Combining support vector machines with linear quadratic regulator adaptation for the online design of an automotive active suspension system

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Combining Support Vector Machines with Linear Quadratic Regulator Adaptation for the Online Design of an Automotive Active Suspension System

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Abstract. As a powerful machine-learning approach to pattern recognition problems, the support vector machine (SVM) is known to easily allow generalization. More importantly, it works very well in a high-dimensional feature space. This paper presents a nonlinear active suspension controller which achieves a high level performance by compensating for actuator dynamics. We use a linear quadratic regulator (LQR) to ensure optimal control of nonlinear systems. An LQR is used to solve the problem of state feedback and an SVM is used to address the question of the estimation and examination of the state. These two are then combined and designed in a way that outputs feedback control. The real-time simulation demonstrates that an active suspension using the combined SVM-LQR controller provides passengers with a much more comfortable ride and better road handling.

1. Introduction

Support vector machines (SVMs) have been very successful in pattern recognition and function estimation problems. In terms of standard SVM approaches, the dual form of a quadratic programming problem plays an important role in the generation of nonlinear discriminate functions. In order to obtain a nonlinear discriminate function, we assume a nonlinear projection $\phi$ which maps the data point $x$ onto a higher, often infinitely, dimensional feature space $F$ where linear discriminate functions are developed. It is crucial that, in the feature space $F$ we can formulate the problem without involving the calculations of the vectors $\phi(x) \in F$. The Wolfe dual formulation enables us to express the quadratic programming problem only in terms of the value of the inner product $(\phi(x_i),\phi(x_j))$ which can be computed directly from the original data points, $x_i$ and $x_j$, by means of the kernel functions. Indeed, kernel-based nonlinear discriminate functions have been successfully applied to a number of real world problems [1]. [2] introduced the use of a least squares support vector machine (LS-SVM) to ensure optimal control of nonlinear systems. A team led by Wang et al. (2007) [3] proposes the elaboration of an adaptive inverse control algorithm by combining a fast online support vector machine regression (SVR) algorithm with a straight inverse control algorithm. The resulting adaptive algorithm is easy to employ; on the basis of a kernel cache-based method, it functions well in real time; and it performs well in the control of time-varying systems. Friedrichs and

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Igel [4] proposed an evolutionary approach to the determination of multiple SVM hyperparameters: The covariance matrix adaptation evolution strategy (CMA-ES) is used to determine the kernel from a parameterized kernel space and to control regularization. Such a procedure is applicable to the optimization of non-differentiable kernel functions and arbitrary model selection criteria.

This paper is organized as follows. Section 2 briefly reviews the basic concept of support vector machines. Section 3 briefly reviews the optimal control problem. In section 4, a controlled simulation of an active suspension system is presented to demonstrate the feasibility and effectiveness of a combined SVM-LQR system. The last section concludes this paper.

2. Support Vector Machines

Support vector machines (SVMs) are a classification technique used in the field of machine learning which is based on the statistical learning theory. In the case of a two-class classification problem, an SVM is used to construct the optimal hyperplane that maximizes the margin between the two classes. According to Vapnik’s statistical learning theory [2, 3], the maximization of the margin implies the extraordinary capacity for generalization and the good performance of SVM classifiers. SVMs have so far been successfully applied to many real-world fields. In this paper, we begin by outlining the application of SVMs to the simplest case of a binary classification. From the perspective of the statistical learning theory, the motivation for examining binary classifier SVMs comes from works on the theoretical bounds on the generalization error [1, 4]. These bounds on generalization have two important features. Firstly, the upper bound on the generalization error is independent of the dimension of the space. Secondly, the generalization error bound is minimized by maximizing the margin, the minimal distance, between the hyperplane separating the two classes and the data points closest to the hyperplane (Fig. 1). Let us consider a binary classification task with data points $x_i (i = 1, \ldots, n)$ which have corresponding labels, $z_i = \pm 1$, and let the decision function be $X = \{ (x_i, z_i) | x_i \in R^n, z_i \in \{-1, 1\}, i = 1, \ldots, n \}$.

