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A New Location Method for Reaction–diffusion Process

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Abstract. A new location method is proposed. In this method, the trial function is such chosen that it satisfies the governing equation to some extent. An illustrating example of reaction-diffusion process is given to reveal the effectiveness and convenience of the new method. The results show the obtained solutions are valid for the whole solution domain and are of high accuracy.

1. Introduction
In this paper we consider the solution of a reaction–diffusion process governed by the following nonlinear ordinary differential equation[1,2]:

\[ y'(x) + y^n(x) = 0, \ 0 < x < 1, \ n > 0 \]  
(1)

with boundary conditions:

\[ y(0) = y(1) = 0 \]  
(2)

where \( y(x) \) represents the steady-state temperature for the corresponding reaction–diffusion equation with the reaction term \( y^n \); \( n \) is the power of the reaction term (heat source).

Reaction–diffusion process arises in many fields, such as chemical engineering[3,4], neural networks[5,6,7], predator-prey system[8], electrical reaction-diffusion chain[9]. The discussed equation has no small parameter, so the traditional perturbation methods become invalid [10]. Recently various different analytical methods were applied to nonlinear equations where traditional approaches fail, such as the homotopy perturbation method [11-23], the variational iteration method[24-26], exp-function method[27], and variational methods[28-29], a complete review is available on Ref.[30]. This problem was studied by Lesnic using Adomian method[1], by Mo using variational method[2], and by Ganji and Sadighi using He’s homotopy perturbation method[31]. In this paper, a new location method is suggested to solve effectively the discussed problem. The obtained results show that the method is very effective and simple.

2. New Location Method
In various variational methods, a trial function is such chosen that it satisfies the boundary conditions. For the discussed problem, considering the boundary conditions, Eq.(2), we can choose the following trial function

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\[ y(x) = ax(1-x)(1 + \sum_{i=1}^{m} b_i x^i) \]  

(3)

where \(a\) and \(b_i\) are unknown constants to be further determined. To illustrate the disadvantage of such a choice, we consider the simplest case, i.e,

\[ y(x) = ax(1-x) \]  

(4)

From the trial function, Eq.(4), we have

\[ y''(x) = -2a \]  

(5)

which implies to

\[ y''(0) = y''(1) = -2a \]  

(6)

Eq.(6) contradicts the governing equation, Eq.(1). From Eq.(1), and considering the boundary conditions, Eq.(2), we obtain

\[ y'(0) = y''(0) = 0, \quad y'(1) = y''(1) = 0. \]  

(7)

According to Eq.(7), we can choose traditionally a trial function for \(y''(x)\) in the form

\[ y''(x) = ax(1-x)(1 + \sum_{i=1}^{m} b_i x^i) \]  

(8)

We also consider the simplest case:

\[ y''(x) = ax(1-x) \]  

(9)

leading to the results

\[ y^{(4)}(0) = y^{(4)}(1) = -2a \]  

(10)

Differentiating Eq.(1) with respect to \(x\) twice, and considering the boundary conditions, Eq.(2), we have

\[ y^{(4)}(0) = y^{(4)}(1) = 0 \]  

(11)

Thus Eq.(11) contradicts again the governing equation. In order to satisfy the governing equation, Eq.(1), to the greatest extent, a trial function can be chosen in the form

\[ y''(x) = ax^n(1-x)^n + bx^{2n}(1-x)^{2n} \]  

(12)

where \(a\) and \(b\) are unknown constants to be further determined. Integrating Eq.(13) twice with respect to \(x\), we have

\[ y'(x) = \frac{a}{n+1} x^{n+1}(1-x)^{n+1}(1-x+n) + \frac{b}{2n+1} x^{2n+1}(1-x)^{2n+1}(1-x+2n) + c \]  

(13)

and
\[ y(x) = a \sum_{i=0}^{n} C_n^{i} \times (-1)^i \frac{1}{(n + i + 1)(n + i + 2)} x^{n+i+2} \]
\[ + b \sum_{i=0}^{2n} C_n^{i} \times (-1)^i \frac{1}{(2n + i + 1)(2n + i + 2)} x^{2n+i+2} + cx + d \]

where \( c \) and \( d \) are integral constants.

Incorporating the boundary condition, \( y(0) = 0 \), results in \( d = 0 \), thus Eq.(15) reduces to

\[ y(x) = a \sum_{i=0}^{n} C_n^{i} \times (-1)^i \frac{1}{(n + i + 1)(n + i + 2)} x^{n+i+2} \]
\[ + b \sum_{i=0}^{2n} C_n^{i} \times (-1)^i \frac{1}{(2n + i + 1)(2n + i + 2)} x^{2n+i+2} + cx \]

The constant \( c \) can be determined by incorporating the boundary condition, \( y(1) = 0 \).

Substituting Eq.(16) into Eq. (1), we can obtain the following residual

\[ R(x) = y'(x) + y''(x) \]  

In order to determine the unknown constants in Eq.(16), locating at \( x = 0.5 \) and \( x = 0.25 \), we have

\[ \begin{cases} R(0.5) = 0 \\ R(0.25) = 0 \end{cases} \]

In order to illustrate the effectiveness and convenience of the method, we consider the following different cases.

