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To cite this article: Z-J Zhou et al 2008 J. Phys.: Conf. Ser. 96 012130

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An Improved Fuzzy Kalman Filter for State Estimation of Nonlinear Systems

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Abstract. The extended fuzzy Kalman filter (EFKF) is developed recently and used for state estimation of the nonlinear systems with uncertainty. Based on extension of the orthogonality principle and the extended fuzzy Kalman filter, an improved fuzzy Kalman filters (IFKF) is proposed in this paper, which is more applicable and can deal with the state estimation of the nonlinear systems better than the EFKF. A simulation study is provided to verify the efficiency of the proposed method.

1. Introduction

There exist a lot of nonlinear systems with fuzzy uncertainty, so many studies have focused on the fuzzy theory and its application [1]-[7].

The traditional extended Kalman filter (EKF) is implemented using the probabilistic techniques, which can provide an accurate solution to state estimation problem based on the appropriate initialization of the uncertainty matrices. However, EFK is not applicable to nonlinear systems with fuzzy uncertainty. On the basis of the fuzzy logic [1] and the Kalman filter, the extended fuzzy Kalman filter (EFKF) is proposed to tackle fuzzy uncertainty [7].

Though the EFKF can deal with the fuzzy uncertainty, it seems to be inapplicable when there exist model mismatches in the nonlinear systems. Fortunately, based on the orthogonality principle and the EKF, Zhou et al. propose the strong tracking filtering (STF), which can solve the above problems [8]. However, the STF is also on the basis of the probabilistic theory and can not deal with the fuzzy uncertainty. So it is necessary to combine the orthogonality principle and the EFKF to propose a new method which is named after the improved fuzzy Kalman filter (IFKF) to estimate the states.

This paper is organized as follows. In Section 2, some preliminaries about the possibility distribution are briefly reviewed. The original EFKF of the nonlinear systems is given in Section 3. In Section 4, the IFKF for state estimation of the nonlinear systems is proposed. The effectiveness of the IFKF method is demonstrated in Section 5. Finally, Section 6 draws the conclusions.

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2. Preliminaries
A fuzzy variable $p$, defined over the universe of discourse $P$, using a trapezoidal possibility distribution $\pi_p(p)$, such as Fig. 1, which means

$$\pi_p(p) = \begin{cases} 
1 & \forall p \in [p_2, p_3] \\
0 & \forall p \notin [p_1, p_4] 
\end{cases} \quad (1)$$

Fig. 1. Possibility distribution

Eqs. (2) to (5) are defined as the expectation, the area of the distribution, the center of gravity and the uncertainty of the distribution, respectively [7].

$$E \{ p \} = \Pi (p_1, p_2, p_3, p_4) \quad (2)$$

$$\gamma_p = \int \pi_p(p) dp \quad (3)$$

$$\tilde{p} = C \{ p \} = \frac{\int p \pi_p(p) dp}{\gamma_p} \quad (4)$$

$$U \{ p \} = C \{ (p - \tilde{p})^2 \} = \frac{\int (p - \tilde{p})^2 \pi_p(p) dp}{\gamma_p} \quad (5)$$

In case of a multivariable system, the similar definitions are given in [7].

3. The extended fuzzy Kalman filter (EFKF) of nonlinear systems
Suppose that the process and the observation models of nonlinear systems are described as follows:

$$\begin{cases} 
    x(k+1) = f(k, u(k), x(k)) + v(k+1) \\
    y(k+1) = h(k+1, x(k+1)) + w(k+1)
\end{cases} \quad (6)$$

where $k \geq 0$ is the discrete time variable. $x$, $u$ and $y$ the state, the input and the output with the appropriate dimensions respectively. $f$ is the nonlinear system function and $h$ the nonlinear measurement function. $v(k+1)$ and $w(k+1)$ are the process noise and measurement noise. $\hat{x}(k | k)$ and $\hat{x}(k+1 | k)$ denote the estimation and one step prediction of $x(k)$ respectively. $\hat{y}(k+1)$ denotes the estimation of $y(k+1)$. $\hat{x}(k | k), v(k+1)$ are independent, and $\hat{x}(k+1 | k), w(k+1)$ are also independent. Moreover, these variables satisfy the following possibility distributions.
\[
E\{v(k+1)\} \sim \Pi(v_1(k+1), v_2(k+1), v_3(k+1), v_4(k+1))
\]
\[
\tilde{v}(k+1) = 0
\]
\[
U\{v(k+1)\} = Q(k+1)
\]
\[
E\{w(k+1)\} \sim \Pi(w_1(k+1), w_2(k+1), w_3(k+1), w_4(k+1))
\]
\[
\tilde{w}(k+1) = 0
\]
\[
U\{w(k+1)\} = R(k+1)
\]
\[
E\{\hat{x}(k|k)\} \sim \Pi(\hat{x}_1(k|k), \hat{x}_2(k|k), \hat{x}_3(k|k), \hat{x}_4(k|k))
\]
\[
U\{\hat{x}(k|k)\} = P(k|k)
\]
\[
E\{\hat{y}(k+1)\} \sim \Pi(\hat{y}_1(k+1), \hat{y}_2(k+1), \hat{y}_3(k+1), \hat{y}_4(k+1))
\]
\[
U\{\hat{y}(k+1)\} = S(k+1)
\]

where \(P(k|k)\) and \(P(k+1|k)\) are the estimation and one step prediction of distribution uncertainty respectively. \(S(k+1)\) denotes the uncertainty distribution of the observation’s estimation.