The optimal hyperplane of $X$ is defined as $f(x) = 0$ where

$$f(x) = \text{sign}(w \cdot x + b). \quad (1)$$

In the SVM literature, is often referred to as the weight vector, and $b$ is called the bias. We choose the separating hyperplane $w \cdot x_i + b = 0$ that is farthest away from the data points $x_i$; that is to say that this choice has the maximum margin. Maximizing the margin is equivalent to maximizing the equation $2/\|w\|^2$, and it turns out that the optimal separating hyperplane can be defined as the solution to the equivalent optimization problem:

$$\min_{w,b} \frac{1}{2} \|w\|^2 \quad \text{subject to} \quad Z_{m,i} (w \cdot S_{m,2,i} + b) \geq 1, \quad (2)$$

Constructing the optimal hyperplane is therefore a convex quadratic problem. This is the crucial property that allows generalization to the non-linear case. Differentiation on the basis of the Lagrange function is achieved by substituting $w, b$ and $\alpha_i \geq 0$ for each of the constraints in (1) to get the following Lagrangian function:

$$L(w, b, \alpha) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^{n} \alpha_i (z_i (w \cdot x_i + b) - 1) \quad (3)$$
Thus, it is possible to equivalently solve the dual optimization problem of maximization (3), such that the gradient of $L$ with respect to $w$ and $b$ vanishes, which requires that $\alpha_i \geq 0$. This can be expressed as follows [5, 6].

$$\begin{align*}
\frac{\partial}{\partial b} L(w, b, \alpha) &= 0, \quad \Rightarrow \sum_{i=1}^{n} z_i \alpha_i = 0 \\
\frac{\partial}{\partial w} L(w, b, \alpha) &= 0, \quad \Rightarrow \quad w = \sum_{i=1}^{n} z_i \alpha_i x_i
\end{align*}$$

By inserting (3) into (4), the dual form of the optimization problem is derived as follows.

$$M(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} z_i z_j \alpha_i \alpha_j x_i x_j,$$

Subject to $\sum_{i=1}^{n} z_i \alpha_i = 0$; $Q \geq \alpha_i \geq 0$, $i = 1, 2, \ldots, n$.

To complete the process described above, we apply the concept of an SVM to a crisp nonlinear regression. The basic idea is that a nonlinear regression is achieved by simply pre-processing input patterns $x_i$ by means of a map $\phi: R^d \rightarrow F$ into some feature space $F$, and then applying the standard ridge regression learning algorithm. It is necessary only to directly apply $K(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle$ rather than define $\phi(\cdot)$ explicitly. Thus, we obtain the following dual optimization problem:

\[\text{Fig.1} \quad \text{The perpendicular distance between the separating hyperplane and a hyperplane through the closest points (the support vectors of opposite sign) is called the margin.}\]
Maximum \( M(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j K(x_i \cdot x_j), \)

Subject to \( \sum_{i=1}^{n} \alpha_i = 0; \quad Q \geq \alpha_i \geq 0, \quad i = 1, 2, \ldots, n, \)

where \( Q \) is a positive constant and \( K(x_i \cdot x_j) \) is a conventionally-called kernel function satisfying the Mercer theorem. The kernel function that this paper uses is a Gaussian function. The constant \( b \) is given by \( b = z_j - \left( x_i \sum_{j=1}^{n} \alpha_j x_j \right) \). Substituting \( w \) in (4) for \( w \) in (1), we have

\[
f(x_i) = \sum_{i=1}^{n} \alpha_i K(x_i \cdot x_j) + b
\]

from sensors: position and velocity

Fig.2 The active suspension system of a quarter-car.

3. The Optimal Control Problem
The objective of control makes use of state feedback to stabilize the system and prevent flutter. In this paper, this approach leads to the use of a full-state feedback controller with the input signal to maintain stability. We also consider a control law given as by relating to SVMs. Because SVM methodology is not a parametric modeling approach, this is less straightforward than is the case for standard neural networks such as the multilayer perceptron (MLP) and the Gaussian radial basis function (RBF).

