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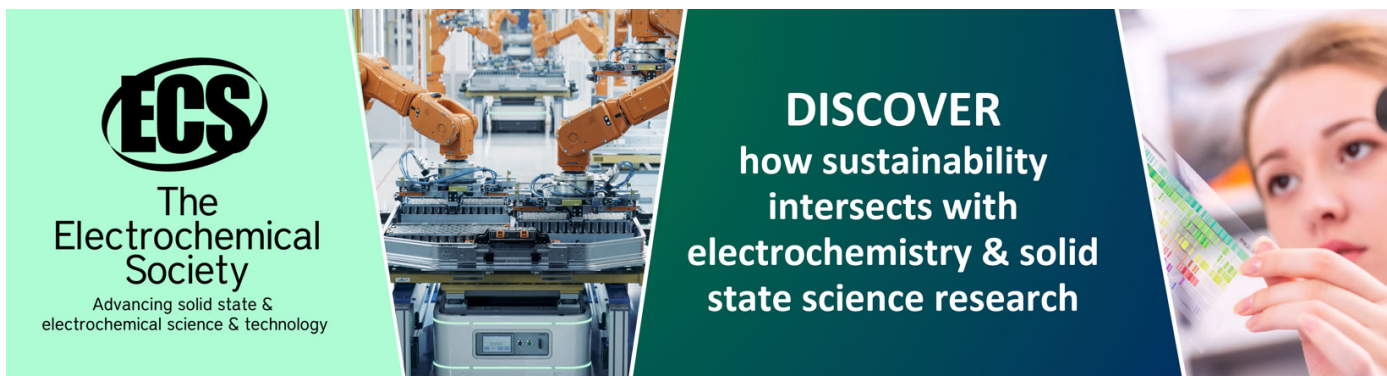
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Analytical solution of Schrödinger equation in minimal length formalism for trigonometric potential using hypergeometry method

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Abstract. The Schrödinger equation has been extended by applying the minimal length formalism for trigonometric potential. The wave function and energy spectra were used to describe the behavior of subatomic particle. The wave function and energy spectra were obtained by using hypergeometry method. The result showed that the energy increased by the increasing both of minimal length parameter and the potential parameter. The energy were calculated numerically using *MatLab*.

1. Introduction

One of the main objectives in quantum mechanics is to obtain exact solutions of the relativistic and non-relativistic equations for some potentials [1]. The Schrödinger equation (SE) is one of non-relativistic quantum mechanics systems that give the exact solutions for shape invariant potential group. The most popular system in SE is a hydrogen atom (Coloumb) and a harmonic oscillator represents two typical examples in quantum mechanics [2-5]. For studying the quantum mechanical systems, it is necessary to study bound states to take the necessary information about the system and also solving scattering states for a system under the effect of a potential [6].

Since many type of research observation, with the other potential and providing analytic solution of the SE. The main problem in this SE is how to solve the SE to obtaining the wave function and the energy spectrum. The wave function and energy spectra were used to describe the behavior of subatomic particle.

Solution of the SE were obtained by inserting the potential into SE, then we used variable separation and substituting the suitable variable to reduce the SE become second-order differential equations such as Hermit function, Legendre, Laguerre, hypergeometric or confluent hypergeometric [7].

The quantum mechanical problems implying a generalized modified, there more piece of evidence for a minimal length (ML) of the order of Planck length [8]. The modified relation includes an MLF or Generalized Uncertainty Principle (GUP) [9]. In this paper we study the SE within the minimal length formalism using trigonometric potential.

The organization of the paper is as follows: in the next section we present an introduction to the MLF which is followed by the description of the SE within the MLF in subsection. In section 3, we explain the hypergeometric method. We use the standard hypergeometric method to obtain accurate numerical approximations to the energy. Finally in section 4, we investigate the result and discussion. The SE has



been extended by applying the MLF for trigonometric cotangent potential. The wave function and energy spectra were obtained by using hypergeometric method. The energy was calculated numerically using *MatLab*.

