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Analysis of minimal length effect on the non-relativistic energy and wave function for the exponential type potential using Supersymmetric Quantum Mechanics

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Abstract. In this research, the minimal length effect was analysed on the energy and wave functions of the non-relativistic mechanical quantum system by studying the Schrodinger equation. The three dimensional Schrodinger equation with minimal length for the exponential type potential was solved by using supersymmetric quantum mechanics. To get the solution, the exponential type potential was reduced to the Morse potential. The non-relativistic energy was calculated numerically for diatomic molecules, LiH and HCl by using the Matlab software. From the results were shown that the presence of minimal length gives effect to the non-relativistic energy, it was seen clearly in the higher value of angular momentum l and quantum number n.

1. Introduction

The Schrodinger equation plays the main role in studying the non-relativistic system [1] that is used to analyse the non-relativistic energy and non-relativistic wave function which give the information of the quantum system [2]. Analytic solutions of the Schrodinger equation have been extensively studied with various potentials, as Eckart and Manning-Rosen [3], Morse [4, 5], and Scarf Trigonometry [6]. In this research, the Schrodinger equation with minimal length is solved for the exponential type potential, where the exponential type potential is reduced to the Morse potential.

The Schrodinger equation with minimal length for the exponential type potential can be solved by using the various methods, such as Nikiforov–Uvarov (NU) [7], Asymptotic Iteration Method (AIM) [8, 9], supersymmetry quantum mechanics [10, 11]. Supersymmetry quantum mechanics is one of the most well-liked methods which based on the shape-invariance concept. Supersymmetry appears based on the theory case in studying quantum mechanics at a higher dimension to unite the four basic interaction in nature, that are electro, weak, strong and gravitational interactions [11].

In recent years, the Schrodinger equation with minimal length has interested many researchers. The effect of minimal length has modified the Heisenberg uncertainty principle into generalised uncertainty principle (GUP) [12-14]. By applying minimal length term in the Schrodinger equation, the Schrodinger equation is modified with an additional form of p^4 [15,16] and become complex. For this reason, the modified Schrodinger equation is solved without minimal length. To get the usual non-relativistic energy, the linear momentum quadratic is only considered a function of r. The studies of the Schrodinger equation with the minimal length effect have been solved to scattering states of

Woods-Saxon interaction [14], Square Well Potential [5], hydrogen atom [16], and Bouncing Particle Spectrum [17].

In this paper, the Schrodinger equation is solved to obtain the non-relativistic energy and non-relativistic wave functions with the minimal length effect. Numerical non-relativistic energy is calculated to LiH and HCl molecule by using Matlab. The paper is structured as follows: The second and third section, the supersymmetry quantum mechanic and shape invariant are reviewed respectively. Section 4 gives the introduction of the Schrodinger equation with minimal length. In Section 5, the solution of the Schrodinger equation with minimal length for the exponential type potential using supersymmetric quantum mechanics is presented. Numerical result and discussion are presented in Section 6. Finally, the conclusions are given in Section 7.

2. Supersymmetry Quantum Mechanics

The main character in supersymmetric is a supercharge operator that commute with a Hamiltonian. For $N=2$, the supercharge operators are Q and $Q+$. Commutation relation of the operators with supersymmetric Hamiltonian Hss is satisfied the equation [11, 18].

$$\{Q, Q^+\} = Q^+Q + QQ^+ = H_{ss} \tag{1}$$

Equation (1) shows that supercharges operators commute with supersymmetric Hamiltonian

$$\{Q^+, H_{ss}\} = \{Q, H_{ss}\} = 0 \tag{2}$$

And Fermionic operators anti-commute with itself

$$(Q^+, Q^+) = (Q, Q) = 0 \tag{3}$$

The supercharge operators are [11]

$$Q = \begin{pmatrix} 0 & 0 \\ A^- & 0 \end{pmatrix}, \qquad Q^+ = \begin{pmatrix} 0 & A^+ \\ 0 & 0 \end{pmatrix} \tag{4}$$

