Huang [36]. They found that the correlation functions are not all independent when $n = 4$ and higher. They also deduced the connecting relations expressing crossing correlations in terms of non-crossing correlations, thus resolving the discrepancy.

Nowadays the interest in integrable systems largely transcends the realm of equilibrium statistical physics. Important and fundamental applications have appeared in out-of-equilibrium physics, in combinatorics, and in the study of certain dualities between string theories and gauge theories known under the common epithet of AdS/CFT duality. This last trend is represented in this issue by the contribution of Kostov [56].

Other interests of Wu in both quantum and classical systems are reflected in the article by Barry[12], Muttalib and Tanaka [57], and in the paper by Bauer, Bernard and Benoist on iterated stochastic measurements [58]. This latter paper appears very timely, since it is inspired by the experiments carried out in the group of Serge Haroche who earned the 2012 Nobel Prize in Physics.

Wu met his wife Ching Tse (Jane) in Taipei. They married in 1963 in St. Louis, Missouri. They have three daughters; Yvonne, a Professor of Child Neurology at the University of California San Francisco, Yolanda, a women's rights lawyer and a teacher of Suzuki violin, and Yelena, a postdoc in Child Clinical Psychology at Cincinnati Children's Hospital. Fred and Jane have five grandchildren. Wu left four siblings behind when he left China in 1949, and reunited with them after a 30-year separation for the first time in 1979. Two sisters and one brother are now deceased, and his younger brother, who also has three daughters, lives a comfortable life in retirement in Kunming, China.

It has been a pleasure to assemble this collection of papers on the occasion of Fred's 80th birthday, and we wish to thank him for providing much of the biographical information on which this introduction is based. We are also grateful to all the contributors for providing such a diverse and decidedly very modern panorama of the topic of lattice models and integrability, and for meeting the strict deadlines necessary to ensure the completion of this issue before the year 2012 draws to an end.

Fred Wu continues to be a highly productive, imaginative scientist, and we look forward to a continuing body of excellent work. Meanwhile, we wish him many more years of a happy and healthy life.

A J Guttmann, *University of Melbourne, Australia*
J L Jacobsen, *Ecole Normale Supérieure de Paris, France*
Guest Editors

[11] Wu's acquaintance with Jesper Jacobsen goes back to this period, when the latter pointed out this fact in a comment to Wu's first paper on this subject. They have since crossed paths on various occasions, most recently at a 2008 workshop at the Isaac Newton Institute in Cambridge.

[12] Jerry Barry is another long-term collaborator of Wu's. They have met at numerous conferences and workshops. In one meeting in 1989, Barry called Wu's attention to a three-dimensional spin model on the pyrochlore lattice that appeared to be soluble. They soon solved the Ising model on that lattice. In 1997 they collaborated on a paper obtaining the phase diagram of a ternary polymer model.

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