2. Minimal Length Formalism

The theory of minimal length formalism (MLF) was motivated by Heisenberg algebra [8]. In the framework this formalism, the principal uncertainty can describe as:

$$[X, P] = [x, p] \quad (1)$$

With

$$X = x$$

$$P = [(1 + 2\alpha' p^2 + p^2 \alpha'^2) p^2]^{1/2} \quad (2)$$

and parameter α' is minimal length parameter which is determined from a fundamental theory and varies in the interval $0 \leq \alpha' \leq 1$. For the $\alpha'^2 = 0$ then equation (2) becomes:

$$P^2 = (-\hbar^2 \nabla^2 + 2\alpha' \hbar^2 \nabla^4) \quad (3)$$

2.1. The Schrodinger Equation in MLF

The original schrodinger equation is described as

$$E\psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(r)\psi \quad (4)$$

The Schrodinger equation in minimal length formalism is obtained by inserting the minimal length formalism in equation (3) into equation (4) which is,

$$E\psi = -\frac{\hbar^2}{2m} (\nabla^2 - 2\alpha' \hbar^2 \nabla^4) \psi + V(r)\psi \quad (5)$$

without the presence of minimal length equation (5) reduces to,

$$E^0 \psi = -\frac{\hbar^2}{2m} \nabla_0^2 \psi + V(r)\psi \quad (6)$$

In this condition, eq (6) is just the usual SE with the spectrum energy is given as E^0 . From the equation (6), we obtained ∇_0^2 as

$$\nabla_0^2 = -\frac{2m}{\hbar^2} (E^0 - V(r)) \quad (7)$$

Then by putting equation (7) into equation (8) so we get

$$E\psi = \left[-\frac{\hbar^2}{2m} \nabla^2 + 4\alpha' m (E^0)^2 - 8\alpha' m E^0 V(r) + 4\alpha' m (V(r))^2 \right] \psi + V(r)\psi \quad (8)$$

with

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right) \quad (9)$$

Now, equation (8) is expressed SE with $V_{\text{eff}} = 4\alpha' m (V(r))^2 - 8\alpha' m E^0 V(r) + V(r)$.

3. Hypergeometric Method

Hypergeometric Differential equation was introduced by C.F.Gauß [7] in term

$$z(1-z) \frac{\partial^2 \Phi}{\partial z^2} + (c - (a+b+1)z) \frac{\partial \Phi}{\partial z} - ab\Phi = 0 \quad (10)$$

the solution of hypergeometric is described as,

$${}_2F_1(a, b; c; z) = \Phi_1(z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(1)_n (c)_n} z^n = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{n! (c)_n} z^n \quad (11)$$

To obtain the spectrum energy we set,

$$a = -n \text{ or } b = -n \quad (12)$$

by appropriate substitution of variable eq (8) is reduced to the standard hypergeometric equation, and so we get the spectrum energy and the corresponding wave function.

4. Result

4.1. Solution of SE in MLF

In this section we discussed the trigonometric potential model. By substitution of variable $\psi = \frac{\varphi}{r}$ to equation (8) we get the corresponding wave function

$$\left[\frac{\partial^2}{\partial r^2} - \left(\frac{L(L+1)}{r^2} \right) + \frac{16\alpha' m^2}{\hbar^2} E^0 V(r) - \frac{8\alpha' m^2}{\hbar^2} (V(r))^2 - \frac{2m}{\hbar^2} V(r) - \frac{8\alpha' m^2}{\hbar^2} (E^0)^2 + \frac{2m}{\hbar^2} E \right] \varphi = 0 \quad (13)$$

Equation (13) is the corresponding wave function for SE in MLF.

with the trigonometric potential cotangent,

$$V(r) = V_0 \cot \gamma r \quad (14a)$$

$$V(r)^2 = V_0^2 \cot^2 \gamma r \quad (14b)$$

then by putting eq (14a) and (14b) into eq (13) so we get

$$\left[\frac{\partial^2}{\partial r^2} - \left(\frac{L(L+1)}{r^2} \right) + \frac{16\alpha' m^2}{\hbar^2} E^0 V_0 \cot \gamma r - \frac{8\alpha' m^2}{\hbar^2} (V_0 \cot \gamma r)^2 - \frac{2m}{\hbar^2} V_0 \cot \gamma r - \frac{8\alpha' m^2}{\hbar^2} (E^0)^2 + \frac{2m}{\hbar^2} E \right] \varphi = 0 \quad (15)$$

