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New stability criterion for delayed neural networks with impulses

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Abstract. This paper studies the exponential stability of impulsive delayed neural networks (IDNN). By employing the standard Lyapunov function and the modified Halanay inequalities, two new sufficient conditions guaranteeing the global exponential stability of the origin of IDNN are established. The proposed results characterize the effects of impulses, delay and the exponential convergence rate of the impulse-free DNN in an aggregated form.

1. Introduction
It is well known that many evolution processes in nature are characterized by the fact that at certain moments of time they experience the abrupt changes of states through short-term perturbations. Since the durations of the perturbations are negligible in comparison with the duration of each process, it is quite natural to assume that these perturbations act in terms of impulses, and therefore, the study of impulsive dynamical systems is assuming a greater importance [1-4]. In 1999, Guan and Chen [5] introduced impulsive perturbations into delayed Hopfield-type neural network model, which is assumed to well describe certain evolutionary processes with impulsive dynamical behaviors. In this pioneering work, the authors investigated several fundamental issues, i.e., global exponential stability, existence and uniqueness of the equilibrium point of such networks. In 2000, Guan, Lam and Chen [6] proposed the impulsive auto-associative neural network model formulated by impulsive ordinary differential equations. Recently, several results for the stability issues of the impulsive delayed neural networks have been reported in the open literature [7-9]. However, all the results in [5-9] require that the impulse-free neural network, i.e., without impulse effects, should be stable. This can be easily deduced by the fact that the derivatives of associated Lyapunov-like functions/functionals, which does not increase in the impulse-moments, are less than or equal to zero. In fact, certain impulsive perturbations may make unstable systems uniformly stable even uniformly asymptotically stable. The readers are referred to [10] for theoretical analysis and illustrated examples.

Motivated by the above discussion, this paper will further study the global stability of the delayed neural networks with impulses and tries to establish several criteria characterizing the effects of impulses, delay and the exponential convergence rate of the origin of impulse-free DNN on the stability of the origin of the whole system in an aggregated form.

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Throughout this paper, we denote by $P^T$ the transpose of matrix $P$, by $P > 0 (< 0, \leq 0)$ the symmetrical and positive (negative, semi-negative) definite matrix $P$ and by $\| \|$ the Euclidian norm of a square matrix or a vector. If a matrix $P$ is reversible, then $P^{-1}$ presents its inverse matrix. By $\lambda_{\text{min}}(P)$ and $\lambda_{\text{max}}(P)$, we denote the minimal and maximal eigenvalues of a real and symmetric matrix $P$, respectively.

2. Problem statement and main results

In this paper, we consider the following IDNN [7-9]

$$
\begin{align*}
\dot{x}(t) &= -Cx(t) + Af(x(t)) + Bg(x(t - \tau)), \quad t \in [t_{k-1}, t_k), \\
\Delta x(t_k) &= B_k x(t_k^-), \quad k = 1, 2, \ldots,
\end{align*}
$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ denotes the state vector in time $t$, the diagonal matrix $C = \text{diag}(c_i) \in \mathbb{R}^{n \times n}$ presents the decay rate, $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ and $B = (b_{ij}) \in \mathbb{R}^{n \times m}$ are connection weight matrix and delayed connection matrix, $f(x) = (f_1(x_1), \ldots, f_n(x_n))^T$ and $g(x) = (g_1(x_1), \ldots, g_n(x_n))^T$ are the activation vector-value functions, $\tau > 0$ is the transmission delay. $\Delta x(t_k) = x(t_k^-) - x(t_k^-)$ are impulses at moments $t_k$, $x(t_k^-) = \lim_{t \to t_k^-} x(t)$, $B_k = \text{diag}(b_{1k}, b_{2k}, \ldots, b_{nk})$ is the impulse rate of the states. We also assume impulse moment $t_k$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, for $k = 1, 2, \cdots$, and $\lim_{k \to \infty} t_k = \infty$.

Mathematically, this is an impulsive delayed differential equation where impulses occur at the discrete time instants $t_k$. Meanwhile, it is a special case of hybrid systems in the control community because it contains both the continuous-system component and discrete-system component.