$A^\pm$ is Bosonic operator that written

$$A^\pm = \mp \frac{\hbar}{\sqrt{2m}} \frac{d}{dx} + \phi(x) \tag{5}$$

Furthermore, supersymmetric Hamiltonian is defined by

$$H_{SS} = \begin{pmatrix} A^-A^+ & 0 \\ 0 & A^+A^- \end{pmatrix} = \begin{pmatrix} H^+ & 0 \\ 0 & H^- \end{pmatrix} \tag{6}$$

Where supersymmetric partner Hamiltonian $H^\pm$,

$$H_\pm = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_\pm(x) \text{ with } V_\pm = \phi^2 \pm \frac{\hbar}{\sqrt{2m}}\phi' \tag{7}$$

3. Shape invariance

Shape invariant potential system is two potentials that have the same shape. Two potential is said to be invariant if satisfies the following equation [19].

$$V_+(x, a_j) = V_-(x, a_{j+1}) + R(a_{j+1}) \tag{8}$$

with $j=0,1,2,3....$ Supersymmetric partner potentials are defined by

$$V_+(x, a_j) = \phi^2(x, a_j) + \frac{\hbar}{\sqrt{2m}}\phi'(x, a_j) \tag{9}$$

and

$$V_-(x, a_j) = \phi^2(x, a_j) - \frac{\hbar}{\sqrt{2m}}\phi'(x, a_j) \tag{10}$$

By using super-algebra a given Hamiltonian is written as

$$H_{\pm} = H^- + E_0 = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{\pm}\left(x, a_j\right) + E_0 \tag{11}$$

Based on equation (8), equation (11) is reduced to

$$V(x) = V_-\left(x, a_j\right) + E_0 = \phi^2\left(x, a_j\right) - \frac{\hbar}{\sqrt{2m}}\phi'\left(x, a_j\right) + E_0 \tag{12}$$

V(x) is an effective potential of the system. In general, k^{th} Hamiltonian is written as

$$H_k = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_-\left(x; a_k\right) + \sum_{i=1}^{k} R\left(a_i\right), \quad k = 0, 1, 2... \tag{13}$$

where

$$R\left(a_i\right) = V_+(x; a_{i-1}) - V_-(x; a_i) \tag{14}$$

So, the general non-relativistic energy from H_- is

$$E_n^{(-)} = \sum_{k=1}^{n} R(a_k) \tag{15}$$

Then, the Hamiltonian from equation (11) is changed to the form

$$E_n = E_n^{(-)} + E_0 \tag{16}$$

En is total non-relativistic energy, and E_o is ground state non-relativistic energy

To get non-relativistic wave function, the ground state non-relativistic wave function is operated by lowering operators and the result must be zero [2]. The equation is written as

$$A\psi_0^{(-)} = 0 \tag{17}$$

The ground state non-relativistic wave function is obtained as

$$\psi_0^{(-)}\left(x, a_0\right) = N\exp\left[-\frac{\sqrt{2m}}{\hbar}\int^x \phi\left(x, a_0\right)dx\right] \tag{18}$$

N is the normalisation constant. The upper-level of non-relativistic wave function $\psi_n^-\left(n; a_0\right)$ can be determined by using a rising operator $\left(A^+\right)$ to the non-relativistic ground state wave function. In general, the excited non-relativistic wave equation of the nth level becomes

$$\psi_n^-\left(x; a_0\right) \sim A^+\left(x; a_0\right)\psi_{n-1}^-\left(x; a_1\right) \tag{19}$$

4. The Schrodinger Equation with minimal length

Generalised modified canonical commutation relation with minimal length is given as[12-13]

$$[X, P] = i\hbar\left(1 + \alpha P^2\right) \tag{20}$$

where

$$X_i = x_i, \quad P_i = \left(1 + \alpha p^2\right)p_i \tag{21}$$

α a very small positive parameter of the minimal length, in an interval $0 \leq \alpha \leq 1$. Here, p_i is the momentum operator at low energies and P_i is the momentum operator at high energies. At low energies, energies much smaller than the Planck mass and the second term of the right-hand side of equation (20) vanishes. The equation was well-known as Heisenberg uncertainty principle [12,14]. According to the equation (21), energy kinetic operator is written as

$$T = \frac{P^2}{2m} = \frac{\left(1 + 2\alpha p^2\right)p^2}{2m}, \quad p^2 = -\hbar^2\nabla^2 \tag{22}$$