A quarter-car suspension system (Fig. 2) is the object of study in the context of this paper.
It is assumed that the tire does not leave the ground and that $z_s$ and $z_u$ are measured in the position of static equilibrium. In addition, the velocity of sprung mass $\dot{z}_s$ and the relative velocities of unsprung and sprung mass $\dot{z}_s - \dot{z}_u$ are assumed to be measurable. The dynamic equations are calculated as follows:

$$m_s \ddot{z} = k_s(z_u - z_s) + b_s(\dot{z}_u - \dot{z}_s) + f_u$$  
(8)

$$m_u \ddot{z}_u = -k_s(z_u - z_s) - b_s(\dot{z}_u - \dot{z}_s) - f_u + k_r(z_r - z_u)$$  
(9)

where $k_s$ is the constant referring to the spring, $k_r$ is the spring constant for the tire, $b_s$ stands for damping, and $z_r$ is a measure of road surface interference. The active suspension control system can be expressed in state-space form as: $\dot{X}(t) = AX(t) + Bu(t)$.

It is essential to design a controller to reduce the acceleration amplitude of the sprung mass. The optimal linear quadratic regulator (LQR) control is applied to the flutter suppression problem. A system can be stabilized if there exists a state feedback control $u(t) = -kx(t)$ such that the closed loop system is exponentially stable. If the system can be stabilized, then there exists a solution to the LQR problem. In the context of LQR theory, a performance index or a cost function is first defined. Note that the control input $u$ is a scalar here. We seek a control $u(t)$ that minimizes the performance measure

$$J(u) = \int_0^\infty \left\langle QX(t), X(t) \right\rangle + \left\langle Ru(t), u(t) \right\rangle dt$$  
(10)

Here $Q = Q^T \geq 0$ and $R = R^T > 0$ are weighting matrices. It is well known that if an optimal control $u(t)$ exists, $k$ is a constant gain matrix. Moreover, the closed loop system is stable:

$$\dot{X}(t) = AX(t) - BkX(t) = (A - Bk)X(t)$$  
(11)

where $X = [z_s - z_u, \dot{z}_s, z_u - z_r, \dot{z}_u]^T$.

The $R > 0$ assumption ensures the energy of the control to be finite. Next, we perform an LQR synthesis via the Riccati equation to determine the feedback gain matrix $k$. If $A^T P + PA + Q - PB R^{-1} B^T P = 0$ and if $k = R^{-1}B^T P$ then the closed loop system in (11) is asymptotically stable.

![Fig.3 The SVM control system.](image-url)
Table 1. The tested rough road is effectively improved by the SVM-LQR.

<table>
<thead>
<tr>
<th>Control Method</th>
<th>Passive</th>
<th>LQR</th>
<th>SVM-LQR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS</td>
<td>RMS</td>
<td>RMS</td>
</tr>
<tr>
<td>Suspension deflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m)</td>
<td>0.0173</td>
<td>0.0158</td>
<td>0.0095</td>
</tr>
<tr>
<td>(improvement %)</td>
<td>(0%)</td>
<td>(8.67%)</td>
<td>(45.09%)</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Suspension deflection</td>
<td>0.0498</td>
<td>0.0452</td>
<td>0.0264</td>
</tr>
<tr>
<td>(m)</td>
<td>(0%)</td>
<td>(9.24%)</td>
<td>(46.99%)</td>
</tr>
</tbody>
</table>

|                         | RMS     | RMS       | RMS       |
| Suspension acceleration |         |           |           |
| (m/sec^2)               | 1.1853  | 1.0824    | 0.6041    |
| (improvement %)          | (0%)    | (8.68%)   | (49.03%)  |
| Maximum                 |         |           |           |
| Suspension acceleration | 4.8470  | 4.5660    | 3.2052    |
| (m/sec^2)               | (0%)    | (5.80%)   | (33.87%)  |

|                         | RMS     | RMS       | RMS       |
| Tire-ground contact     |         |           |           |
| (m)                     | 0.0021  | 0.0019    | 0.0012    |
| (improvement %)          | (0%)    | (9.52%)   | (42.86%)  |
| Maximum                 |         |           |           |
| Tire-ground contact     | 0.0134  | 0.0129    | 0.0107    |
| (m)                     | (0%)    | (3.73%)   | (20.15%)  |