Then the main steps of the EFKF proposed in [7] can be obtained.

1) Prediction:
\[
\hat{x}_i(k+1|k) = f(k, u(k), \hat{x}_i(k|k)) + v_i(k+1)
\]
\[
P(k+1|k) = F(k)P(k|k)F^T(k) + Q(k+1)
\]
\[
S(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1)
\]

where \(l = 1, \ldots 4\) and
\[
F(k) = \frac{\partial f(k, u(k), x(k))}{\partial x}\bigg|_{x(k)=c[\hat{x}(k|k)]}
\]
\[
H(k+1) = \frac{\partial h(k+1, x(k+1))}{\partial x}\bigg|_{x(k+1)=c[\hat{x}(k+1|k)]}
\]

2) Observation and matching:
When the observation \(y(k+1)\) is obtained, a possibility criterion to accept or reject the observation is
\[
\pi_y\{y(k+1)\} \geq \beta
\]
where \(\beta\) is a confidence value which is given by expert knowledge and it shows the uncertainty degree of the observation.

3) Correction:
\[
\gamma_i(k+1) = y_i(k+1) - h(k+1, \hat{x}_i(k+1|k)) - w_i(k+1)
\]
\[
\hat{x}_i(k+1|k+1) = \hat{x}_i(k+1|k) + K(k+1)\gamma_i(k+1)
\]
\[
P(k+1|k+1) = \left[ I - K(k+1)H(k+1) \right] P(k+1|k)
\]

where \(\gamma_i(k+1)\) is the estimation error of the observation and
The filter gain $K(k+1)$ can be obtained by minimization $P(k+1|k+1)$. Thus there is:

$$K(k+1) = P(k+1|k)H^T(k+1)S^{-1}(k+1)$$

(21)

4. An improved fuzzy Kalman filter (IFKF) of nonlinear systems

4.1. Extension of orthogonality principle

Based on the orthogonality principle [8], its extension for the IFKF is given:

$$C\left[\hat{x}(k+1|k+1) - x(k+1)\right]\left[\hat{x}(k+1|k+1) - x(k+1)\right]^T = \min$$

(22)

$$Dep\left[\gamma(k+1+j)\gamma^T(k+1)\right] = 0$$

(23)

Remark 1. For the EFKF, Eq.(22) is naturally satisfied. Eq.(23) requires that the errors at different time maintain the orthogonality. It means that when model mismatches exist, the gain matrix should be on-line adjusted such that Eq.(23) can be satisfied.

4.2. An improved fuzzy Kalman filter (IFKF) of the nonlinear system

According to the extension of the orthogonality principle, $K(k+1)$ should be adjusted on-line in order to satisfy Eq.(23). Then the following regulations can be obtained:

$$P(k+1|k) = \lambda(k+1) F(k) P(k|k) F^T(k) + Q(k+1)$$

(24)

$$K(k+1) = P(k+1|k)H^T(k+1)\left[H(k+1)P(k+1|k)H^T(k+1) + R(k+1)\right]^{-1}$$

(25)

where $\lambda(k+1)$ is the fading factor. So the main goal is to determine the fading factor. Using the algorithm given in [8], $\lambda(k+1)$ can be determined as follows:

$$\lambda(k+1) = \frac{tr\left[N(k+1)\right]}{tr\left[M(k+1)\right]}$$

(26)

where $tr$ denotes the trace of the matrix and

$$M(k+1) \propto H(k+1)F(k)P(k|k)F^T(k)H^T(k+1)$$

(27)

$$N(k+1) \propto V_0(k+1) - R(k+1) - H(k+1)Q(k+1)H^T(k+1)$$

(28)

$$V_0(k+1) = \begin{cases} C\left\{\gamma(1)\gamma^T(1)\right\}, & k = 0 \\ \rho V_0 (k) + C\left\{\gamma(k+1)\gamma^T(k+1)\right\}, & k \geq 1 \end{cases}$$

(29)

$$\lambda(k+1) = \begin{cases} \lambda_0, & 0 \leq \lambda_0 < 1 \\ 1, & \lambda_0 \geq 1 \end{cases}$$

(30)

where $0 \leq \rho \leq 1$ is a forgetting factor. One often selects $\rho = 0.95$ and

$$\lambda_0 = \frac{tr\left[N(k+1)\right]}{tr\left[M(k+1)\right]}$$

(31)

Thus we give the following IFKF algorithm.