The Schrodinger equation with the minimal length effect $\left(\alpha \neq 0\right)$ leads to

$$\left\{\frac{-\hbar^2}{2m}\nabla^2 - \frac{\alpha\hbar^4}{m}\Delta^2 + V(r)\right\}\chi = E\chi \tag{23}$$

To obtain the analytic solution of the second order differential, equation (23) is solved for $\alpha = 0$

$$H^0 = \frac{-\hbar^2}{2m}\Delta + V(r) \tag{24}$$

From the equation (24) we get

$$\Delta^2 = \left(\frac{2m}{\hbar^2}\right)^2 \left[V(r) - E^0\right]^2 \tag{25}$$

Known that, electron revolute to the nuclei on polar shaped shall. It causes the non-relativistic energy and non-relativistic wave functions of an electron are employed by a solution of the Schrodinger equation in polar coordinate [20]. The Laplacian operator in polar coordinate is explained as follows

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\Delta_\Omega \tag{26}$$

where

$$\Delta_\Omega = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} \tag{27}$$

Since the used potential only contains the variable r then the solution is only solved on the radial form. Where Δ_Ω is the angular part of the Laplacian operator. Then, the angular part changed to

$$\Delta_\Omega Y_{\ell m}\left(\theta,\varphi\right) = -l\left(l+1\right)Y_{\ell m}\left(\theta,\varphi\right) \tag{28}$$

by substituting equation (25) and (26) to equation (23), the Schrodinger equation is rewritten as

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2}l\left(l+1\right)\right] + 4\alpha m\left[V(r) - E^0\right]^2 + \left(V(r) - E\right)\right\}\chi(r) = 0 \tag{29}$$

Using the new non-relativistic wave function

$$\chi(r) = F\left(r\right)Y_{\ell m}\left(\theta,\varphi\right) \tag{30}$$

Equation (29) becomes

$$\left\{\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\hbar^2}{2m}\frac{1}{r^2}l\left(l+1\right) - \left[4\alpha m\left(V\left(r\right) - E^0\right)^2 + \left(V\left(r\right) - E\right)\right]\right\}F(r) = 0 \tag{31}$$

with $F\left(r\right) = \dfrac{\Psi\left(r\right)}{r}$, the Schrodinger equation is reduced to one dimensional only in radial function

$$\left\{\frac{\hbar^2}{2m}\frac{d^2}{dr^2} - \frac{\hbar^2}{2m}\frac{1}{r^2}l\left(l+1\right) + \left[-4\alpha m\left(V\left(r\right) - E^0\right)^2 - \left(V\left(r\right) - E\right)\right]\right\}\Psi(r) = 0 \tag{32}$$

Equation (32) is the Schrodinger equation in the presence of minimal length.

5. Solution of the Schrodinger equation with minimal length for the exponential type potential using supersymmetric quantum mechanics

The used exponential type potential is a part of the Morse potential. The potential is written as

$$V = De^{-\delta x} \tag{33}$$

with $x = \dfrac{r - r_0}{r_0}$ stand $\delta = a r_e$. D is dissociation energy, δ is the Morse parameter, r_0 is the equilibrium inter-nuclear distance. The exponential type potential in equation (33) is substituted into equation (32)

$$\frac{\hbar^2}{2m}\frac{d^2\Psi(r)}{dr^2} - \frac{\hbar^2}{2r}\frac{l(l+1)}{r^2}\Psi(r) + \left[\begin{array}{c}\left(-4\alpha mD^2\right)e^{-2\delta x} + \left(8\alpha mDE^o - D\right)e^{-\delta x} \\ + \left(E - 4\alpha m\left(E^o\right)^2\right)\end{array}\right]\Psi(r) = 0 \quad (34)$$

To get the exact solution of the Schrodinger equation for $l \neq 0$, we need a suitable approach. In this research, we used the Pakeris approximation. The approximation is selected because of the centrifugal term in the exponential series base on the internuclear distance. To get the solution, the centrifugal term is simplified around $x=0$. Then, the second term in equation (34) becomes [7].