The initial condition associated with system (1) is defined by

$$
x(s) = \phi(s), \quad -\tau \leq s \leq 0,
$$

where $\phi$ is continuous over $[-\tau, 0]$. As usual, we say that $x(t) = x(t, \phi)$, $t \in [-\tau, +\infty)$ is a solution of system (1) with (2) if $x(t)$ is right-continuous, i.e., $x(t_k^+) = x(t_k^-)$ and satisfies Eq. (1) for $t > 0$.

Throughout the paper, we assume that the activation functions satisfy the following conditions:

(H1) there exist positive constants $l_i'$ and $l_i^g$ such that $|g_i(\alpha)| \leq l_i^g |\alpha|$ and $|f_i(\alpha)| \leq l_i' |\alpha|$ for any $\alpha \in \mathbb{R}$ and $i = 1, 2, \ldots, n$. In the sequel, we denote $L_f = \text{diag}(l_i')$ and $L_g = \text{diag}(l_i^g)$.

(H2) there exist positive constants $T'$ and $T$ such that $\tau < T^- \leq t_k - t_{k-1} \leq T^+$, $k = 1, 2, \ldots$.

The first equation of system (1) is the basic model for the delayed recurrent neural networks, which is the continuous component of system (1) and called the impulse-free delayed neural networks [11-14].

The following lemmas as the modified versions of Halanay inequality are easy to be proved

**Lemma 1.** Let $\omega$ be a nonnegative function defined on the interval $[t_0 - \tau, \infty)$ and be continuous on the subinterval $[t_0, \infty)$. Assume that

$$
\dot{\omega}(t) \leq -a \omega(t) + b \omega(t - \tau), \quad a, b > 0, \quad t \geq t_0.
$$
Then,
\[ \omega(t) \leq \bar{\omega}_0 \exp \left\{ -\lambda(t-t_0) \right\}, \quad t \geq t_0, \]
where \( \bar{\omega}_0 = \sup_{t_0-t \in \mathbb{R}} \omega(t) \), and \( \lambda > 0 \) satisfies \( -a + b \exp \{ \lambda \tau \} = -\dot{\lambda} \).

Now, we state the main result.

\textbf{Theorem 1.} Suppose that (H1) and (H2) hold. If there exist a symmetric and positive definite matrix \( P \), positive definite diagonal matrices \( Q_1 \) and \( Q_2 \), and positive constants \( \alpha \), \( g \), and \( h \) satisfying \( g > h \) such that

(i) \( \Omega_1 = -PC - CP + PAQ_1^{-1} A^T P + L_f Q_f L_f + PBQ_2^{-1} B^T P + gP \leq 0 \),
(ii) \( L_e Q_e L_e - hP \leq 0 \),
(iii) \( (E + B_1)^T P(E + B_1) - \alpha P \leq 0 \),
(iv) \( T^T - \ln \alpha > 0 \),

where \( \alpha = \max \{ \exp (\gamma \tau), \alpha \} \), \( \gamma > 0 \) is the root of the equation \( g - \gamma - h \exp (\gamma \tau) = 0 \), then the origin of system (1) is globally exponentially stable.

\textbf{Proof.} Consider the Lyapunov function
\[ V(x) = x^T Px. \]  

The time derivative of \( V \) along the trajectories of the system (1) with (2) on \([t_k, t_{k+1})\) is
\[
\dot{V}(x(t)) = 2x^T(t) P \left[ -Cx(t) + Af(x(t)) + Bg(x(t-\tau)) \right] \\
\leq x^T(t) \left[ -PC - CP + PAQ_1^{-1} A^T P + PBQ_2^{-1} B^T P \right] x(t) \\
+ f^T(x(t))Q_f(x(t)) + g^T(x(t-\tau))Q_g(x(t-\tau)) \\
\leq x^T(t) \left[ -PC - CP + PAQ_1^{-1} A^T P + PBQ_2^{-1} B^T P + L_f Q_f L_f + gP \right] x(t) \\
- gV(x(t)) + x^T(t-\tau)(L_e Q_e L_e - hP)x(t-\tau) + hV(x(t-\tau)) \\
\leq -gV(x(t)) + hV(x(t-\tau)).
\]

This implies by Lemma 1 that
\[ V(x(t)) \leq F(x(t_k)) \exp \{ -\gamma(t-t_k) \}, \quad t \in [t_k, t_{k+1}), \quad k = 0, 1, 2, \ldots. \]  

where \( F(x(t_k)) = \sup_{t_k-t \in \mathbb{R}} V(x(t)) \).

