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Coupled Nonlinear Oscillator arising in Two-Strand Yarn Spinning

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Abstract. The variational iteration method is applied to a coupled nonlinear oscillator arising in two-strand yarn spinning. The critical condition for the resonance of the system is obtained.

1. Introduction
Two-strand or Sirospun yarns[1-5] are produced on a conventional ring frame by feeding two rovings, drafted simultaneously, into the apron zone at a predetermined separation. Emerging from the nip point of the front rollers, the two strands are twisted together to form a two-ply structure(see Fig. 1). Compared with air-jet spinning [6], air-vortex spinning [7], electrospinning[8,9] or other spinning processes[10,11], the mechanical character of the produced two-strand yarn can be dramatically improved over its parent yarns.

2. Nonlinear Dynamical Model
Assume that the convergence point (equilibrium position) leaves to an instantaneous position (see Fig.2), and the distance $x$ and $y$ are measured from the equilibrium position, the motion equations in $x$- and $y$-directions can be expressed as

$$m\frac{d^2x}{dt^2} + F_x \cos \alpha - F_z \cos \beta = 0,$$

$$m\frac{d^2y}{dt^2} + F_y \sin \alpha + F_z \sin \beta - F = 0.$$

Here $m$ is total mass of a fixed control volume, the control volume is chosen in such a way that mass center locates on the convergent point ($O$) of the two strands.

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Fig. 1 Two-strand Yarn Spinning

Fig. 2 The dynamical illustration of two-strand spun.
Expanding the trigonometric functions into series of x and y, we can obtain a coupled nonlinear oscillator [2,5].

In this paper we consider the following special case:

\[
\begin{align*}
\dot{x} + \omega_1^2 x + \epsilon_1 y^2 x &= 0 \\
\dot{y} + \omega_2^2 y + \epsilon_2 x^2 y &= 0,
\end{align*}
\]

with the initial condition \( x(0) = A, \dot{x}(0) = 0; \ y(0) = B, \dot{y}(0) = 0 \).

The physical interpretation of the coefficients are available in Refs. [2]. In our study \( \epsilon_1 \) and \( \epsilon_2 \) do not need to be small [12]. We will apply the variational iteration method to solve Eq. (1).

Recently, the variational iteration method first proposed by He [13-15] has been successfully applied to various kinds of nonlinear problems [16-21].

3. Variational iteration method

Applying the variational iteration method [13-15], we can easily construct the following iteration formulations:

\[
\begin{align*}
x_{n+1} &= x_n + \frac{1}{\omega_1} \int_0^t \sin \omega_1 (s-t) \left[ \frac{d^2 x}{ds^2} + \omega_1^2 x + \epsilon_1 y^2 x \right] ds, \\
y_{n+1} &= y_n + \frac{1}{\omega_2} \int_0^t \sin \omega_2 (s-t) \left[ \frac{d^2 y}{ds^2} + \omega_2^2 y + \epsilon_2 x^2 y \right] ds.
\end{align*}
\]

We begin with the initial solutions:

\[
\begin{align*}
x_0 &= A \cos \Omega_1 t, \\
y_0 &= B \cos \Omega_2 t,
\end{align*}
\]

where \( \Omega_1, \Omega_2 \) are the frequencies in the x and y directions, respectively.

According to the iteration formulations (4) and (5), we obtain

\[
x = A \cos \Omega_1 t + \frac{1}{\omega_1} \int_0^t \sin \omega_1 (s-t) \left[ A \left( \omega_1^2 - \Omega_1^2 \right) \cos \Omega_1 s + \epsilon_1 AB^2 \cos \Omega_2 s \cos \Omega_2 s \right] ds
\]

\[
= A \cos \Omega_1 t + A \left( \omega_1^2 - \Omega_1^2 \right) \cos \Omega_1 t + \epsilon_1 AB^2 \cos \Omega_2 t
\]

\[
+ \frac{1}{4} \left[ \frac{1}{\omega_1^2 - (2\Omega_1 + \Omega_2)^2} + \frac{1}{\omega_1^2 - (2\Omega_1 - \Omega_2)^2} \right] \cos \Omega_1 t
\]