$$\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} = \frac{\hbar^2}{2m}\frac{l(l+1)}{r_0^2}\frac{1}{(1+x)^2} = \gamma\left(1 - 2x + 3x^2 + ...\right) \quad (35)$$

where

$$\gamma = \frac{\hbar^2}{2m}\frac{l(l+1)}{r_0^2} \quad (36)$$

With assumption fourth term and so on are very small, the approximation in equation (35) is only used until the third term. Then, the rotational term is replaced to

$$1 - 2x + 3x^2 + ... = D_0 + D_1 e^{-\delta x} + D_2 e^{-2\delta x} \quad (37)$$

Furthermore, equation (37) is rewritten in

$$1 - 2x + 3x^2 + ... = D_0 + D_1\left(1 - \delta x + \frac{\delta^2 x^2}{2!}\right) + D_2\left(1 - 2\delta x + \frac{4\delta^2 x^2}{2!}\right) \quad (38)$$

By combining the right and left term, the relation between the coefficient and the parameter δ is obtained as

$$D_0 = 1 - \frac{3}{\delta} + \frac{3}{\delta^2}, \quad D_1 = \frac{4}{\delta} - \frac{6}{\delta^2}, \quad D_2 = -\frac{1}{\delta} + \frac{3}{\delta^2} \quad (39)$$

So, equation (35) becomes

$$\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} = \gamma\left(D_0 + D_1 e^{-\delta x} + D_2 e^{-2\delta x}\right) \quad (40)$$

By substituting equation(40) into equation (34), the equation (34) is rearranged as

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(r)}{dr^2} + \gamma\left[D_0 - D_1 e^{-\delta x} + D_2 e^{-2\delta x} + ...\right]\Psi(r)$$
$$+ \left[\left(+4\alpha mD^2\right)e^{-2\delta x} - \left(8\alpha mDE^o - D\right)e^{-\delta x} + 4\alpha m\left(E^o\right)^2\right]\Psi(r) = E\Psi(r) \quad (41)$$

By introducing the following parameter

$$A_0 = \gamma D_0 + 4\alpha m\left(E^o\right)^2, \quad A_1 = \gamma D_1 + \left(8\alpha mDE^o - D\right), \quad A_2 = \gamma D_2 + \left(4\alpha mD^2\right) \quad (42)$$

Equation (41) was changed to the form

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(r)}{dr^2} + \left(A_0 - A_1 e^{-\delta x} + A_2 e^{-2\delta x}\right)\Psi(r) = E\Psi(r) \quad (43)$$

Finally, the effective potential is found as

$$V_{eff} = A_0 - A_1 e^{-\delta x} + A_2 e^{-2\delta x} \quad (44)$$

Based on the effective potential equation on equation (44), the corresponding superpotential is

$$\phi = C_1 - C_2 e^{-\delta x} \tag{45}$$

Using equation (45), equation (12) becomes

$$V_{eff} = C_1^2 - \left(2C_1 C_2 + \frac{\hbar}{\sqrt{2m}} \delta C_2 \right) e^{-\delta x} + C_2^2 e^{-2\delta x} + E_0 \tag{46}$$

By combining equation (44) and (46), the value of c_1, c_2 and E_0 are defined by

$$C_2 = \sqrt{A_2} \ , \quad C_1 = \frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}} \delta \tag{47}$$

and

$$E_0 = A_0 - \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}} \delta \right)^2 \tag{48}$$

By inserting equation (47) into equation (45), the new superpotential becomes

$$\phi = \frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}} \delta - \sqrt{A_2} e^{-\delta x} \tag{49}$$

Using equation (49), equation (9) and (10) is written as

$$V_+(x;a_0) = A_2 e^{-2\delta x} - \left(A_1 - \frac{2\hbar\sqrt{A_2}}{\sqrt{2m}} \delta \right) e^{-\delta x} + \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}} \delta \right)^2 \tag{50}$$

$$V_-(x;a_0) = A_2 e^{-2\delta x} - A_1 e^{-\delta x} + \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}} \delta \right)^2 \tag{51}$$