Note that
\[
V(x(t_k)) = x^T(t_k) Px(t_k) \\
= x^T(t_k) \left[ (E + B_1)^T P(E + B_1) \right] x(t_k) \\
= x^T(t_k) \left[ (E + B_1)^T P(E + B_1) - \alpha P \right] x(t_k) + \alpha V(x(t_k)) \\
\leq \alpha F(x(t_k)) \exp \{ -\gamma(t_k-t_{k-1}) \}, \quad k = 1, 2, \ldots.
\]  

and
\[ \bar{V}(x(t_k)) = \max \left\{ \sup_{\tau_k \leq \theta \leq \theta_k} V(x(\theta)), V(x(t_k)) \right\} \]
\[ \leq \alpha \bar{V}(x(t_{k-1})) \exp(-\gamma(t_k - t_{k-1})), \quad k = 1, 2, \ldots. \]  

(5)

Using (4) and (5) successively on each interval excluding impulse instants leads to the result. More precisely, one has

(a) when \( t \in [t_0, t_1) \),
\[ V(x(t)) \leq \bar{V}_0 \exp\{-\gamma(t-t_0)\}, \quad \text{and} \quad \bar{V}(x(t_1)) \leq \alpha \bar{V}_0 \exp\{-\gamma(t_1 - t_0)\}, \]
where \( \bar{V}_0 = \sup_{\tau \geq \theta \geq 0} V(x(\theta)) \).

(b) when \( t \in [t_1, t_2) \),
\[ V(x(t)) \leq \left[ \sup_{n \geq \theta \geq 0} V(x(n \theta)) \right] \exp\{-\gamma(t-t_1)\} \leq \alpha \bar{V}_0 \exp\{-\gamma(t-t_0)\}. \]

(c) Generally, for \( j = 1, 2, \ldots \), when \( t \in [t_j, t_{j+1}) \),
\[ V(x(t)) \leq \alpha \bar{V}_0 \exp\{-\gamma(t-t_0)\} = \bar{V}_0 \exp\{-\gamma(t-t_0) + j \ln \alpha\}. \]

Note that \( t-t_0 = \sum_{k=1}^{j-1}(t_k - t_{k-1}) + (t-t_j) \geq jT^- \), namely, \( j \leq \frac{t-t_0}{T^-} \), which leads to
\[ V(x(t)) \leq \bar{V}_0 \exp\left\{ -\frac{\gamma T^- - \ln \alpha}{T}(t-t_0) \right\}. \]

(6)

One observes from (3) that
\[ \lambda_{\min}(P) \|x(t)\|^2 \leq V(x(t)) \leq \lambda_{\max}(P) \|x(t)\|^2. \]

(7)

Substituting (7) into (6) yields
\[ \|x(t)\| \leq \left[ \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \|\phi\| \exp\left\{ -\frac{1}{2T^-(\gamma T^- - \ln \alpha)}(t-t_0) \right\} \right]^{1/2}. \]

where \( \|\phi\| = \sup_{\tau \geq \theta \geq 0} |\phi(\theta)| \). This concludes the proof.

**Remark 1.** In Theorem 1, the conditions (i) and (ii) are to ensure the global exponential stability of (1) without impulses. Condition (iii) is to characterize the impulse magnitude \( \alpha \), and the condition (iv) presents the relationship between the exponential convergence rate \( \gamma \) and impulse magnitude \( \alpha \). Note that the exponential convergence rate \( \gamma \) is of delay dependence. Therefore, Theorem 1 characterizes the effects of impulse, delay and the exponential convergence rate \( \gamma \) of the impulse-free DNN on the stability of the whole system (1) in an aggregated form. Additionally, it follows from the conditions (iii) and (iv) that the requirement \( \|E + B_k\| < 1 \) in [7-9] is not necessary.

3. **Conclusion**

In this paper, we have established a general theorem to characterize the effects of impulse, delay and the exponential convergence rate \( \gamma \) of the impulse-free DNN on the exponential stability of the whole system in an aggregated form. Compared with the existing results, the present theorem does relax the requirement of the impulse magnitude.
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