**Case 1** \( n = 2 \)

Under such case, Eq.(16) reduces to

\[ y(x) = \left( \frac{1}{30} x^6 - \frac{1}{10} x^5 + \frac{1}{12} x^4 - \frac{1}{60} x \right) a \]
\[ + \left( \frac{1}{90} x^6 - \frac{1}{18} x^5 + \frac{3}{28} x^4 - \frac{2}{21} x^3 + \frac{1}{30} x^2 - 7.9365 \times 10^{-4} x \right) b \]

Eq.(18) becomes

\[ \begin{cases} \frac{1}{16} a + \frac{1}{256} b + (-\frac{11}{1920}) a - 2.9917 \times 10^{-4} b)^2 = 0 \\ \frac{9}{256} a + \frac{81}{65536} b + (-0.0039 a - 1.9465 \times 10^{-4} b)^2 = 0 \end{cases} \]

Solving Eq.(20), we have

\[ \begin{cases} a = -0.0122 \times 10^5 \\ b = -0.0161 \times 10^6 \end{cases} \]

We, therefore, obtain the following approximate solution
Fig. 1 illustrates the remarkable accuracy of the approximate solution, Eq.(22). It maximal error is 0.0924%.

\[
y(x) = -0.0122 \times 10^5 \times [x(1-x)] - 0.0161 \times 10^5 \times (x^2 (1-x)^2) + \\
\left[ \left( \frac{1}{30} - \frac{1}{10} x + \frac{1}{12} x^4 - \frac{1}{60} x \right) \times (-0.0122 \times 10^5) + \left( \frac{1}{90} x^6 - \frac{1}{18} x^9 \right) \right] \\
+ \frac{3}{28} x^8 - \frac{2}{21} x^7 + \frac{1}{30} x^6 - 7.9365 \times 10^{-4} x \times (-0.0161 \times 10^5) ]^2
\]  

(21)

Case 2  \( n = 3 \)

Proceeding the same way as illustrated in Case 1, we can easily identify the values of \( a \) and \( b \):

\[
\begin{align*}
\begin{cases}
a = -0.0118 \times 10^5 \\
b = -0.0132 \times 10^7 
\end{cases}
\end{align*}
\]  

(22)

Its approximate solution reads

\[
y(x) = -0.0118 \times 10^5 \times \left( - \frac{1}{56} x^8 + \frac{1}{14} x^7 - \frac{1}{10} x^6 + \frac{1}{20} x^5 \right) - 0.0194 \times 10^5 \times \left( \frac{1}{182} x^{14} - \frac{1}{26} x^{13} + \frac{5}{44} x^{12} \right) \\
- \frac{2}{11} x^{11} + \frac{1}{6} x^{10} - \frac{1}{12} x^9 + \frac{1}{56} x^8 \right) \left( \frac{1}{280} \times (-0.0118 \times 10^5) + 4.1625 \times 10^5 \times (-0.0194 \times 10^5) \right) x
\]

(23)

which is of excellent accuracy, see Fig.2. Its accuracy arrives at as high as 0.1404%.
Case 3  $n = 4$

Under such case, solving Eq.(18), we have

\[
\begin{align*}
    a &= -0.0229 \times 10^5 \\
    b &= -0.0194 \times 10^8
\end{align*}
\]

We obtain the following approximate solution

\[
y(x) = -0.0229 \times 10^5 \times (\frac{1}{90} x^{10} - \frac{1}{18} x^9 + \frac{3}{28} x^8 - \frac{2}{21} x^7 + \frac{1}{30} x^6 ) \\
-0.0194 \times 10^8 \times (\frac{1}{306} x^{18} - \frac{1}{34} x^{17} + \frac{7}{60} x^{16} - \frac{4}{15} x^{15} + \frac{5}{13} x^{14} - \frac{14}{39} x^{13} + \frac{7}{33} x^{12} - \frac{4}{55} x^{11} ) \\
+ \frac{1}{90} x^{10} - [7.9365 \times 10^{-4} \times (-0.0229 \times 10^5) + 2.2853 \times 10^{-6} \times (-0.0194 \times 10^8)]x^{10}
\]

From Fig.3, we can see that the approximate solution, Eq.(26), agrees remarkably well with the exact solution, its maximal error is only 0.5629%.
Case 4 other cases

By same manipulation as illustrated above, we have

\[
\begin{align*}
   a &= -0.0051 \times 10^6 \\
   b &= -0.0313 \times 10^9 \\
   &\quad \text{for } n = 5
\end{align*}
\]  \hspace{1cm} (26)

\[
\begin{align*}
   a &= -0.0120 \times 10^6 \\
   b &= -0.0511 \times 10^9 \\
   &\quad \text{for } n = 6
\end{align*}
\]  \hspace{1cm} (27)

\[
\begin{align*}
   a &= -0.0028 \times 10^7 \\
   b &= -0.0836 \times 10^{11} \\
   &\quad \text{for } n = 7
\end{align*}
\]  \hspace{1cm} (28)

Comparison of approximate solutions with exact ones is given in Figs. 4-6.
Fig. 4 Comparison of approximate solution with exact one when $n = 5$. Maximal error is 0.8338\%.

Fig. 5 Comparison of approximate solution with exact one when $n = 6$, Maximal error is 1.1187\%.
7.

3. Conclusion
We suggest a new location method, a trial function is such chosen that it satisfies to some extent the
governing equation, the solution procedure is of utter simplicity, while the results are of remarkable
accuracy. The method can be easily extended to other nonlinear problems without any difficulties.

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