Our online SVM control system can be described as Fig. 3. Given the following training sample sets,

\[
X = \{ \tilde{x}_i, \tilde{z}_j \} = \{ (x_i, u_j) \} \in \mathbb{R}^n, z_j \in [-1,1], i = 1, 4, j = 1, \ldots, n, \]

we can construct a linear regression function (as in 7), where \( X_{LQR} = [x_1, x_2, x_3, x_4]^T \) implies that the active suspension system in (11) has been
calculated on the basis of a reference LQR. \( E_j = \left( \| e_j \| + \| de_j \| \right) < \epsilon, z_j = \text{sign}(E_j) = \pm 1 \) where \( \epsilon \) is the error value.

Finally, the following model is obtained in the dual space defined in (7), where the kernel function \( K \) corresponds to \( K(E_j, u_j) = \phi(E_j)^T \phi(u_j) \) according to Mercer's condition. Several options exist for this kernel function, such as the linear, polynomial, radial basis functions (RBF). In a follow-up paper we focus on RBF kernels. The support vectors are obtained through training and those below the average \( \bar{X}_{SVM} = \frac{1}{n} \sum_{j=1}^{n} \alpha_i K(E_j, u_j) \), where \( n \) is a scalar, are used to calculate the interpolation law. \( h(x_i) = f(x_i) - \bar{X}_{SVM} \) and \( Q_{SVM} = Q + \text{diag}(h(x_i)) \), substituting \( Q_{SVM} \) for \( Q \) in (10), we obtain \( J(u) = \int_0^\infty \left( \langle Q_{SVM} X(t), X(t) \rangle + \langle Ru(t), u(t) \rangle \right) dt \) and the new feedback control is \( u(t) = -k_{SVM} x(t) \).

4. Simulation
The active suspension control system of an automobile is currently of great interest [7, 8], both academically and in the automobile industry worldwide. In 1999, Kuo and Li [9] use a GA-based fuzzy PI/PD controller in an automotive active suspension system (AASS). For the quarter-car suspension system discussed in this section, the typical parameters for the suspension model are set as \( m_s = 30000 \text{ kg}, m_i = 250000 \text{ kg}, k_s = 150000 \text{ N/m}, k_i = 150000 \text{ N/m}, b_s = 1000 \text{ N/s/m} \).

This is done in order to impose a set of constraints on the choice of the interconnection weights of a MLP controller. Based on the continuous time model presented in (11), we design a reference LQR controller for \( R = 1 \), where \( Q = \text{diag}[100, 2.0e + 5, 1, 1000] \) is inserted in the LQR cost function \( \int_0^\infty (x^T Q x + u^T Ru) dt \). The given initial state is defined as \( x = [0, 0, 0, 0] \) and \( u = [0, 0]^T \).

In this example, the learning performance of the SVM for an AASS is superior to that of both the passive suspension system and the optimal linear feedback control law of the AASS.

A pseudorandom road is the so-called power spectral density (PSD) [9, 10]. This paper proposes that a combination SVM-LQR may change the controller to the best possible state quickly, thus minimizing system error. An advantage of SVMs is their capability to classify and output the worst situation and make proper adjustments to the system parameters. The simulation results are presented in Tables 1.

5. Conclusions
In this paper we introduce the use of a combined SVM-LQR to solve problems of optimal control. The active suspension control problem is formulated for use with a nonlinear state feedback controller consisting of SVMs. The cost functions for the control problem and for the least squares SVM are combined within one objective function. This study was designed to develop a combination SVM-LQR controller for use in an active suspension system. It demonstrated the ability to change the controller to the best state quickly, thus minimizing system error. An advantage of SVMs is the ability to classify and output the worst situation and make proper adjustments to system parameters. All the computer simulations that were performed demonstrate that the proposed controller achieves a much better performance than in the case of a passive suspension and linear optimal control, regardless of the type of road profile.
Fig.4 Time response to a rough road (passive: Red dashed line; SVM-LQR: Blue dotted line; LQR: Green dash-dot line). (a) Suspension deflection. (b) Sprung mass acceleration. (c) The beating in the distance between the tire and the ground.

References