To obtain the exact solution of equation (15) we used centrifugal approximation

$$\frac{1}{r^2} = \frac{\gamma^2}{\sin^2 \gamma r} = \gamma^2 \csc^2 \gamma r \quad (16)$$

With suitable substitution variable then insert equation (16) to the equation (15) were reduced to differential hypergeometric is given as,

$$\left[\left(z(z-1) \frac{d^2}{dz^2} + (1-2z) \frac{d}{dz} - \left(L(L+1) + \frac{8\alpha' m^2}{\hbar^2 \gamma^2} (V_0)^2 \right) \right) + \left(\frac{V_0 (1-2z) (16E^0 \alpha' m^2 - 2m) - 8\alpha' m^2 V_0^2}{4\gamma^2 z(1-z) \hbar^2} \right) + \left(\frac{2mE - 8\alpha' m^2 E^0}{4\gamma^2 z(1-z) \hbar^2} \right) \right] \varphi = 0 \quad (17)$$

then we set

$$\delta(\delta-1) = \left(L(L+1) + \frac{8\alpha' m^2}{\hbar^2 \gamma^2} (V_0)^2 \right) \quad (18)$$

$$2q = \frac{V_0 (16E^0 \alpha' m^2 - 2m) - 8\alpha' m^2 V_0^2}{\hbar^2 \gamma^2} \quad (19)$$

$$-E' = \frac{2mE - 8\alpha' m^2 E^0}{\hbar^2 \gamma^2} \quad (20)$$

by inserting eq (18), (19), and (20) into the eq (17) we obtained

$$\left[z(1-z) \frac{d^2}{dz^2} + (1-2z) \frac{d}{dz} + \delta(\delta-1) + \frac{2q(1-2z)}{4z(1-z)} - \frac{E'}{4z(1-z)} \right] \varphi = 0 \quad (21)$$

The solution of wave function in eq (21) described as

$$\varphi = z^\alpha (1-z)^\beta f(z) \quad (22)$$

if we put eq (22) into eq (21) we obtained

$$z(1-z) z^\alpha (1-z)^\beta \left[\frac{\alpha(\alpha-1)}{z^2} f(z) - \frac{2\alpha\beta}{z(1-z)} f(z) + \frac{\beta(\beta-1)}{(1-z)^2} f(z) + \frac{2\alpha}{z} f'(z) - \frac{2\beta}{(1-z)} f'(z) + f''(z) \right] + (1-2z) z^\alpha (1-z)^\beta \left[\frac{\alpha}{z} f(z) - \frac{\beta}{(1-z)} f(z) + f'(z) \right] + \left(\delta(\delta-1) - \frac{4\alpha^2}{4z} - \frac{4\beta^2}{4(1-z)} \right) z^\alpha (1-z)^\beta f(z) = 0 \quad (23)$$

by dividing Eq (23) with $z^\alpha (1-z)^\beta$ then eq (23) can reduce as which is

$$z(1-z) f''(z) + [2\alpha + 1 - (2\alpha + 2\beta + 2)z] f'(z) - \left[(\alpha + \beta + \frac{1}{2})^2 - \left(\delta - \frac{1}{2} \right)^2 \right] f(z) = 0 \quad (24)$$

Now, we can see from eq (24) the parameter hypergeometry as,

$$\begin{aligned} a' &= \left(\delta - \frac{1}{2} \right) + \left(\alpha + \beta + \frac{1}{2} \right) = (\alpha + \beta + \delta) \\ b' &= - \left(\delta - \frac{1}{2} \right) + \left(\alpha + \beta + \frac{1}{2} \right) = (\alpha + \beta + \delta - 1) \\ c' &= 2\alpha + 1 \end{aligned} \quad (25)$$

where

$$4\alpha^2 = (-2q + E') \quad (26)$$

$$4\beta^2 = (2q + E') \quad (27)$$

The energy of SE in MLF is obtained by inserting eq (18), (19), and (20) into eq (26) and eq (27)

$$E = \frac{\hbar^2 \gamma^2}{2m} \{ L(L+1) + n^2 \} - V_0 (8\alpha' B_m E^0 + 1) - 4\alpha' B_m E^{0^2} \quad (28)$$