By comparing equation (50) and (51), we get

$$a_0 = A_1 \ , \quad a_1 = A_1 - \frac{2\hbar\delta\sqrt{A_2}}{\sqrt{2m}} \ , \quad a_2 = A_1 - \frac{4\hbar\delta\sqrt{A_2}}{\sqrt{2m}} \tag{52}$$

From equations (14), (50), (51), and (52), we get the constant value as

$$R(a_1) = \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar\delta}{2\sqrt{2m}} \right)^2 - \left(\frac{A_1}{2\sqrt{A_2}} - \frac{3\hbar\delta}{2\sqrt{2m}} \right)^2 \tag{53}$$

$$R(a_2) = \left(\frac{A_1}{2\sqrt{A_2}} - \frac{3\hbar\delta}{2\sqrt{2m}} \right)^2 - \left(\frac{A_1}{2\sqrt{A_2}} - \frac{5\hbar\delta}{2\sqrt{2m}} \right)^2 \tag{54}$$

Then the general form of constant value $R(a_n)$ is

$$R(a_n) = \left(\frac{A_1}{2\sqrt{A_2}} - \frac{(2n-1)\hbar a}{2\sqrt{2m}} \right)^2 - \left(\frac{A_1}{2\sqrt{A_2}} - \frac{(2n+1)\hbar\delta}{2\sqrt{2m}} \right)^2 \tag{55}$$

So equation (15) becomes

$$E_n^{(-)} = \sum_{k=1}^n R(a_k) = \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar\delta}{2\sqrt{2m}} \right)^2 + \left(\frac{A_1}{2\sqrt{A_2}} - \frac{(2n+1)\hbar\delta}{2\sqrt{2m}} \right)^2 \tag{56}$$

$E_n^{(-)}$ is ground state non-relativistic energy from nth Hamiltonian. By inserting equations (56) and (48) into equation (16), the total non-relativistic energy of the system is

$$E_n = A_0 + \left(\frac{A_1}{2\sqrt{A_2}} - \frac{(2n+1)\hbar\delta}{2\sqrt{2m}} \right)^2 \tag{57}$$

Equation (57) shows that total non-relativistic energy is influenced by quantum number (n), momentum angular (l), minimal length parameter (α). The analytic solution in equation (57) is used to get the numerical calculation of non-relativistic energy. The calculation is not allowed to having the value A_2 in equals zero, so we have to set the value both l and α cannot be zero at the same time.

To calculate the non-relativistic wave function, equation (49) is substituted into equation (18) and we obtain

$$\psi_0^{(-)} = N\, Exp\left(-\frac{\sqrt{2m}}{\hbar} \int \left(\left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}}\delta \right) - \sqrt{A_2}\, e^{-\delta x} \right) dx \right) \tag{58}$$

From equation (58), the ground state non-relativistic wave function is found as

$$\psi_0^{(-)} = N \exp\left(-\frac{\sqrt{2m}}{\hbar} \left(\frac{A_1}{2\sqrt{A_2}} - \frac{\hbar}{2\sqrt{2m}}\delta \right) x \right) \exp\left(-\frac{\sqrt{2m}}{\hbar} \frac{\sqrt{A_2}}{\delta} e^{-\delta x} \right) \tag{59}$$

With N is normalisation factor. Equation (59) shows that the non-relativistic wave function is influenced by momentum angular (l) and minimal length parameter (α).

6. Numeric Result

The numerical results of non-relativistic energy for the diatomic molecules were obtained by Matlab. The diatomic molecules were used because the Morse potential describes the vibration of the molecules with two atoms. The used molecules in the calculation were LiH and HCl as the example of diatomic molecules. In this paper, we used the natural unit for calculating the value of Non-relativistic energy. The result for the various quantum numbers and the minimal length parameter could be seen in Table 1 and 2. Table 1 and Table 2 showed the non-relativistic energy of LiH and HCl respectively.