Algorithm 1. IFKF algorithm

Step 1: When $k = 0$, set the initial one step prediction values $P(1|0)$ and $\hat{x}(1|0)$.
Step 2: When the new observation comes, Eq.(16) is firstly checked. If it is satisfied, Eqs.(17) to (20) and (31) are used to correct the initial values. Otherwise, the next new observation is waited for.

Step 3: Eqs.(11), (24) and (13) to (15) are used to calculate the new prediction values.

Step 4: Let \( k + 1 \rightarrow k \), go to step 2.

Due to the introduction of the fuzzy uncertainty, it is natural that the estimated results of the EFKF and the IFKF are both the four points of the corresponding trapezoidal distribution. Now two criterions to check the estimation algorithm are given:

1. The real values must be located in the trapezoidal distribution zone if the estimation algorithm is usable.
2. The trapezoidal distribution zone given by the estimation algorithm is smaller, the algorithm is more effective.

5. Simulation studies

A modified model of the three-tank experimental setup—DTS200, which is made by the Automatic Company of Amira in Germany, can be modeled by:

\[
\begin{align*}
A\dot{h}_1 &= Q_1 - Q_{13} \\
A\dot{h}_2 &= Q_3 + Q_{12} - Q_{20} \\
A\dot{h}_3 &= Q_{13} - Q_{12}
\end{align*}
\]

with

\[
\begin{align*}
Q_{13} &= a_1 s \text{sgn}(h_1 - h_3) \sqrt{2g|h_1 - h_3|} \\
Q_{12} &= a_2 s \text{sgn}(h_1 - h_3) \sqrt{2g|h_1 - h_2|} \\
Q_{20} &= a_3 \sqrt{2gh_2}
\end{align*}
\]

where \( a_i \) and \( h_i \) (\( i = 1, 2, 3 \)) are the outflow coefficients and the liquid level (m), \( \text{sgn}(z) \) the sign of the argument \( z \), \( s \) the section of connection pipe (m²), \( A \) the section of tank (m²), \( Q_1 \) and \( Q_2 \) the supplying flow rates (m³/s), \( T \) the sampling period. The nominal technical data of the setup is given in Table 1.

Define

\[
\begin{bmatrix}
Q_1 \\
Q_3 \\
Q_2
\end{bmatrix}, \quad
\begin{bmatrix}
h_1 \\
h_2 \\
h_3
\end{bmatrix}^T, \quad
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}^T
\]

The noises obey the following possibility and the center of \( w \) is not at zero.

\[
\begin{bmatrix}
E\{w\} & \sim \Pi(0.0001, 0.0, 0.0, 0.0001) \\
E\{v\} & \sim \Pi(-0.00016, -0.00004, -0.00004, 0.00008)
\end{bmatrix}
\]

Then DTS2000 can be transformed to the nonlinear model as shown in Eq.(6). Assume that DTS2000 works normally and \( \beta = 0.1 \). Fig. 2 exhibits the state estimation results by the IFKF, which shows that the IFKF can estimate the states of the nonlinear system accurately. In all the figures of this paper, solid and dot lines denote the true and estimation values of liquid levels, respectively.

In order to test the robustness of the IFKF, assume there exist some model mismatches, i.e., the parameters of the state estimator are same as ones given in Table 1, but the parameters of the real plant become \( a_1 = 0.48 \), \( a_2 = 0.62 \), \( a_3 = 0.52 \). Fig. 3 and Fig. 4 exhibit the simulation results of the IFKF and the EFKF, respectively. From these two figures and the above checking criterions, we can see that the proposed IFKF has the strong robustness against the model mismatches, but the EFKF is not usable in this case.
Table 1. Some technical parameters of DTS2000

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.154 m²</td>
</tr>
<tr>
<td>s</td>
<td>0.00005 m²</td>
</tr>
<tr>
<td>Q₁</td>
<td>0.000026 m³/s</td>
</tr>
<tr>
<td>Q₂</td>
<td>0.000008 m³/s</td>
</tr>
<tr>
<td>g</td>
<td>9.8 cm/s²</td>
</tr>
<tr>
<td>a₁</td>
<td>0.5</td>
</tr>
<tr>
<td>a₂</td>
<td>0.6</td>
</tr>
<tr>
<td>a₃</td>
<td>0.5</td>
</tr>
<tr>
<td>Tₛ</td>
<td>1 s</td>
</tr>
</tbody>
</table>

6. Conclusions

In this paper, an improved fuzzy Kalman filter is proposed to estimate the states of the nonlinear systems with uncertainty on the basis of the extension of the orthogonality principle and the EFKF. Computer simulation results illustrate that, compared with the EFKF, the IFKF has the better estimation ability and stronger robustness.

![Fig. 2. State estimation by the IFKF in the normal condition](image1)

![Fig. 3. Simulation results of the IFKF under the model mismatches](image2)
Fig. 4. Simulation results of the EFKF under the model mismatches

7. References