E is energy of the SE in MLF (eV) and E^0 is ground state energy of the SE in MLF (eV) which is given as

$$E^0 = \left[\frac{\hbar^2}{2B_m} \{ L(L+1) \gamma^2 + \gamma^2 n^2 \} + V_0 \right] \quad (29)$$

4.2. Discussion

The SE which is based in MLF for a trigonometric potential cotangent was solved by using the hypergeometry method. The non-relativistic energy spectrum and the wave function were obtained by using hypergeometry method. In Table 1, we can see the result for sub-atomic particle (electron) by inserting $m = 9.11 \times 10^{-31} \text{ kg}$ and a variation of n -number. For natural units, $\hbar = 1$ and $V_0 = 5 \text{ eV}$, and α is minimal length parameter which was determined from interval $0 \leq \alpha' \leq 1$, so the value of energy was given in Table 1.

Table 1. Energy of SE in MLF.

ENERGI (eV)											
(n.l)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
(1.0)	1E+10	2E+10	2E+10	2.4E+10	2.9E+10	3.3E+10	3.8E+10	4.2E+10	4.7E+10	5.1E+10	5.6E+10
(2.0)	4E+10	1E+11	2E+11	2.6E+11	3.3E+11	4E+11	4.7E+11	5.4E+11	6.1E+11	6.8E+11	7.6E+11
(3.0)	1E+11	5E+11	8E+11	1.2E+12	1.5E+12	1.9E+12	2.3E+12	2.6E+12	3E+12	3.3E+12	3.7E+12
(4.0)	2E+11	1E+12	2E+12	3.6E+12	4.7E+12	5.9E+12	7E+12	8.1E+12	9.3E+12	1E+13	1.2E+13
(5.0)	3E+11	3E+12	6E+12	8.6E+12	1.1E+13	1.4E+13	1.7E+13	2E+13	2.3E+13	2.5E+13	2.8E+13

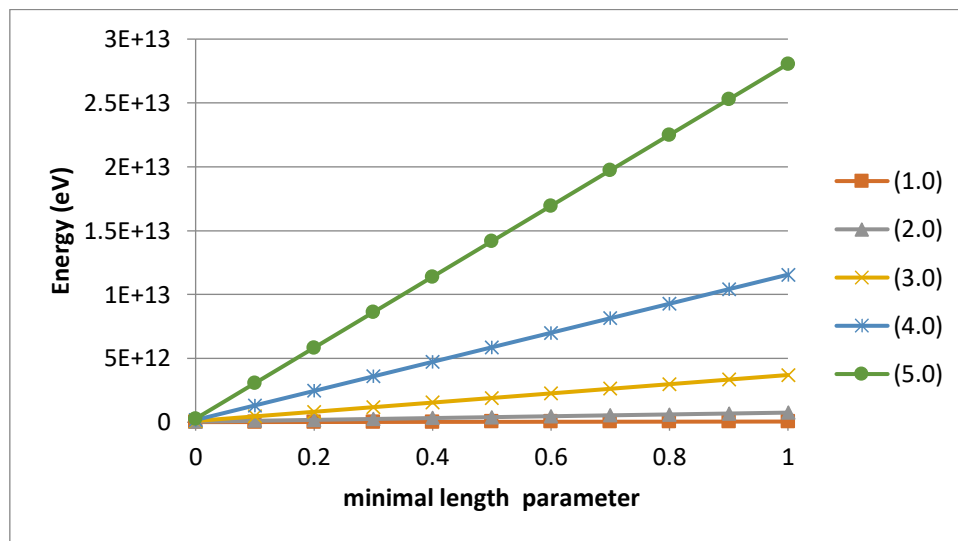


Figure 1. The graph of Energy spectrum SE in MLF.

5. Conclusion

We have studied the SE in MLF and have obtained the energy of this problem. The energy is calculated numerically using *MatLab*. The graph of energy for (n,l) with various of n number was described in Figure 1. The x-axis is parameter MLF and the y-axis is the Energy spectrum of SE in MLF (eV). The result showed that the energy increased by the increasing of both the minimal length parameter and the potential parameter. Based on the result of nonrelativistic energy is increased due to the increasing quantum number value and the increasing potential constant.

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