Table 1 and 2 informed that the minimal length parameter affects the non-relativistic energy value both LiH and HCl. For the higher quantum number, the higher minimum length parameter increased the non-relativistic energy. The results showed that the non-relativistic energy with minimal length has the higher value than non-relativistic energy without minimal length. When $\alpha=0$, the non-relativistic system was not influenced by minimal length and has very small energy than the system with minimal length. So, we know that the presence of minimal length which investigated by the Schrodinger equation upgraded the relativistic energy system significantly.
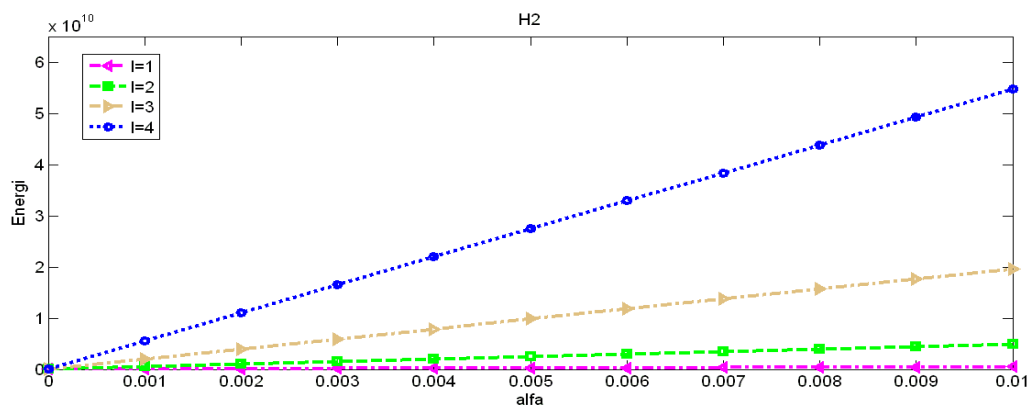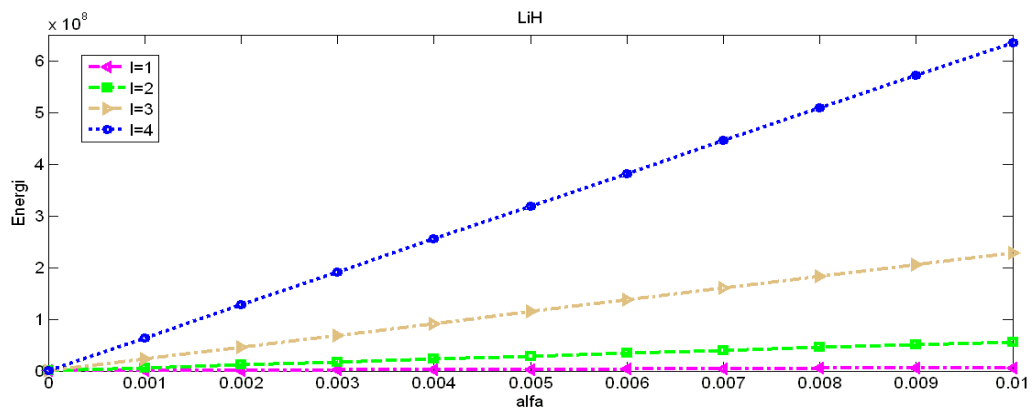
Table 1. The non-relativistic energy of LiH for various quantum numbers (n) ($D = 20287$ (cm^{-1}), $m = 0.8801221$ (amu), $r_0 = 1.5956$ (Å), $a = 1.7998368$).

	E(eV)		
n,l	$\alpha = 0$	$\alpha = 0.005$	$\alpha = 0.01$
2,1	4.423E+02	3.100E+06	6.039E+06
3,1	4.429E+02	3.108E+06	6.056E+06
4,1	4.434E+02	3.116E+06	6.072E+06
5,1	4.440E+02	3.125E+06	6.088E+06
6,1	4.446E+02	3.133E+06	6.105E+06

Table 2. The non-relativistic energy of HCl for various quantum numbers (n) (D = 37255 (cm^{-1}), m = 0.9801045 (amu), $r_{0 =}$ 1.2746 (Å), a = 2.38057042).