\[
\quad - \frac{\epsilon_1 AB^2}{4} \left[ \frac{2 \cos \Omega_1 t}{\omega_1^2 - \Omega_1^2} \cos (2\Omega_1 + \Omega_2) t + \frac{\cos (2\Omega_1 - \Omega_2) t}{\omega_1^2 - (2\Omega_1 - \Omega_2)^2} \right]
\]

\[
(8)
\]
\[ y_1 = B \cos \Omega t + \frac{1}{\omega_2} \int_0^s \sin \omega_2 (s-t) \left\{ B \left( \omega_1^{-2} - \omega_2^{-2} \right) \cos \Omega s + \varepsilon_2 B A^2 \cos \Omega s \cos^2 \Omega s \right\} ds \]

\[ = B \cos \omega_2 t + B \left( \omega_1^{-2} - \omega_2^{-2} \right) \cos \omega_2 t - \cos \omega_1 t + \frac{\varepsilon_2 B A^2}{2} \left( \cos \omega_1 t - \cos \omega_2 t \right) \]

\[ + \frac{\varepsilon_2 B A^2}{4} \frac{\cos \omega_1 t - \cos \left( 2\Omega_1 + \Omega_2 \right) t}{\omega_1^{-2} - (2\Omega_1 + \Omega_2)^{-2}} + \frac{\varepsilon_2 B A^2}{4} \frac{\omega_1^{-2} \omega_2^{-2}}{\omega_2^{-2} - (2\Omega_2 - \Omega_1)^{-2}} \]

\[ = \left\{ B + \frac{\varepsilon_2 B A^2}{2 \left( \omega_1^{-2} - \omega_2^{-2} \right)} + \frac{\varepsilon_2 B A^2}{4} \left[ \frac{1}{\omega_1^{-2} - (2\Omega_1 + \Omega_2)^{-2}} + \frac{1}{\omega_2^{-2} - (2\Omega_1 - \Omega_2)^{-2}} \right] \right\} \cos \omega_2 t \]

\[ - \frac{\varepsilon_2 B A^2}{4} \left[ \frac{2 \cos \Omega_1 t}{\omega_1^{-2} - \omega_2^{-2}} + \frac{\cos \left( 2\Omega_1 + \Omega_2 \right) t}{\omega_1^{-2} - (2\Omega_1 + \Omega_2)^{-2}} + \frac{\cos \left( 2\Omega_1 - \Omega_2 \right) t}{\omega_2^{-2} - (2\Omega_1 - \Omega_2)^{-2}} \right] \] (10)

Eliminating the secular terms in \( x_1 \) and \( y_2 \), we require

\[ A + \frac{\varepsilon_1 A B^2}{2 \left( \omega_1^{-2} - \omega_2^{-2} \right)} + \frac{\varepsilon_2 A B^2}{4} \left[ \frac{1}{\omega_1^{-2} - (2\Omega_1 + \Omega_2)^{-2}} + \frac{1}{\omega_2^{-2} - (2\Omega_2 - \Omega_1)^{-2}} \right] = 0, \] (11)

\[ B + \frac{\varepsilon_1 B A^2}{2 \left( \omega_1^{-2} - \omega_2^{-2} \right)} + \frac{\varepsilon_2 B A^2}{4} \left[ \frac{1}{\omega_1^{-2} - (2\Omega_1 + \Omega_2)^{-2}} + \frac{1}{\omega_2^{-2} - (2\Omega_2 - \Omega_1)^{-2}} \right] = 0, \] (12)

from which the values of \( \Omega_1 \) and \( \Omega_2 \) can be determined.

4. Resonance

From (9) and (10), we can obtain the resonance condition of the coupled oscillator, which reads

\[ \omega_1^{-2} - (2\Omega_2 - \Omega_1)^{-2} = 0, \] (13)

\[ \omega_2^{-2} - (2\Omega_1 - \Omega_2)^{-2} = 0. \] (14)

5. Conclusion

The condition for resonance can be obtained easily when the parameters are chosen. Resonance occurs when \( \omega_1 = 2\Omega_2 - \Omega_1 \) or \( \omega_2 = 2\Omega_1 - \Omega_2 \), where \( \Omega_1 \) and \( \Omega_2 \) can be determined from Eqs.(11) and(12).

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References