n,l	E(eV)		
	$\alpha = 0$	$\alpha = 0.005$	$\alpha = 0.01$
2,1	1.367E+02	2.857E+05	5.495E+05
3,1	1.394E+02	2.991E+05	5.764E+05
4,1	1.420E+02	3.127E+05	6.036E+05
5,1	1.446E+02	3.266E+05	6.314E+05
6,1	1.473E+02	3.407E+05	6.595E+05

The non-relativistic energy also depended on the value of angular momentum. figure 1 and 2 showed the relation of non-relativistic energy H2 and LiH as a function of minimal length parameter for various angular momentum. The figure showed the higher value of minimal length increases the non-relativistic energy both molecules. The effect becomes sensible for higher angular momentum. These results were appropriate with ref. [12,13], that the non-relativistic energy increased with increasing angular momentum, and at higher minimal length values the non-relativistic energy increased visibly.

Figure 1. The non-relativistic energy of H$_2$ as the function of minimal length parameter for arbitrary angular momentum (l).

Figure 2. The non-relativistic energy of LiH as the function of minimal length parameter for arbitrary angular momentum (l).

The results showed that there was the difference value of non-relativistic energy both LiH and HCl. Table 1 and 2 informed that non-relativistic energy of LiH was higher than HCl, and figure 1 showed the different molecules have a slightly different slope. The difference occurred because of the input of molecules parameter. Where, dissociation energy (D), Morse parameter (δ), and mass (m) of HCl were higher than LiH while the equilibrium inter-nuclear distance (r_0) of HCl was lower then LiH.

7. Conclusion

In this research, the Schrodinger equation in the presence of minimal length was soluted. The solutions were obtained for the exponential type potential by using the supersymmetric quantum mechanics. The research was aimed to analyse the minimal length effect on the non-relativistic energy and non-relativistic wave function for diatomic molecules, LiH and HCl. The numerical results were calculated for molecules LiH and HCl by using Matlab. The results showed that the presence of the minimal length influenced the non-relativistic energy system for the various quantum number and angular momentum.

Acknowledgement

References

[1]   Feizi H, Rajabi A A and Shojaei M R 2004 *J. Act. Phys. Polonica B* **42** 2143
[2]   Cari, Suparmi and Marini H 2012 *Indonesian J. App. Phys.* **2** 112
[3]   Farizky N M, Suparmi A, Cari C and Yunianto M 2016 *J. Phys. Conf. Series* **776** 1
[4]   Chen Gang 2004 *J. Phys. Lett. A* **326** 55
[5]   Roy A K 2013 *J. Results in Phys.* **3** 103
[6]   Deta U A, Suparmi and Cari 2013 *Padjadjaran Int. Phys. Symp.* (Bandung: AIP Publishing LLC) 190-193
[7]   Berkdemir C and Han J 2005 *Int. At. Energy Agency (IAEA)* **37** 10
[8]   Bayrak O and Boztosun I 2007 *J. Mol. Struc. TEOCHEM* **802** 17
[9]   Bayrak O and Boztosun I 2006 *J. Phys. A: Math. Gen.* **39** 6955
[10]   Aktas M and Sever R 2004 *Mod. Phys. Lett. A* **19**, 2871
[11]   Filho E D and Ricotta M R 2000 *Phys. Lett. B* **269** 269
[12]   Alimohammadi M and Hassanabadi H 2017 *J. Nucl. Phys. A* **957** 439
[13]   Chabab M, El Batoul A, Lahbas A and Oulne M 2016 *J. Phys. Lett B* **758** 212
[14]   Hassanabadi H, Zarrinkamar S and Maghsoodi E 2012 *J. Phys. Lett. B* **718** 678
[15]   Haouat S 2014 *J. Phys Lett B* **727** 33
[16]   Brau F 1999 *J. Phys. A: Math Gen.* **32** 44
[17]   Nozari K and Pedram P 2010 *Europhysics Lett.* **92** 5
[18]   Suparmi and Cari 2011 *Mekanika Kuantum II* (Surakarta: Universitas Sebelas Maret)
[19]   Saregar A, Suparmi, Cari and Yuliani H 2013 *Int. J. Eng. Sci.* **2** 14
[20]   Suparmi and Cari 2011 *Mekanika Kuantum I* (Surakarta: Universitas Sebelas Maret)